PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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OF

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PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XXVII. 1906-7.

THE 124TH SESSION.

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I.—The Influence of an Excessive Meat Diet on the Osseous System. By Chalmers Watson, M.D. (*From the Physiological Laboratory of Edinburgh University.*) *Communicated by* Professor Schäfer, F.R.S. (With Four Plates.)

(MS. received November 20, 1906. Read December 3, 1906.)

In a communication given to this Society in December 1905, I described the clinical results obtained in an experimental investigation on the influence of an excessive meat diet on the growth and nutrition of rats. It was there shown that the progeny of meat-fed rats are usually poorly developed and show a high mortality in early life. The present record comprises an account of the naked-eye and microscopic appearances observed in the osseous system of these meat-fed subjects. The *material employed* consisted in the young of mothers fed for some weeks or months prior to pregnancy, and during pregnancy and lactation, on a diet of ox-flesh, the animals, after weaning, being continued on the meat *régime*, an equal number of controls being taken from the young of rats fed on an exclusively bread-and-skim-milk diet. Both diets were given in unrestricted amount, and with the meat diet water was given *ad libitum*. Over a hundred meat-fed rats were utilised for the investigations, their ages ranging from one day to three months, the majority being under three weeks old at the time of death. A record was made of the naked-eye appearances of the skeleton, special attention being directed to the consistence as well as to the general appearance of the long bones, ribs, and flat bones. The tissues were fixed in formalin (5 per cent.), decalcified in weak nitric acid solution, and stained in the ordinary manner with hæmatoxylin and eosin. Sections were made through the anterior part of the cranium so as to demonstrate the conditions present in the frontal, malar, and maxillary bones, while, in the case of rats æt. one day, three weeks, and two months respectively, sections were also made of the tibia, humerus, and ribs. Similarly prepared sections from control animals were, for comparison, mounted on the same slide.

Macroscopic Appearances.

The macroscopic conditions noted in the bones of the meat-fed rats vary according to the age of the animals, but show throughout, in a more or less marked degree, the same general characteristics. The most striking feature

is the marked general softness of the whole osseous system, this condition being present in every meat-fed subject. The long bones of the flesh-fed rats are distinctly softer and more pliable than those of the bread-and-milk-fed animals; a similar condition is observed in the ribs, short bones, and cranial bones of the meat-fed rats. This soft condition is present in the bones at birth, and becomes accentuated as age advances. A second striking appearance in the meat-fed animals is the darker colour of the long bones, more especially of the ribs, this being due to increased vascularity. This condition is present, in a greater or less degree, in all the meat-fed subjects which died or were killed after the second week of life. In some cases nothing further was observed in the bones of the flesh-fed rats killed even as late as three months after birth, but in the majority of cases a third feature shows itself. During the second month various curvatures of the spine and long bones occur. These consist in marked scoliosis and lordosis, with bending of the ribs at their angles, while curving of the bones of the limbs is present in a less degree. This condition of the bones is usually associated with an enlargement of the costo-chondral junction. In a small percentage of cases (about 15 per cent.) an additional feature is the presence of small white nodules in the bony ribs, these nodules standing out as pale bead-like prominences in the substance of the dark bone of the rib. On section these nodules are composed chiefly of cartilage (see fig. 7). In the more pronounced cases the skeletal changes generally are similar to those seen in advanced cases of rickets in the human subject. Microscopically, however, this similarity is not borne out.

Microscopic Appearances.

Owing to the uniformity of the bony changes throughout the whole series of meat-fed animals, it was unnecessary to make a histological examination of each subject. Sections were accordingly made from forty out of the hundred meat-fed rats and from an equal number of control bread-and-milk-fed animals. In this examination special attention was directed to the following points:—

(*a*) *Long bones.*—The state of their development, by intra-membranous and intra-cartilaginous ossification.

(*b*) The histological appearances of the *cranial bones.*

(*c*) *Ribs.*—The minute structure of the nodules present in the bony ribs.

(*a*) *Long Bones.*—The ossification of the long bones of meat-fed rats is delayed and imperfect, the defect involving both the endochondral and periosteal bone formation. The epiphyses are for the most part normal;

in some instances there is a slight irregularity in the size and arrangement of the cartilage cells at the bone-forming margin. The minute structure of the epiphyses of the long bones may be normal, even in animals in which pronounced rachitic-like changes are present in the skeleton. The bone marrow of the meat-fed rats æt. six weeks and onwards shows, in some subjects, a great excess of fat.

(*b*) *Cranial Bones.*—In the meat-fed rats, ossification, both intra-membranous and intra-cartilaginous, is less advanced than in the control animals, the bony trabeculæ in the former being less numerous and enclosing a marrow excessively rich in red blood corpuscles. While intra-membranous and intra-cartilaginous ossification are both affected, the defect is in some cases more pronounced in the periosteal bone-formation.

There is a striking difference in the degree of development of the frontal, malar, and maxillary bones in the bones of the meat-fed animals at birth, the contrast becoming accentuated as age advances. The bones of the jaws in the meat-fed subjects are of a different shape from the controls, the former being wider and more square-shaped. This condition is associated with an extreme thinness of the bones and a great increase in the number of cells—red blood corpuscles and leucocytes—in the medullary cavity in the meat-fed rats (see figs. 1 and 2). In a very few animals æt. three weeks, the difference between the development of the bones in meat-fed and bread-and-milk-fed animals is very slight. The average state of bone development in the two series is further illustrated in figs. 4 and 6 for animals æt. three weeks and three months. The bony trabeculæ in the meat-fed rats are extremely thin, and the medullary spaces show a great increase in the number of thin-walled vessels, which are distended with red blood corpuscles. This increase in the number of red blood corpuscles in the medulla of meat-fed rats is, in the great majority of subjects, a very striking feature (see fig. 2).

(*c*) *The Structure of the White Nodules in the Bony Ribs.*—These nodules present a striking histological picture. They are composed mainly of cartilage cells, which are derived from the periosteum; at the periphery of these nodules the nodules are undergoing transformation into bones (see fig. 7). It is of interest to note that I have recently observed a similar histological appearance in the bones of an infant æt. fifteen months, whose mother—a tuberculous subject—was fed during gestation, and for some time prior to it, on a diet containing a great excess of meat (see fig. 8).

To sum up:—The results show that the bones of animals fed on an excessive meat diet present an appearance of delayed and imperfect ossification, with increased vascularity, and an increase in the number of

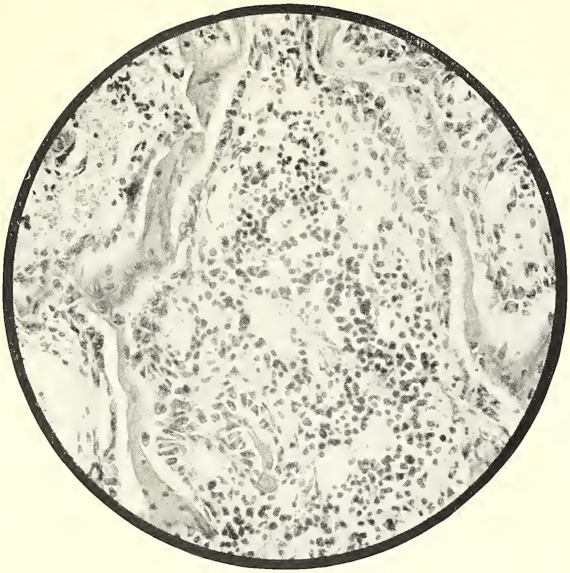

FIG. 1.—The Marrow of a Rat æt. three weeks, unweaned. Mother fed on a bread-and-milk diet. (×200.)

Note the leucoblastic type of marrow, the white blood corpuscles being in excess of the red blood corpuscles. *Cf.* fig. 2.

FIG. 2.—The Marrow of a Rat æt. three weeks, unweaned. Mother fed on an ox-flesh diet. (×200.)

Note the erythroblastic type of marrow, the red blood corpuscles being more numerous than the leucocytes. *Cf.* fig. 1.

Fig. 3.—From a vertical section of the Cranium of a Rat æt. three weeks, unweaned. Mother fed on a bread-and-milk diet. The photograph shows the frontal bone, and the upper part of the nasal septum and nasal cavities. (×75.)

Note the normal state of development of the frontal bone. *Cf.* fig. 4.

Fig. 4.—From a vertical section of the Cranium of a Rat æt. three weeks, unweaned. Mother fed on an ox-flesh diet. For comparison with fig. 3. (×75.)

Note the imperfect development of the frontal bone and increase in the size of the medullary cavity.

FIG. 5.—From a vertical section of the Cranium of a Rat æt. three months, fed on bread-and-milk.

Note the well-developed frontal bone. *Cf.* fig. 6.

FIG. 6.—From a vertical section of the Cranium of a Rat æt. three months, fed on ox-flesh.

Note the imperfect development of the frontal bone and the increase in the size of the medullary cavity. The mucous membrane of the nose is in a state of catarrh. *Cf.* fig. 5.

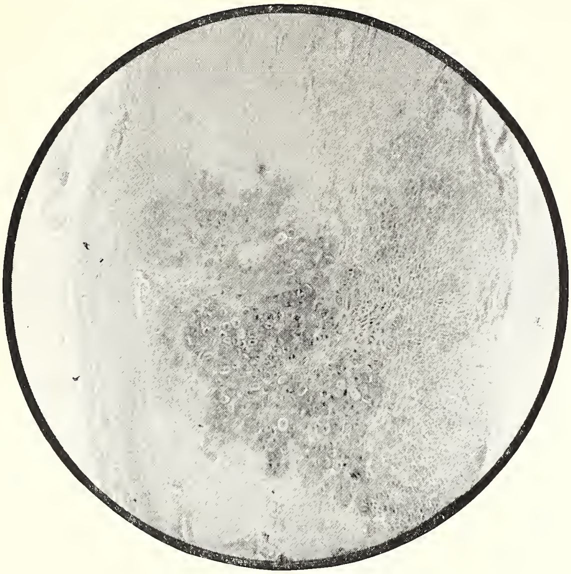

Fig. 7.—Longitudinal section of the Bony Rib of a Meat-fed Rat (second generation), æt. two months, the section taken through one of the nodules described in the text. (× 50.)

Note the area of cartilage cells in the bone. *Cf.* fig. 8.

Fig. 8.—Longitudinal section of the Shaft of the Radius of an Infant æt. sixteen months, whose mother was fed on an excessive meat diet. (× 90.)

Note the area of cartilage cells in the bone. *Cf.* fig. 7.

red blood corpuscles. Associated with this there is, in a number of cases, the presence in the bony ribs of nodules of cartilage, developed from the periosteum, with direct transformation of these cartilage cells into bone. It is noteworthy that, while the naked-eye appearances of the skeleton may closely simulate those present in advanced cases of rickets in the human subject, the microscopic appearances in the epiphysial junctions of long bones of meat-fed rats are quite distinct from those present in that disease.

I have pleasure in expressing my indebtedness to Dr Dingwall Fordyce for much assistance in this investigation.*

* The expenses of this research were defrayed by grants from the Moray Fund of the University, and from the Carnegie Trust.

(*Issued separately February* 11, 1907.)

II.—The Effect of a Meat Diet on Fertility and Lactation. By B. P. Watson, M.D., F.R.C.S.E. (*From the Physiological Laboratory, University of Edinburgh.*) *Communicated by* Professor Schäfer, F.R.S. (With a Plate.)

(MS. received November 21, 1906. Read December 3, 1906.)

In a paper on "The Influence of Diet on Growth and Nutrition," in the *Journal of Physiology* (vol. xxxiv., p. iii), Dr Chalmers Watson showed that in rats a diet of ox-flesh begun when the animals were weaned interfered with the development of pregnancy, none of the four flesh-fed animals having young, whereas the control animals from the same litter all became pregnant. On the other hand, it is stated of three families fed on horse-flesh from the age of 2½ months approximately, that all became pregnant, from which he concludes that "the use of this diet in animals of this age appears not to affect the supervention of pregnancy."

It was further found in the case of the meat-fed animals which became pregnant and suckled their young that the mammary tissue was less developed than in the control bread-and-milk-fed rats.

At Dr Chalmers Watson's suggestion I have extended these observations, and with a larger amount of material at my disposal am able to amplify his statements, and as regards the question of fertility slightly to modify them.

The method of conducting the investigation was as follows:—Twelve female rats and several males were put on a bread-and-milk diet, and the females were continued on this throughout pregnancy and lactation. These served as the controls. Seventeen females and five males were put upon an ox-flesh diet, but were otherwise under exactly the same conditions as the bread-and-milk animals. The animals were begun on the meat diet at various ages, from the second up to the fourth month, and some of them were kept on the diet for as long as five months.

1. *Effect of a Meat Diet on Fertility.*—Of the 17 animals fed upon a meat diet only 8 became pregnant, and of these 4 bore young within 21 days—the usual gestation period in the rat—of being put on the diet, so that only 4 actually conceived while on the diet. Of these latter one had been 24, one 25, one 27, and one 30 days on an exclusive ox-flesh regimen.

The other 9 animals, although kept for several months. did not conceive, and this in spite of the fact that they were seen to copulate freely right

up to the end of the experiment. This reservation must, however, be made, that one of these 9 animals probably had young which were eaten, and it is just possible that this happened in other cases. It is not at all probable, however, as the animals were frequently carefully examined, and if there was any indication of their being pregnant, were at once transferred to separate cages.

Of the 12 animals fed on a bread-and-milk diet, all became pregnant and had young, so that we may conclude that a meat diet is decidedly prejudicial to the occurrence of pregnancy in rats when the diet is begun when the animals are from 2 to 4 months old.

In order to determine whether the fault resided in both sexes or in only one, a fresh male which had been fed on bread-and-milk was put beside the sterile females which had been on meat for several months. When the animals were killed, some time after, one of them was found to be in an early stage of pregnancy, and must have been impregnated by the bread-and-milk male.

This would appear to indicate that the cause of the sterility is partly due to the male, but we have not had sufficient material to form any more definite conclusion regarding this.

2. *Effect of a Meat Diet on Lactation.*—For this part of the investigation the same animals were used, viz., the 12 controls fed on bread-and-milk and the 8 meat-fed animals which became pregnant. The point specially attended to was the weight of the mammary tissue of the animals killed after suckling their young for varying periods.

In the nursing rat the mammary tissue forms a continuous sheet spread under the skin of the abdomen on each side of the middle line. In addition there are extensions of it into the axillæ, up along the neck and into the groins, while in some cases it spreads out so much laterally as almost to reach the back. The nipples are in a double row extending from thorax to the groins.

The animals were killed at different periods during lactation. The weight of the mother and the number and weight of the young at the time of death were ascertained. The skin and subcutaneous tissue of the mother's abdomen was immediately removed down to the muscle, care being taken that no mammary tissue was left behind in the axillæ or in the groins. The skin was put into 5 per cent. formalin for a few days, when it was quite easy to separate the mammary tissue from the skin on one hand and from the areolar tissue on the other (see fig.). If an attempt were made to strip the mammæ before first fixing in formalin, it was found that a great deal of milk was squeezed out. The immersion in formalin

prevented this, although it extracted a small quantity of milk, as shown by its cloudy appearance at the end of twelve hours. It is to be understood, then, that the mammæ were weighed out of formalin.

The basis of comparison between the meat- and the bread-and-milk-fed rats is the percentage weight of the mammary tissue to that of the mother.

By a reference to the tables below it will be seen that there are wide individual variations in this percentage among the bread-and-milk animals.

Table giving the Lactation History of Eight Rats fed on an exclusive Ox-Flesh Dietary.

No. of days on meat before parturition.	Weight of animal at death.	Weight of mammæ.	Per cent.	Time in lactation when killed.	Number of young.	Weight of young.
	grms.	grms.				grms.
30	180	13	7·2	1st day	10	50
9	140	8·5	6	1st day	10	49
27	230	19·5	8·4	1st day	...	...
25	150	13	8·6	7th day	5	48
12	115	10·5	9·1	20th day	6	120
15	130	12	9·2	21st day	7	185
21	110	10·1	9·1	21st day	6	110
24	140	11·5	8·2	22nd day	4	82

Average percentage = 8·2.

This is explained by the different times during the course of lactation at which the animals were killed, and also by the varying numbers of young which they suckled. Thus there is a fairly uniform rise in the percentage from the first up to the twenty-first day, after which there is again a fall; and taking animals killed at the same lactation period, the percentage is higher in those which nursed the larger number of young.

We may therefore take it that the mammary gland in the rat is most actively functioning about the twenty-first day of lactation, and after this, as the young begin to feed themselves, it undergoes atrophy.

It will be seen that the average percentage weight of the mammæ of all the bread-and-milk animals is 9·6, while that of the meat-fed ones is only

8·2. This difference would in all probability have been greater but for the fact that the average number of young suckled by the bread-and-milk animals was only 4·6, while that of the meat-fed animals was 6·3.

As before stated, the longest period before the birth of the young during which the mother had been on a meat diet was thirty days. Taking this into consideration, the difference is sufficiently marked to point to the conclusion that the result of a meat diet is to diminish the amount of mammary tissue in nursing mothers.

Table giving the Lactation History of Twelve Rats fed on an exclusive Bread-and-Milk Dietary.

Weight of animal at death.	Weight of mammæ.	Per cent.	Time during lactation when killed.	Number of young.	Weight of young.
grms. 147	grms. 12·5	8	1st day	7	grms. 32
140	10	7·1	3rd day	7	45
180	18	10	6th day	7	70
225	18·5	8·2	10th day	7	115
120	17	14·1	21st day	5	150
140	19	13·5	21st day	7	220
115	7·5	6·5	21st day	3	120
130	11·5	8·8	22nd day	4	100
160	19	11·2	25th day	7	145
160	15	9·3	30th day	6	175
0	13	10	...	6	...
150	13·5	9	27th day	...	...

Average percentage = 9·6.

The effect of this relatively poor mammary development on the young of the meat-fed rats is shown by a comparison of their weights with those of the young of the control animals towards the end of the lactation period. Thus the average weight of each of the young of the meat rats at the twentieth to the twenty-first day is 21·6 grms., in contrast to 29·4 grms., the average weight of each of the bread-and-milk young. It is thus evident that the young of the animals fed on meat suffer in genera

nutrition and growth, as compared with those whose mothers are on a bread-and-milk diet.

Whether this is due to a mere deficiency in the amount of milk available for their use, or to some alteration in its constituents, has not been determined, but the probability is that the former plays a large part.

On microscopic examination there are no marked differences in the histological characters of the mammary tissues. The character of the glandular tissue varies according as the acini are full of secretion or empty, and while the mammæ of the meat-fed animals show a preponderance of closely packed empty lobules, this may only be due to the fact that the mothers had been killed shortly after the gland had been emptied by suckling.

Conclusions.

1. That a meat diet is prejudicial to the occurrence of pregnancy in rats.

2. That in rats fed on a meat diet the mammary development of nursing mothers is less than in rats fed on bread-and-milk.*

* The expenses of this investigation were defrayed by grants from the Moray Fund of the University, and from the Carnegie Trust.

(*Issued separately February* 11, 1907.)

Mammary Tissue from a Female Rat during Lactation.

Dr B. P. Watson.

III.—The Effects of Diet on the Development and Structure of the Uterus. By Malcolm Campbell, M.B., Ch.B., F.R.C.S. Edin.

(*From the Physiological Department, University of Edinburgh.*)
Communicated by Professor Schäfer, F.R.S. (With Four Plates.)

(MS. received November 27, 1906. Read December 3, 1906.)

The investigation was undertaken to discover what changes, if any, were produced in the development and structure of the uterus by various diets.

The animals employed in the research were rats. The uteri of 86 animals were examined, both macroscopically and microscopically. The tissue for microscopic examination was removed in all cases from, as nearly as possible, the same area, viz., the junction of the distal and middle thirds of the uterus.

Eight wild rats, in various stages of development, from the immature to the adult animal, were examined in order to form an opinion as to the structure of the uterus in animals living, presumably, under natural conditions.

The remaining animals were divided into five series, viz.:—

(*a*) A series of 13 animals fed from weaning, for periods of from 9 to 14 weeks, on an exclusively milk diet. In one section of this series plain milk was used; in another, Pasteurised milk; in the third, sterilised milk.

(*b*) A series of 27 animals fed for periods of from 21 days to 9 months on bread soaked in milk.

(*c*) A series of 11 animals fed for periods of from 4 to 14 weeks on a rice diet.

(*d*) A series of 5 animals fed for varying periods on a diet of porridge or oats.

(*e*) A series of 22 animals fed for periods of from 21 days to 8 months on a raw meat diet.

In the cases of the raw meat and rice diets, some animals were put on the diet as soon as weaned, others after they had reached various stages of development.

The uterus of the adult wild rat is lined by columnar epithelium. There are glands lined by epithelium which varies from low cubical to columnar in type. The mucous coat is bounded externally by a muscular coat.

In the mucous layer three varieties of cells are found:

(1) A cell with a large round or oval, relatively faintly staining nucleus. This appears to be a young connective tissue type of cell.

(2) A cell with a small, round, darkly staining nucleus, comparable to a lymphoid cell.

(3) A cell with an elongated, very darkly staining nucleus, similar to cells in fibrous tissue.

In the wild rat the large cells are most numerous (fig. 5). The cells of the other types are few in number, and are found only in that part of the mucosa near the muscular coat.

From the examination of this series of 86 animals, it is evident that in animals of the same age, and approximately of the same weight, living under similar conditions, the uteri may vary, within a limited range, in size and development. The muscular coat is relatively uniform; the mucosa shows the greatest variations. While in most cases the epithelium lining the cavity is columnar, in some it is cubical. There are also found marked variations as to the position of the nucleus, and also as to its staining capacity.

The animals fed in groups (1) and (2), on milk, and bread soaked in milk, approximate most nearly to the type of structure seen in the wild rat. The only difference is that the cells are not quite as large as in the wild rat (compare fig. 5 with fig. 6). In the other groups, fed on what we may term "abnormal diets," viz., rice, porridge or oats, or raw meat, there is found a relatively constant departure from the normal. The type of change is common to all abnormal diets; its severity varies.

The severity of the changes induced are found to be in proportion to the ages of the animals at the time when the abnormal diet was begun. The changes are most marked in the animals put on the diet at weaning, they are less marked the more mature the animals at the time of the commencement of the abnormal diet. In a fully-developed animal any abnormal diet may fail to materially change either the size or structure of the uterus.

In regard to the development of the uterus, an abnormal diet appears to arrest its growth (compare figs. 1 and 2, also figs. 3 and 4). This arrest of development is most marked in animals fed from weaning on ox-flesh, but is also very well seen in animals fed on rice or on porridge or oats.

In regard to structure, all the abnormal diets lead to a diminution of the number of the large connective tissue type of cells and a relative increase in the small cells.

FIG. 1.—Uterus of Young Rat, æt. 10 weeks, Bread-and-Milk-fed. (×30.)

FIG. 2.—Uterus of Young Rat, æt. 10 weeks, Meat-fed. From same litter as fig. 1. (×30.)

Fig. 3.—Uterus of Adult Rat, Bread-and-Milk-fed. (×30.)

Fig. 4.—Uterus of Adult Rat, Meat-fed. (×30.)

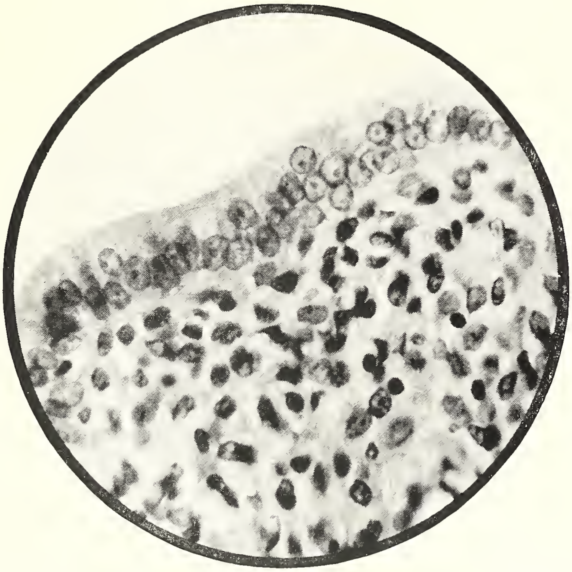

Fig. 5.—From Uterus of Wild Rat, showing large cells. (×500.)

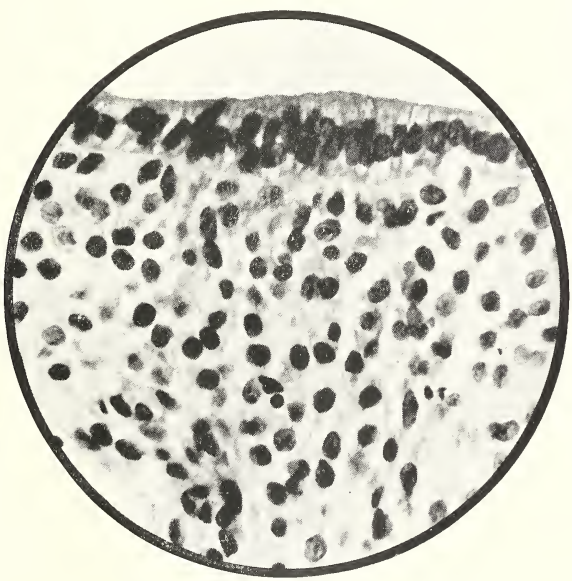

Fig. 6.—From Uterus of Bread-and-Milk-fed Rat, showing large cells. (×500.)

Fig. 7.—From Uterus of Meat-fed Rat, showing fibrotic change in cells of mucosa. (×500.)

This change, which may be described as a fibrosis (compare fig. 7 with figs. 5 and 6), was most marked in animals fed from weaning on an ox-flesh diet, for periods of from four to five months; in this group of animals none became pregnant, while controls from the same litters, fed on bread and milk, all had young.

From these observations it seems justifiable to state:

(1) The use of a non-physiological diet, *e.g.* exclusive flesh, rice, or porridge, induces, in the great majority of cases, a modification in the structure of the uterine mucous membrane. This modification consists in a diminution in the number of the large connective tissue type of cells, which appear to be important constituents in a physiologically active mucosa.

(2) The structural change is most profound in animals fed from weaning on an exclusively flesh diet. In such animals the development of the uterus is also most interfered with.

(3) The structural change in (2) is associated with sterility.*

* The expenses of this research were in part defrayed by a grant from the Carnegie Trust.

(*Issued separately February* 11, 1907.)

IV.—The Temperature of the Fresh-water Lochs of Scotland, with special reference to Loch Ness. By E. M. Wedderburn, M.A. *Communicated by* Sir John Murray, K.C.B.

(Read May 28, 1906. MS. received February 5, 1907.)

(*Abstract.*)

The communication of which this is an abstract has been published in the Society's *Transactions* (Vol. XLV. Part II. No. 16), with an appendix containing observations made in Loch Ness by members of the Scottish Lake Survey and of the Order of Saint Benedict at Fort Augustus.

Numerous temperature observations have been made in the lochs which have been surveyed by the Scottish Lake Survey, and these observations are published in the Lake Survey Reports appearing in the *Geographical Journal*. A great number of observations were made in Loch Ness between July 1903 and May 1905. Generalising from these observations, it appears that, under conditions similar to those which exist in Loch Ness, a loch gains heat gradually throughout its whole depth until July or August. In August or September it begins to lose heat. There appears at the surface a gradually increasing layer of water of uniform temperature, and while the surface temperature decreases, the total quantity of heat in the loch may for a time increase, owing to various mixing influences heating the waters at considerable depths. This layer of water of uniform temperature, while decreasing in temperature increases in depth, and at the time when the quantity of heat in the loch is least, the temperature of the loch is nearly uniform throughout.

When this surface layer becomes distinct, *i.e.* when the *Sprungschicht* appears, the temperature seiche described by Mr Watson * becomes evident. The observations now available confirm Mr Watson's general conclusions. Observations made at the two ends of Loch Ness show opposition between the phases of the seiche, and a binodal seiche is also apparent. The observed value for these temperature seiches agrees well with values arrived at by rough calculations. Records obtained by the Callendar Electrical Recorder also support strongly the theory of the temperature seiche.

Rough calculations of the amount of heat which Loch Ness gains during

* *Geographical Journal*, October 1904.

the year give $1{\cdot}9 \times 10^{16}$ gram calories. Dr Knott states the total quantity of heat supplied as about $7{\cdot}2 \times 10^{16}$ gram calories, or about four times the quantity of heat which is actually stored up.

It was not found possible to measure directly radiation into or out of the loch. The temperature observations at the surface, and in the neighbourhood of the *Sprungschicht*, show the presence of convection currents, and these currents are a very large factor in the heating and cooling of a loch.

(*Issued separately April* 4, 1907.)

V.—Note on the Change produced in the Conductivity and Density of Lead Wires by Permanent Stretching. By James A. Donaldson and Robert Wilson, Natural Philosophy Laboratory, Edinburgh University. *Communicated by* Professor J. G. MacGregor.

(MS. received February 22, 1907. Read March 4, 1907.)

The wires used were of good commercial lead and well drawn.

The special difficulty in experimenting with lead is due to its softness, involving initial lack of uniformity in section, which is increased on stretching. That the variability of the section of the wire might be allowed for in the computation of specific resistance, etc., the diameter was measured at the ends, at two points respectively one inch from each end, and at eight other points dividing the rest of the length of the wire into equal parts. The ends of the wire were soldered into copper pieces to prevent serious thinning near the ends when it was stretched.

The diameter was measured by means of a screw gauge, made by Elliot, which reads to ·0001 inch.

In computing the specific resistance two methods were used, both of which require that the lack of uniformity between the points of measurement of the diameter be small. In the first method we considered the parts of the wire between the successive points of measurement as forming truncated cones, and thus got $R=\frac{\rho}{\pi}\Sigma\frac{l_1}{r_1r_2}$, where R is the total resistance of the wire, ρ is the specific resistance, and l_1 is the length and r_1 and r_2 the radii of the ends, of one of the cones. In the other method we made a graph for length along wire against diameter, and thus determined the mean diameter, which we used in the formula $R=\frac{\rho}{\pi}\frac{l}{r^2}$, where l is whole length of wire and r is mean radius.

The arrangement of the apparatus and the method of hanging the wires was as follows:—The copper piece, into which the upper end of the lead wire was soldered, was clamped to a block of copper containing a mercury cup. This block of copper, again, was fixed into a block of wood, which was sunk into the wall of the room. To the lower end of the wire to be stretched was attached a copper rod, with a disc for weights. This rod passed through the disc and dipped into a mercury cup, fixed on a screw

jack which could be adjusted to the required height. In order to prevent jerking, the first weights, which were made of brass and contained a slot for the copper rod, were put on while the disc rested on the screw jack; the last one or two were put on after stretching had begun, and were carefully adjusted by hand. By previous tests we knew how many weights were required to produce any considerable stretch. After the removal of the weights, the wire was always allowed to hang freely for several hours before any measurements were made.

The instruments used to determine the resistance of the wire were a Carey Foster Bridge by Nalder (N.C.S. pattern) and a Nalder's Four Coil Astatic Galvanometer of Kelvin type (N.C.S. pattern), which was provided with a "scissors" magnet control. The bridge wires were detachable, and of different resistances, varying from very thin to about $\frac{1}{8}$ inch in thickness. In the experiments, the resistances forming the arms of the bridge were the lead wire that was stretched, and a second lead wire of the same diameter and length approximately. The wires hung close together, and the second wire carried no weights. From the mercury cups with which the ends of the lead wires were in contact, thick insulated copper cables, of the same lengths and diameters, led to the bridge.

Several of the detachable bridge wires were used, and these were calibrated by Strouhal and Barus' method. The bridge wire was divided into parts of equal resistance by comparing a length on the bridge wire with the same one of several approximately equal resistances, joined in series by mercury cups, this resistance being moved forward one place after each determination. Then, by a graph, we computed the relative resistances of any two lengths on the bridge wire. We then determined the absolute resistance of the bridge wire between two points, one near each end, and, from the graph, got the resistance of the part between any two points lying in this interval. In making the absolute determination, one arm of the bridge was a thick copper piece of negligible resistance, and the other arm was, first, a definite resistance, and, secondly, this definite resistance in parallel with a standard ohm.

The resistance of the unstretched wire was measured on one of the Carey Foster wires before the other lead wire was stretched, and also between the stretches. The difference of the resistances of the two lead wires was measured after each stretch, and the whole resistance of the wire under investigation determined. By this means we insured that the change in specific resistance, if any, should be due to the stretching. The wires were hung in a long cupboard to prevent temperature changes, and the whole of the apparatus guarded from air currents by means of screens.

The length of the lead wire under investigation was measured by means of a steel scale stretched vertically beside the wire before any permanent stretch had been made. For some time previously a small weight was allowed to hang on the wire in order to remove any bends. The measurement of the increase of length after stretching was made by means of a cathetometer, there being a plate-glass window in front of the cupboard to admit of this reading.

As precision measure for the different readings, we took the maximum deviation from the mean, divided by the square root of the number of readings. In the case of the cross-section this corresponds to about 3 in 1600, in the total resistance to 1 in 1000, and in the length to 1 in 2000. That component of the error in the specific resistance, due to the error in the length, we found to be negligible.

Considerable difficulty was experienced in obtaining good results, owing mainly to the lack of uniformity in some of the wires when stretched. At first also we did not make so many readings of the diameter along the wire, and the error in the section due to this was large.

The results for the stretching of the wire which we considered most reliable are given in the following table. The first three columns give the length, cross-section, and total resistance of the lead wire, and the fourth column gives the specific resistance as computed from these values.

Length in Cms.	Section in Sq. Cms.	Resistance in Ohms.	Sp. Res.
196·2	·01688	·2207	$18{\cdot}99 \times 10^{-6}$
204·4	·01622	·2395	$19{\cdot}01 \times 10^{-6}$
213·4	·01555	·2592	$18{\cdot}93 \times 10^{-6}$
221·4	·01493	·2812	$18{\cdot}96 \times 10^{-6}$

The precision measure of the specific resistance, computed from the precision measures of the length, cross-section, and total resistance, is equal to $\cdot 04 \times 10^{-6}$. The above values of the specific resistance show no change greater than can be accounted for by the errors in the section and total resistance.

Tomlinson * has studied permanent change of specific resistance produced by stretching. He determined the value of the section by finding the specific gravity at the beginning and end, and assuming the change of section at intermediate stages to be proportional to increase of length.

The percentage increase of specific resistance per unit of percentage increase of length of wire is, for lead, by the above results, not greater than $\pm \cdot 017$.

* *Phil. Trans.*, 174, 1, 1883.

Tomlinson gives the following corresponding values:—

Silver	from	·0094	to	·0262
Copper	„	·063	to	·013
Iron (one specimen)	„	·034	to	·041
Iron (another „)	„	− ·0183	to	− ·0091
Nickel	„	− 2·366	to	+ ·509

He did not study lead.

Gray * determined the corresponding changes in the weight specific resistance, *i.e.* resistance of unit length of wire of unit weight, and gave the following results:—

Copper	·5	to	·6
Iron	·7	to	·8
German Silver	·50	to	·55

Compared with most of these results, the above determination would seem to show that the change, if any, in the specific resistance of lead, due to permanent stretching, is small as compared with the similar changes in copper, nickel, iron, etc.

The volume of the lead wire was computed, from the readings given above, for the various stages. We obtained the following results:—

Volume in Cubic Cms.	Length of Wire in Cms.
3·312	196·2
3·315	204·4
3·312	213·4
3·306	221·4

The precision measure of the volume, calculated as before, is ·006. The values found above for the volume show no change that cannot be accounted for by the errors in the determination of section and length.

The percentage increase of volume per unit percentage increase of length is, from the above results, not greater than ±·012.

Gray and Henderson † give the change in density of a lead wire from 7·695 to 7·637, the wire being stretched until it broke, and the amount of elongation being 3·6 per cent. This corresponds to a ·22 per cent. decrease in density per unit percentage increase of length. In our experiments the lead wire broke during the next stretch after it was of length 221·4 cms., the last length for which the results were obtained. Gray and Hender-

* *Trans. R S.E.*, xxx. 369, 1880. † *Proc. Roy. Soc.*, liv. 283, 1893.

son's large value for change of density for a small stretch up to breaking-point, compared with our results, would suggest that the change of density for the most part occurred at or near the rupture.

H. Tomlinson * has studied the effect of stretching to near breaking-point on the density of silver, copper, iron, and other wires, but not of lead. He found that the percentage change of specific gravity, for a permanent increment of length of 1 per cent., ranged from −·0082 for aluminium to −·0620 for platinum, being −·0156 for silver. †

* *Phil. Trans.*, 174, 1, 1883.

† The above experiments were carried out by the aid of a grant from the Moray Endowment for the Promotion of Research.

(*Issued separately April* 4, 1907.)

VI.—On the Discovery of a new Genus of Thread-Bacteria (Spirophyllum ferrugineum, Ellis). By David Ellis, D.Sc., Ph.D., F.R.S.E., Lecturer in Botany and Bacteriology, Glasgow, and West of Scotland Technical College, Glasgow. (With a Plate.)

(MS. received February 23, 1907. Read March 18, 1907.)

Introduction.

Throughout Lanarkshire and Renfrewshire many streams and rivulets have beds which are deep reddish-brown in colour. This colour is due to the deposition of ferric hydroxide, and in some cases the deposition is so great that the course of the stream (if a small one) is hindered, and therefore it must be periodically cleared.

The cause of the deposition is well known. The underlying rocks are partly composed of bands of iron-stone which contain a very large proportion of ferrous carbonate. This iron-stone is subject to denudation, and the carbonate in the form of the soluble bicarbonate $FeH_2(CO_3)_2$ is set free. The consequence is that when water oozes out from the surface it contains a large quantity of iron. By the time, however, that the surface is reached, oxidation and precipitation have taken place, with the result that ferric hydroxide is formed, which is deposited on the bed of the stream or rivulet into which the water issuing from the earth flows. This deposit is found to consist of organisms belonging to the thread-bacteria or Chlamydobacteriaceæ. Four of these, viz., *Leptothrix ochracea*, *Crenothrix polyspora*, *Cladothrix dichotoma*, and *Gallionella ferruginea*, are well known. The brownish-red colour is due to the fact that these organisms are coated, often to an extent greater than their own diameter, with ferric hydroxide, so that the size of each cell is abnormally increased. Hitherto no other iron-bacteria have been known. In Lanarkshire and Renfrewshire I find *Leptothrix ochracea* and *Gallionella ferruginea* to be exceedingly common. Usually the former preponderates, though *Gallionella* is present; sometimes the latter is absent, and, very rarely, a sample may be found in which *Gallionella* is the predominant factor. About a mile from Renfrew, water permeated with ferric hydroxide oozes from the earth and runs along a ditch until it is emptied into a small stream which flows into the Clyde. The grass in the neighbourhood of the ditch is quite red in colour owing to the deposit.

On examination the deposit was found to consist of iron-bacteria which have not hitherto been described. Although I have examined samples of iron-water from several places, this is the only place in which I have found this particular organism. The question of its distribution must be left to another paper. I propose to call the genus *Spirophyllum,* and this particular species *Spirophyllum ferrugineum.*

General Structure.

A typical example of this plant is seen in fig. 1. The body of the cell is flattened like a leaf, and spirally twisted. In this figure two turns are shown; but there is great variety in this respect, for, while there may be only half a turn in some, in others there may be fifteen or more complete turns. By a complete turn is meant one in which, as a result of the turn, the lower surface has become the upper surface. The name "Spirophyllum" is meant to indicate the above facts, viz., a spirally wound, flattened structure.

With regard to the dimensions, it is necessary to state them as regards length, width, and thickness. In addition, in this case we must also state the length of a turn or a twist in the same direction. The width is least immediately after germination from the conidia, *e.g.* figs. 16, 18, in which the width is 1 μ. In fig. 2, which shows a portion of a spiral which had fourteen turns, the width is nearly 6 μ.

Between these two dimensions all sizes are seen, showing a perfect gradation, leaving us in no doubt as to the inclusion of all these forms under one species. The maximum length, of course, cannot be stated. In fig. 3 we see what is very probably the first stage in the germination, in which the length is not more than about 2 μ. On the other hand, fig. 2 in its entirety would reach up to 180–200 μ. I have no doubt that still longer threads exist, because, as is explained later, this organism probably relies on the germination of conidiospores, and not on vegetative division, for increase in numbers. The thickness of the bands is not easy to determine accurately, because, though the threads are often seen with the edges facing the observer, as must be the case in a spirally twisted organism, yet the edges are thicker than the middle portions. The edges attain a thickness extending to approximately $\frac{1}{2}$ μ; the middle portion can be roughly estimated at $\frac{1}{4}$–$\frac{2}{5}$ μ.

The closeness of the spiral turns finds its highest expression in fig. 4, which is, however, extremely exceptional. In this case the length of a twist is very little greater than the width. The average is seen in fig. 1,

in which the twist-length is roughly four times the width. In exceptional cases, as in figs. 5, 6, a twist-length may be nine to twelve times longer than the width, and it may be said that nineteen out of twenty of the organisms have twist-lengths three or four times greater than the width.

A noticeable feature of the twist is that it may be towards the right or towards the left, and it may exhibit both kinds, even in the same individual. The internal structure can be followed only in those organisms in which no deposit has yet taken place, or else in which it is so slight as not to interfere with the staining. There is no indication of a definite cell-wall such as is found, for instance, in *Leptothrix ochracea.* In place of it we find that the organism is thicker at the edges than at the middle, and there does not seem to be a sharp line of demarcation separating the two parts, though, owing to its greater thickness, the thickened edge is easily distinguishable when stained with methylene-blue or any other dye (fig. 11).

By examination of stained specimens, and by carefully examining the edges at the parts where they are turned up, it is seen that a transverse section would appear somewhat like fig. 7. This is different to anything known among Bacteria, and is found only in the genus *Spiromonas*, which was found by Warming on the coasts of Denmark, and which is described as having a similarly thickened edge. Now, as *Spiromonas Cohnii,* which is the only known representative of the genus, is also a flat leaf-like organism, which curls in the same way, it would appear that this development has become necessary to subserve the purposes of twisting without danger of tearing. In the case of *Spirophyllum ferrugineum* there is the additional necessity of strength imposed upon it, because each individual has to bear the weight of a large quantity of ferric hydroxide. In one case the edge of an organism had become torn away from the middle portion, showing quite clearly its nature, and bearing out what has been stated above on this point.

The extremities of the individuals are very different from anything yet known in this division of the vegetable kingdom. Where the organism has made only half a turn, the extremities appear much narrower than the remaining portions, but this is due to the fact that at this point the organism is somewhat turned up (fig. 18). Where the end is flattened out, as in figs. 5, 10, 20, it is seen that the ends are as wide as the rest of the organism. The remarkable feature, however, is the fact that the ends are not neatly rounded off as we see, for example, in bacilli, but are unsymmetrical and angular. Examples are well shown in figs. 5, 9, 18, 27. When individuals are examined in which no deposition of iron has taken

place, the appearance of the cells is somewhat transparent and grey-coloured. Not the slightest evidence was obtained to show that it was other than homogeneous. In most cases, however (figs. 5, 13, 24), especially when the water was examined immediately after it had been collected, all stages in the formation of conidia could be seen, and the beginning of this formation is marked, as will be explained later, by the appearance of a number of large dark dots (figs. 5, 11) which really represent very slight protuberances from the surface of the organism, and at first give one the impression that they are cell contents.

The Spiral Twisting of the Individuals.

The most characteristic feature of this organism is that the individuals, at a very early stage, begin to twist. This phenomenon must be regarded as a spontaneous vital movement, similar, for example, to the amœboid movement of zoospores of the Myxomycetes. It does not appear to be influenced by any external factors; it is universally found, except in the very young stages, *i.e.* stages immediately subsequent to germination. Examples of stages in which twisting has not begun are shown in figs. 12, 14. In fig. 14 is shown the youngest condition in which the twisting is observable. Others, showing a slightly older condition, are shown in figs. 15, 16, 17. At this period the individual has no deposit of iron on its surface, and in consequence is grey and transparent. Also, although the edge is thickened, it is not very pronounced, and would thus present no hindrance to the operation. That the period of twisting can be prolonged beyond this condition is seen in fig. 18, in which the twisting is only just commencing. This condition, however, is far less common than the one just mentioned, and it may be accepted that twisting begins almost immediately after germination and before deposition of iron has taken place. One curious instance of a case in which twisting had been dispensed with was seen, which had grown to a comparatively large size. In this, however, there was also an absence of deposition of iron and an absence of conidia formation, so that very probably the individual was not in a healthy condition. When once begun, the twisting goes on, until the whole length has been traversed. It does not always take place in the same direction, for there may be two or three twists to the right, followed by one or two to the left. Thus in fig. 1 there is one right and one left twist. In this paper I regard a right twist as one of the same kind as in a coil of wire round a stick in which the wire on the upper part of the stick goes to the right, *e.g.* fig. 22 is a coil to the right, and fig. 23 one to the left. On examining the various diagrams, it will be

seen that right and left turns are fairly evenly distributed. Again, two individuals twining round each other are as common as those that are free. When once they have caught on, twisting takes place till the whole course has been traversed. Examples are given in figs. 5, 6, 24. In fig. 24 the process is shown in the case of two long threads. As is evident in this figure, twining round another organism is carried on until the length of the shorter is exhausted, after which the remainder of the longer thread twines on its own accord. Usually, however, the twining individuals are of the same length (figs. 5, 6). At first I thought it somewhat remarkable that more than two individuals do not interlock, as the organisms in the deposit on the bed of the stream are always massed very closely together, and it would appear that one organism would touch several others. It seems to me, however, that when they are thickly massed together in the deposit, the time of twisting is past, and it is doubtful whether the organism, apart from the conidia, is alive. Again, the twisting must be accomplished before a thick deposit is laid down, otherwise a very large inertia, probably greater than the capacity of the plant, has to be overcome.

Vegetative Division.

So far as I have observed, vegetative division is entirely absent in this species. In fig. 25 is shown a case in which division by a transverse slit has apparently taken place, but this was the only case of the kind which I have met, and, whilst admitting the probability of this method of multiplication taking place under other conditions, at present the matter must be left in doubt. The development of conidia, however, is so prolific and so universal, that it is possible that the plant has adopted this method of multiplication to the exclusion of the other.

Conidia Formation.

The formation of conidia in this species, as in *Leptothrix*, is very prolific. I have also found that *Gallionella ferruginea* (or *Chlamydothrix ferruginea*, Migula) can also form conidia in quite as prolific a manner as *Leptothrix*. It has been hitherto unknown in *Gallionella ferruginea*, and I intend publishing the investigation shortly. This conidia formation is thus the commonest method of multiplication in all the iron-bacteria of this neighbourhood, and marks both an advance in development over the other bacteria, and is an important connotative mark indicating the close alliance of these three forms. The first appearance of the conidia is indicated by large dark dots marking the surface of the flat organism (figs. 5, 11). These dots are obviously produced by very slight protrusions, and can be

seen in the youngest stages, when no trace of iron deposition is observable on the organism and before twisting has begun (figs. 11, 21). They are found in individuals on which a very slight deposit is observable and in which twisting is already well advanced, and, judging from the huge numbers that are often found attached to the mother organism (figs. 26, 29), they must be produced in large numbers even when a thick deposit has been laid down. In fact I have traced the various stages of conidia formation in *Leptothrix* (as will be explained in a later publication), from which it was clear that the conidia arise even when a thick deposit of iron is present, and they seem to be able to make their way through it without difficulty. It thus is evident that conidia formation takes place in many cases throughout the whole life of the individual; and often individuals are seen so loaded with conidia—for most still cling to the parent organism—that the original outline of the latter is completely lost, and an appearance similar to fig. 26, which represents a part of an individual, is seen. Thus hundreds, even thousands, of conidia may be attached to an individual. The deposit on the bed of the stream into which the iron-water runs consists chiefly of organisms in this condition, and the cause of the sinking of the organisms to the bottom of the water is evidently due to the increase of weight caused by a further deposit of iron. It is obvious that if the conidia remain attached, the surface upon which iron can be deposited is very much increased thereby, and the density very soon becomes greater than that of water. The further development of the conidium consists in an increase in size of the protrusions from the cell, until it becomes a well-defined protuberance (figs. 21, 26, 27, 29). By examining an individual in which the deposit of iron is very small, and on which the number of conidia is comparatively few, all stages in conidia formation can be readily observed. When the protuberance has attained the normal size, a constriction is formed at its base, and the newly formed conidium, as it can now be called, is physiologically independent of the parent plant. The process is repeated until the conidia are present in large numbers. Every portion of the individual can take part in this process, and in fact in many cases it may be safely stated that every square μ of the organism has upon it a conidium either mature or in process of development. The increase of thickness caused by conidia formation is diagrammatically represented in fig. 29. The reason why the presence of the iron deposit presents no difficulty is most probably due to the fact that the cell is surrounded as in other bacteria by a coating of mucilage, and that the iron is held by this coating so that the iron deposit cannot be very dense and closely compacted. Whatever the explanation, there is no doubt of the capacity of the conidia to be

formed when a thick deposit of iron is present, as I have seen them in all stages in *Leptothrix* making their way through this deposit in the course of their development.

The Structure of Conidia.

The conidium is oval in shape (fig. 26, *a*), measuring about 1 μ in width and about 1·75 μ in length. There is very little variation from this size, and the conidia are thus exactly the same in size as those of *Leptothrix ochracea* and *Gallionella ferruginea*. By careful staining with methylene-blue or fuchsin, the presence of the external membrane of the conidium is easily observable. The contents appear, when stained, as a homogeneous material, but of course, owing to the smallness of these bodies, further information is not possible.

Germination of the Conidia.

This part of the investigation is not complete, for the conidia do not, as a rule, germinate in the deposit of the bed of the stream, and, as they are formed in such huge numbers, it is only a comparatively small number that will be able to germinate at all. If the water, however, be collected before it comes to the surface, or, at any rate, immediately after coming out from the earth, it will be found that the sediment takes a comparatively long time to settle on account of the activity of the enclosed *Spirophyllum*. When examined as soon as possible after collection, a few stages in germination were observed. Fig. 26, *b*, obviously represents the condition in which the conidium membrane has been split at the polar end and the young cell thrust out: it seems to assume its flattened condition at once. In fig. 3 a stage is seen in which the membrane has been thrust away. Even at this stage the characteristic want of symmetry is observable. When stained with methylene-blue, no appearance of a limiting membrane could be observed, nor any differentiation of cell contents. It forms before staining a grey, semi-transparent, angular, flattened portion of protoplasm.

The Young Vegetative Cell.

In freshly collected samples, when sedimentation takes place very slowly, we see numerous examples of young vegetative cells. Fig. 12 shows a very young condition and is a typical example. It is about 1·5 μ in width and about 10 μ in length. It is flat, grey, semi-transparent, and, like older specimens, is unsymmetrical and somewhat angular. In this particular specimen one of the ends is slightly rounded off, but this

is not always, nor even normally, the case. Also no spiral twisting is observable, neither is the edge of the individual thicker than the middle portion. I invariably found this organism *motile* at this stage. The motility consists of a wriggly movement, accompanied by a slow, pendulum movement, as if one end had one or more cilia attached. I have not, as yet, been able to demonstrate the organs of motion. The next stage in the development of the cell is seen in fig. 14. The thinner terminal portion was partly twisted, and in fig. 15 a complete twist has been accomplished. The young cell is still grey and transparent, has no thickened edge, and no deposit of iron. The length is about 14 μ, and the breadth varies from $1\frac{1}{4}$ μ in the widest to 1 μ in the narrowest portion. A still further stage is shown in fig. 16, in which two turns have been accomplished. In this individual no change had taken place except for this extra spiral turn; it moved about very slowly in a forward direction, exhibiting in addition a trembling movement. Later stages are shown in figs. 10, 18, 19. In the individual represented by fig. 18 there was no deposit, but in those shown in figs. 9, 19 a slight deposit had begun to be formed, as shown by the loss of its grey, semi-transparent character and its assumption of a pale, brownish-red colour. Movement had stopped, and during the rest of the life-history of the individual, is not again assumed. The deposition of iron, the formation of conidia, and the thickening of the edges are inimical to movement. From this point on, growth in length and width had evidently taken place, and conidia were rapidly formed. All stages and all sizes were observed, the twisting with the variations in the length, width, and twist-length causing a large variety of forms, which, if one had not seen the intermediate stages, would incline one to believe that we were dealing with several species.

The Culture of Spirophyllum ferrugineum.

I have made numerous experiments with the object of cultivating this form, and have tried most of the methods tried by others in their efforts to obtain pure cultures of other iron-bacteria. Winogradsky gives one method which was not at all successful with this form, nor indeed was it with *Leptothrix* and *Gallionella*.

The methods found to be successful by others, resulted, in my cultures, in a preponderance of an undesired bacillus or coccus and a consequent extinction of the desired organism. This must often be the case in bacteriological experiments, for the preponderance of any one form depends on the sum total of conditions being favourable to it, and the sum total is never the same in any two localities. It is not necessary to state all

the experiments which were unsuccessful, it will be sufficient to describe the only one out of many which was successful in cultivating *Spirophyllum ferrugineum.* A small portion was inoculated into a solution made up of sterilised well-water, to which freshly precipitated ferric hydroxide had been added, and contained in a sterilised flask. The culture was allowed to remain on a table exposed to sunlight. I think the exposure to the sunlight was the determining factor in deciding which form should be preponderant, for, being protected by the iron compound, the light was more harmful to the other forms. In two or three weeks a flocculent red deposit was obtained, in which I found that nearly all the ferric hydroxide which I had added had attached itself to the organisms, which accounted for the very flocculent nature of the deposit, as ferric hydroxide placed in water under the same conditions settles down at the bottom of the flask in the same way that fine sand would. The appearance of the organism differed in no way from that found in nature, consisting of the usual spirally twisted bands, coloured deep brown, and thickly covered in most cases with conidia. As the development took place during vacation time, I could not examine the various phases of growth from day to day. I hope to communicate these results in a later publication.

The Iron Deposit.

The interpretation of the deposition of iron in these bacteria has undergone many phases. Cohn (3), who was the first to observe these bacteria, compared the deposition in *Crenothrix polyspora* to the deposition of silicon in diatoms, and calcium carbonate in certain cell membranes of the Melobesiaceæ. In 1878 Zopf (8) examined the same organism, and came to the conclusion that cell activity had nothing whatever to do with the deposit, and that the deposition was purely a mechanical one, the iron being caught by the mucilaginous layer which surrounds the cell. He maintained that the iron was retained in this mucilaginous layer in the same way that the colouring matter is retained by the gelatine in certain coloured jellies. In 1888 an article appeared by Winogradsky (7) in which a totally different explanation was given. He came to the conclusion that the soluble bicarbonate $FeH_2(CO_3)_2$ in the water was absorbed into the cell, and there oxidised, being changed into ferric hydroxide. The change can be represented by the equation

$$\left.\begin{array}{l}2FeCO_3\\H_2CO_3\end{array}\right\}\begin{array}{l}+3H_2O+O=Fe_2(OH)_6+2CO_3\\ \qquad\qquad\qquad\;\;=H_2O+CO_2\end{array}$$

This oxidation is the source of energy of the plant whereby the vital

activities are rendered possible. The iron-bacteria are thus supposed to respire, and so obtain their energy much in the same way as the sulphur-bacteria had been previously proved by him to obtain their energy. In the one case the free sulphur is oxidised to the sulphate, in the other the ferrous compound is oxidised to the ferric form. This conclusion was based upon the following data:—

1. Deposition of iron occurs only in water containing iron in the ferrous condition.
2. For the growth of these bacteria ferrous carbonate is absolutely necessary.
3. Only living threads possess an iron deposit.

Hence the conclusion that the presence of a large quantity of iron is absolutely necessary to the existence of the organism. The first two of the above data were never proved by him. His publication was meant as a preliminary notice, but, though nineteen years have elapsed, no further publications from his pen on this subject have, to my knowledge, appeared.

It is also remarkable that in his prescription for the cultivation of these bacteria he recommended that freshly precipitated ferric hydroxide be added to the culture medium, presumably to supply the necessary energy. If respiration be dependent on the oxidation of a ferrous to a ferric compound, one would have expected that a ferrous compound should have been added. The improbability of this hypothesis—for he did not carry it beyond this stage—is obvious when it is remembered that when a solution of ferrous carbonate is made, it changes into the hydroxide almost immediately, and very special precautions have to be taken to prevent its doing so, by carefully eliminating the atmospheric oxygen. An important work on this subject was published by Molisch (4), who was able to cultivate bacteria in solutions in which not a particle of iron was present. He rightly explained the reason why living threads are able to store iron, but dead threads were not able to, by stating that living threads were surrounded by a mucilaginous layer, but that dead threads were devoid of it, and, as it is this layer which retains the iron, this substance is obviously absent from dead threads. I have verified this fact in the case of *Leptothrix ochracea*, in which observation on this point is rendered easy by its sharply defined membrane. Again, if it were true that iron is absorbed into the system, we should be able to detect it inside the cell. Winogradsky does not mention that he was able to do this, and neither Molisch nor myself have succeeded. The deposit can very easily be cleared away by adding a little very dilute hydrochloric acid under the coverslip to a microscopic preparation. In the

case of *Leptothrix ochracea* this can be done very gradually, until the deposit has been removed, when it may be presumed that, owing to its dense membrane, the contents are untouched. When now potassium ferrocyanide is added, and given time to penetrate, no blue coloration can be detected. Molisch's explanation certainly covers the facts of the case. He states that the mucilaginous layer round the cells acts like a filter, keeping back the iron compounds, which are later oxidised without entering into the cell. He further points out that the iron-bacteria presents an analogous case to Zygnema, the mucilaginous covering of which has been shown by Klebs to possess a power of attraction for aluminium, chromium, and iron compounds, and in consequence of the deposition of these substances, when presented to it, considerable swelling of Zygnema takes place. This power of attraction for particular inorganic or organic substances is quite common throughout the whole vegetable kingdom, and among other bacteria many instances may be mentioned, *e.g.* the salts of potassium are well known to have an attraction for bacteria, and they may be lured to their destruction by corrosive sublimate. We seem thus to have in the iron-bacteria a case of *chemotaxis*, with this difference, however, that the iron-bacteria, like Zygnema, are the attracting and not the attracted agents. We cannot call them iron-bacteria in any other sense than we should call Zygnema an aluminium or iron alga; we cannot call them iron-bacteria in the same sense as we speak of the sulphur-bacteria or the nitrate-bacteria, attaching to the term a physiological meaning. Again, as pointed out by Beythien, Hempel, and Kraft (2), not only can iron be dispensed with in the cultivation of these bacteria, but it can be replaced by manganese, for which also the organism has a remarkable attraction; and in fact, as the attraction is greater than that of iron, we are more justified in naming them manganese-bacteria. As Molisch points out, nowhere in Nature is iron necessary to a plant and being at the same time replaceable by manganese. Later investigations by Adler (1) show that the addition of substances which hinder the growth of these organisms prolongs the precipitation of ferric hydroxide, so that, according to this investigator, it would seem that in addition to a purely chemical there is also a biological factor at work in effecting the precipitation of the hydroxide. This is difficult to explain in face of the fact that these organisms can thrive very well without the presence of a particle of iron, and that the presence of iron has never yet been demonstrated inside the cell. Further, one would expect that the presence of these bacteria would tend to prolong the precipitation by using up the oxygen dissolved in the water. The only possible way out of it seems to me to be the supposition that in some way these organisms assimilate and thus give

out oxygen. This would explain the fact that their presence expedites the precipitation of the hydroxide. In my experiments I have found that the presence of sunlight does not act as a deterrent to the growth of these forms, and it is just possible that we have before us organisms which beneath their iron deposit possess a certain amount of colouring matter performing the same functions as chlorophyll. I am at present engaged on this problem. Whatever the reason of this enormous attraction for iron may be, it is very thorough, for an analysis of a sample in which the iron had been precipitated showed not a trace of this substance in the water.

Phylogenetic Position of Spirophyllum ferrugineum.

Migula, in his *System der Bacterien* (4), defines the Chlamydobacteriaceæ as follows:—"Zellen cylindrisch zu Fäden angeordnet, die von einer Scheide umgeben sind. Vermehrung verfolgt durch bewegliche oder unbewegliche Conidien, welche direkt aus den vegetativen Zellen hervorgehen und ohne eine Ruheperiode durchzumachen, zu neuen Fäden auswachsen." This new form agrees with this definition in all essentials except that the body is flattened instead of being cylindrical, and is not externally limited by a membrane, which one would hardly expect in an organism of this shape. There is complete similarity in the methods of reproduction with the other iron-bacteria, all of which are included in the Chlamydobacteriaceæ; and in whichever family *Leptothrix ochracea* and *Gallionella ferruginea* are placed, that family must also take in this form, as they obviously form a natural group. Again, *Gallionella ferruginea*, renamed *Chlamydothrix ferruginea* by Migula, possesses, according to this observer, only an extremely fine membrane, which, however, I have not been able to see, and Adler records the same failure; and in another, *Chlamydothrix hyalina*, the membrane is altogether wanting. Again, *Phragmidiothrix* is placed by Migula as a genus of the order Chlamydobacteriaceæ. This is described as consisting of cylindrical cells which later become *flattened* and disc-shaped. Hence the fact that, in the absence of a limiting membrane and in the presence of the flattened condition, we have characters not usually found in this order, does not exclude this new form from that order, in view of the obvious relationship to the other iron-bacteria, and in view of the non-universality of the excluding characteristics. I would therefore suggest that Migula's definition of the order Chlamydobacteriaceæ be altered to read thus:—"Cylindrische oder flache Zellen, die gewöhnlich von einer Scheide umgeben sind," instead of "Zellen cylindrisch zu Fäden angeordnet, die von einer Scheide umgeben sind."

The most interesting feature of *Spirophyllum ferrugineum*, from a phylogenetic point of view, is that it serves as a bridge to connect the iron-bacteria *Leptothrix*, *Gallionella*, *Cladothrix*, and *Crenothrix* on the one hand, with the genus *Spiromonas* on the other hand, which is made a dependent on the thread-bacteria, and is not included in this order. *Spirophyllum* agrees with *Spiromonas* in possessing a flattened body thickened at the edge in the same way. The latter is also spirally twisted, though never showing more than one and a quarter turns, and is motile in the mature condition; whilst *Spirophyllum*, as described above, may exhibit any number of turns, and is only slightly motile immediately subsequent to germination. Hence *Spirophyllum* can be placed midway between the other iron-bacteria and *Spiromonas*, and thus justifies not only the inclusion of *Spirophyllum*, but also of *Spiromonas* into the thread-bacteria.

Summary of Characters of Spirophyllum ferrugineum.

Body of cell elongated, flattened, and spirally twisted. The number of spiral turns may vary from a quarter turn to fifteen and more turns. The width varies from 1 μ to 6 μ. The length may reach to 200 μ and possibly more. The middle portion of the cell has a thickness of $\frac{1}{4}$–$\frac{2}{5}$ μ, but the edge is thickened up to $\frac{1}{2}$ μ. There is no definite membrane, but the edge is thickened so as to form a kind of rampart all round the cell. The ends are usually irregular, angular, and unsymmetrical. The spirals may be very close or very wide apart, but the majority of the individuals have spiral lengths three or four times greater than the width. The organism has, so far, been found only in iron-water, and, except in the youngest stages, is coated with a thick deposit of ferric hydroxide.

Multiplication is effected by the budding off of conidia, the process being exactly the same as in *Leptothrix ochracea* and *Gallionella ferruginea*, viz., a budding off in large numbers. Each conidium has a single coat, is oval in structure, measuring about 1 μ in width and 1·75 μ in length. Each germinates by bursting open the spore membrane, the contents then being protruded, after which the young vegetative cell is freed from the spore membrane and for a short time exhibits a faint movement, partly of a trembling and partly of a pendulum nature. Conidia formation begins when very young, even before the twisting has begun, and may persist throughout the whole life of the individual, even when thickly encrusted with ferric hydroxide. Hitherto only one doubtful case of multiplication by vegetative division has been observed. Immediately

after germination, the vegetative cell becomes flat, irregular in width, and very soon begins to twist spirally. Two individuals often twist round each other, and the twists may be right or left or even both in the same individual.

LITERATURE.

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(2) Beythien, Hempel, and Kraft, *Z. f. Nahrungsmittel-Unters., etc.*, 1904.
(3) Cohn, *Beitrage z. Biol. d. Pflanze*, Heft 3.
(4) Migula, *System der Bakterien*, Jena, 1897.
(5) Molisch, H., *Die Pflanze in ihren Beziehungen zum Eisen*, Jena, 1892.
(6) Warming, E., *Observations sur quelques bactéries qui se rencontrent sur les côtes du Danemark*, 1875.
(7) Winogradsky, *Botanische Zeitung*, 1888.
(8) Zopf, *Entwickelungsgeschichtliche Untersuchungen über Crenothrix polyspora, die Ursache der Berliner Wasserkalamität*, Berlin, 1879.

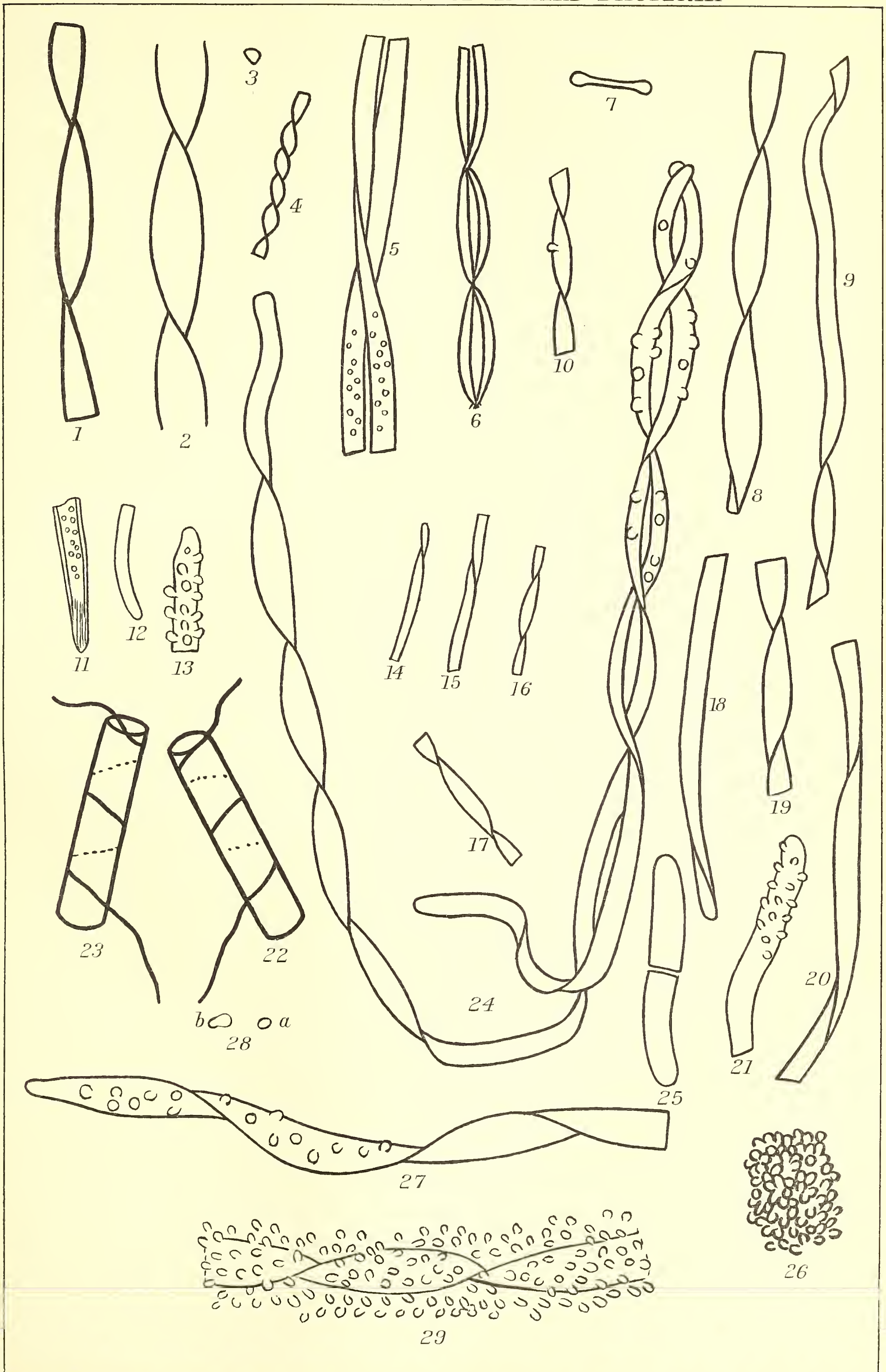
EXPLANATION OF PLATE.

All diagrams, except figs. 7 *and* 29, *drawn to same scale. Magnification* 1650.

Fig. 1. Typical example of *Spirophyllum ferrugineum.*
Fig. 2. Portion of a spiral with fourteen spirals.
Fig. 3. Appearance immediately after germination.
Fig. 4. Individual showing very close spirals.
Fig. 5. Two spirally twisting individuals, both showing the commencement of conidia formation: stained with methylene-blue.
Fig. 6. Two individuals with long spiral lengths.
Fig. 7. Diagrammatic representation of appearance of transverse section of *Spirophyllum ferrugineum.*
Fig. 8. Individual showing two and a half spirals.
Fig. 9. Example showing irregular ends.
Figs. 9–21 show stages in the development of the organism from the conidium stage. Fig. 11, stained with methylene-blue and showing beginnings of conidia formation. In figs. 13 and 21 conidia are well developed. For further explanation see text.
Fig. 22. Example of right-hand twisting.
Fig. 23. Example of left-hand twisting.
Fig. 24. Two individuals twisting round each other, both showing conidia formation.
Fig. 25. A doubtful case of cell division.
Fig. 26. Small portion of a band showing the usual prolific conidia formation.
Fig. 27. Individual showing three and a half turns.
Fig. 28. *a*, conidium; *b*, first stage in the germination of the conidium. Conidium membrane is still attached.
Fig. 29. Diagrammatic representation showing how the size of the organism is normally increased by the formation of conidia.

(*Issued separately April* 24, 1907.)

ELLIS: NEW GENUS OF THREAD BACTERIA



W. & A. K. Johnston, Limited, Edinburgh.

VII.—Prymnothonus Hookeri, Poisson pélagique de l' "Erebus" et de la " Terror " retrouvé par l'Expédition Antarctique Nationale Ecossaise. Note préliminaire, par Louis Dollo, Conservateur au Musée royal d'Histoire naturelle, à Bruxelles. *Présentée par* M. R. H. Traquair, M.D., F.R.S., V.P.R.S.E.

(Read February 4, 1907.)

I. Introduction.

On sait que tous les Poissons recueillis lors de la fameuse Expédition de l'*Erebus* et de la *Terror* (1839–1843) ne sont pas parvenus jusqu'à nous.* Parmi ceux qui ne furent représentés, au retour, que par des dessins, deux des plus intéressants sont, assurément, le *Pagetodes* et le *Prymnothonus.*

On se souvient que le *premier* fut mangé par le chat de l'équipage de la *Terror,*—et que, selon toute probabilité, c'est le *Cryodraco antarcticus* de l'Expédition de la *Belgica* (1897–1899), ainsi que je l'ai signalé,—mais qu'il ne fut point possible de conserver le nom proposé par Richardson, à cause de la diagnose absolument insuffisante de cet ichthyologiste.†

Quant au *second,*—jugé si remarquable par M. A. Günther, Conservateur honoraire au British Museum, qu'il le figura dans son Catalogue des Poissons,‡ bien que cet ouvrage soit généralement dépourvu d'illustrations,—il était réservé à l'Expédition de la *Scotia* (1902–1904) de le retrouver et d'en rapporter les seuls spécimens authentiques qui existent actuellement en Europe. Je dis authentiques, car s'il est bien vrai qu'on a *assimilé* quelques jeunes Poissons pélagiques du *Challenger* § au *Prymnothonus,* les différences d'âge n'ont pas permis d'*identification,* comme nous le verrons plus loin.

* J. Richardson, "Fishes," *Zoology of H.M.S. "Erebus" and "Terror," under the command of Captain Sir James Clark Ross, R.N., F.R.S., during the years* 1839 *to* 1843, p. 51. Londres, 1844–48.

† L. Dollo, "Poissons de l'Expédition Antarctique Belge," *Résultats du Voyage du S.Y. "Belgica" en* 1897, 1898, 1899, *sous le commandement de A. de Gerlache de Gomery,* p. 8. Anvers, 1904.

‡ A. Günther, *Catalogue of the Fishes in the British Museum,* vol. viii. p. 145. Londres, 1870.

§ A. Günther, "Report on the Pelagic Fishes," *Voyage of H.M.S. "Challenger" during the years* 1873–76 : Zoology, vol. xxxi. p. 39. 1889.

Les *Prymnothonus* de la *Scotia* sont donc les seuls pour lesquels on puisse parler de certitude dans la détermination.

Ils sont au nombre de trois, et de tailles diverses. Grâce à eux, nous allons être à même de contrôler, de rectifier et de compléter les observations du vénérable Sir Joseph Dalton Hooker et de Richardson.

Tel est l'objet de la présente Note.

II. Le Prymnothonus de l' "Erebus" et de la "Terror."

La première mention du *Prymnothonus Hookeri* remonte à 1845, quand J. Richardson publia le dessin du jeune chirurgien de l'*Erebus*, en lui dédiant l'espèce nouvelle. Cette figure était accompagnée des commentaires suivants : *

"The figure here introduced is copied from a pencil drawing (No. 217) by Dr Hooker, and we can give little more information than the sketch conveys, the notes made at the time by Dr Hooker having been mislaid. The specimen measured an inch and a quarter in length, but it has perished not having been found in the collection. It is evidently a Muræenoid fish, closely allied to the Congers, but is remarkable in that family for the shortness of the belly, the vent being only a fourth of the whole length distant from the snout. The gill-openings are lateral, but their position will be unusual, if the oblong white mark before and below them be meant to represent a pectoral fin. It seems to be placed too far forward for that member. The generic name is derived from the backward position of the dorsal, from πρύμνα, *puppis*, and ὀθόνη, *velum*, *i.e.* mizen-sail. The caudal and part of the anal are marked as rayed, and a note subjoined to the drawing states that rays were not perceptible in other parts of the fins. It would be unwise to attempt drawing up a generic character without further information, but it appeared advisable to give the figure a name, for the sake of reference."

III. Les Prymnothonus du "Challenger."

La deuxième fois qu'il fut question du genre *Prymnothonus* dans un travail original, ce fut en 1889, lorsque M. Günther lui rapporta trois jeunes Poissons pélagiques de la mémorable Expédition du *Challenger* (1873–1876). †

Ces Poissons furent désignés par les lettres A, B, D,—C étant attribué

* J. Richardson, *Fishes, etc.*, p. 51.

† A. Günther, *Pelagic Fishes, etc.*, p. 39.

au *Prymnothonus* de l'*Erebus* et de la *Terror,** dans le classement par taille.

Ils sont documentés ainsi :

A.—12 mm. de long. Pacifique septentrional.
B.—14 mm. de long. Atlantique septentrional.
D.—44 mm. de long. Sud-est de l'île Kerguelen.†

D'autre part, quoique la planche V les incorpore tous (A, B, C, D) dans le genre *Prymnothonus,* le texte dit : ‡

"That they all are stages of development of the same generic type of fishes is very improbable, but the second and third specimens may well be considered to be the same type, which provisionally may be designated by the name proposed for it by Richardson."

Examinons ces assimilations, biogéographiquement et anatomiquement.

I. *Biogéographiquement :*

Bien que l'habitat du *Prymnothonus* de l'*Erebus* et de la *Terror* nous soit inconnu, il est peu probable,—vu les itinéraires de l'illustre James Clark Ross, §—que les spécimens A et B du *Challenger* puissent avoir quelque chose de commun avec lui.

Quant au spécimen D du *Challenger,* c'est une autre affaire, car Ross s'est également approché de la Terre de Wilkes, pourtant un peu plus à l'est.

II. *Anatomiquement :*

A (*Challenger*).—Rien que des divergences avec C (*Erebus* and *Terror*) :

1. Tête énorme, au museau camus et à la fente buccale oblique,—au lieu d'une tête gracile, au museau légèrement spatuliforme et à la fente buccale horizontale.
2. Pas de pectorales.
3. Dorsale commençant au milieu de la longueur et *déjà rayée* dans sa partie postérieure, pour un Poisson de 12 mm.,—au lieu de commencer au quart postérieur et d'être *dépourvue de rayons,* pour un Poisson de 32 mm.

* Indiqué, à tort, comme provenant de l'Expédition du *Sulphur* (1836–1842) dans l'explication de la planche v de M. Günther.

† Station 156 du *Challenger,* non loin de la Terre de Wilkes, et non au sud-ouest de Kerguelen, comme l'écrit M. Günther.

‡ A. Günther, *Pelagic Fishes, etc.,* p. 41.

§ J. C. Ross, *A Voyage of Discovery and Research in the Southern and Antarctic Regions during the years* 1839–1843. Londres, 1847.—J. D. Hooker, "Summary of the Voyage," *Zoology of H.M.S. "Erebus" and "Terror," under the command of Captain Sir James Clark Ross, R.N., F.R.S., during the years* 1839 *to* 1843 : Part I. Londres, 1844.

4. Anale commençant au delà du milieu du corps, *entièrement rayée* et plus *haute* en son milieu, pour un Poisson de 12 mm.,—au lieu de commencer dans le cinquième antérieur du corps, d'être seulement *rayée* dans une région très *localisée* et d'avoir une hauteur sensiblement *égale* partout, pour un Poisson de 32 mm.

B (*Challenger*).—Importantes divergences avec C (*Erebus* and *Terror*):

1. Tête massive, à la fente buccale légèrement oblique,—au lieu d'une tête gracile, à la fente buccale horizontale.
2. Dorsale commençant au milieu de la longueur et *déjà rayée*, pour un Poisson de 14 mm.,—au lieu de commencer au quart postérieur et d'être *dépourvue de rayons*, pour un Poisson de 32 mm.
3. Anale commençant dans le deuxième quart du corps, *entièrement rayée* et avec un *lobe saillant* vers son milieu, pour un Poisson de 14 mm.,—au lieu de commencer dans le cinquième antérieur du corps, d'être seulement *rayée* dans une région très *localisée* et d'avoir une hauteur sensiblement *égale* partout, pour un Poisson de 32 mm.

D (*Challenger*).—Concordances avec C (*Erebus* and *Terror*):

1. Corps très grêle, allongé.
2. Tête gracile; museau spatuliforme; fente buccale, horizontale et atteignant l'œil; dents pointues, recourbées et espacées.
3. Dorsale, courte, contigue à la caudale, non rayée.
4. Anale rayée, localisée, rejetée en arrière, mais non contigue à la caudale.
5. Caudale, échancrée, rayée.
6. Pectorales, présentes, petites.
7. Ventrales, absentes.
8. Cavité abdominale, égale à la longueur de la tête.

III. *Conclusions:*

1. Les spécimens A et B du *Challenger* (celui-ci, en particulier, malgré l'opinion contraire de M. Günther),—biogéographiquement et anatomiquement,—n'ont rien à faire avec le *Prymnothonus* de l'*Erebus* et de la *Terror*,—puisque, notamment, quoique beaucoup plus petits, ils ont déjà des différenciations qui manquent encore à ce dernier.

2. Le spécimen D du *Challenger* (ici, encore, je suis obligé de me séparer de M. Günther),—biogéographiquement et anatomiquement,—présente les caractères qu'on doit attendre d'un *Prymnothonus* plus âgé que celui de l'*Erebus* et de la *Terror*,—ce qui correspond à sa taille.

IV. Les Prymnothonus de la "Scotia."

I. Les *Prymnothonus* de la *Scotia* sont au nombre de trois et mesurent respectivement:

1.—38 mm. de long.
2.—42 mm. de long.
3.—56 mm. de long.

Ils ont été pêchés ensemble, et leurs caractères, comme leur habitat, indiquent qu'il s'agit bien, ici, de jeunes Poissons appartenant à la même espèce.

II. Caractères qui déterminent l'identification des *Prymnothonus* de la *Scotia* avec celui de l'*Erebus* et de la *Terror*:

Prymnothonus Hookeri, Richardson, 1845.

(*Zoology of H.M.S. "Erebus" and "Terror":* Fishes, vii. p. 51. Londres, 1845.)

1. *Corps* très grêle, allongé.
2. *Tête*, gracile. *Museau*, spatuliforme, légèrement retroussé, qui s'allonge et se redresse avec l'âge. *Fente buccale*, horizontale et atteignant l'œil, bordée seulement par le prémaxillaire supérieurement. *Dents*, pointues, recourbées et espacées. *Ouïes*, larges (comme l'avait supposé, avec raison, M. Günther).
3. *Dorsale*, courte, contigue à la caudale, non rayée.
4. *Anale*, très longue, mais à l'état de frange embryonnaire, sauf une portion rayée, localisée, située très en arrière, mais non contigue à la caudale.
5. *Caudale*, rhipidicerque (homocerque) échancrée, rayée.
6. *Pectorales*, présentes, petites.
7. *Ventrales*, absentes.
8. *Anus*, immédiatement en avant de l'anale embryonnaire.
9. *Cavité abdominale*, courte, refoulée antérieurement.

10. Une *tache noire,* placée directement derrière les ouïes et au dessus de la pectorale.
11. *Longueur totale:* 32 à 56 millimètres.

Type: Dessin No. 217 de Sir Joseph Dalton Hooker, reproduit par J. Richardson (Pl. 30, fig. 6 et 7).

Cette diagnose rectifie celle de Richardson, en ce qui concerne la nature des ouïes; elle la complète pour ce qui touche aux relations du prémaxillaire avec l'orifice buccal, ainsi que pour le changement de forme du museau avec l'âge.

III. Une difficulté, maintenant, pour l'assimilation du spécimen D du *Challenger* au genre *Prymnothonus.*

Dans le *Prymnothonus* de 56 mm. rapporté par la *Scotia,* la longue frange embryonnaire anale existe encore tout entière,—tandis que, dans le spécimen D de 44 mm. recueilli par le *Challenger*, elle aurait disparu, ne laissant que la courte anale rayée.

Cela suffit-il pour empêcher que le spécimen D du *Challenger* soit un jeune *Prymnothonus* ?—Ou s'agit-il de deux espèces différentes du même genre ?—Ou faut-il voir, ici, un simple cas de vie larvaire prolongée, comme on en connait ailleurs chez les Téléostéens ?

Quoiqu'il en soit,—en attendant de nouveaux matériaux,—je crois que le mieux est de laisser le spécimen D du *Challenger* dans le genre *Prymnothonus,*—vu le nombre et l'importance des coïncidences avec le *Prymnothonus Hookeri* de l'*Erebus* et de la *Terror.*

V. Position systématique.

I. *Murænidæ.*—1. *J. Richardson* (1845): "It is evidently a Muræenoid fish, closely allied to the Congers." *

2. *A. Günther* (1870): "Appears to belong to the Murænidæ." †

3. *Discussion.*—Impossible, car:

Le prémaxillaire bordant, seul, supérieurement, l'orifice buccal; les ouïes fort larges; et l'anus très antérieur, — s'opposent à cette interprétation.

II. *Paralepidæ.*—1. *A. Günther* (1889): "I have no doubt that all these specimens represent larval conditions of fishes belonging to *Paralepis* or *Sudis,* or of genera allied to them." ‡

* J. Richardson, *Fishes, etc.*, p. 51.
† A. Günther, *Catalogue, etc.*, vol. viii. p. 145.
‡ A. Günther, *Pelagic Fishes, etc.*, p. 41.

2. *Discussion.*—M. Günther ne donne pas ses raisons, mais, en effet :

1. Le corps allongé ;
2. Le museau spatuliforme ;
3. La bouche bordée uniquement, vers le haut, par le prémaxillaire ;
4. La fente buccale large ;
5. Les dents pointues, recourbées et espacées ;
6. Les larges ouïes ;
7. La dorsale adipeuse au voisinage de la caudale ;
8. L'anale rayée assez longue, située près de la caudale, mais non confluente avec elle ;
9. La caudale rhipidicerque homocerque échancrée ;
10. Les pectorales présentes, petites ;
11. Le péritoine noir ;

sont d'accord avec ce qu'on observe chez les *Paralepidæ.* Quatre points délicats, pourtant,—l'anus très antérieur ; l'absence de ventrales ; la disparition de la première dorsale ; la dorsale adipeuse assez longue et basse,—chez *Prymnothonus.*

Cependant, la *position de l'anus* est très variable, de genre à genre, ou même d'espèce à espèce, chez les *Paralepidæ,* auxquels je joins déjà *Prymnothonus* pour la comparaison :

α. *Prymnothonus Hookeri :* à la limite du premier quart.*

β. *Plagyodus ferox :* un peu en avant ($\frac{1}{12}$) du milieu du corps.†

γ. *Paralepis sphyrænoides :* un peu en arrière ($\frac{1}{15}$) du milieu du corps.‡

δ. *Paralepis Krφyeri :* un peu en arrière ($\frac{1}{51}$) du deuxième tiers.§

De plus, chez les *Paralepidæ,* comme chez *Prymnothonus,* l'anus n'a

* J. Richardson, *Fishes, etc.*, p. 51.

† G. B. Goode and T. H. Bean, "Oceanic Ichthyology," *Smithsonian Contributions to Knowledge*, vol. xxx., pl. xxxviii., fig. 142. Washington, 1895.

Plagyodus, Steller, 1831, n'est, certainement, pas autre chose qu'un Paralépide, de grandes dimensions, adapté à la Vie Nectique Pélagique superficielle, avec Dorsale véliforme émergée : Convergence avec *Histiophorus*, parmi les Acanthoptérygiens Scombriformes.—H. N. Moseley, *Notes by a Naturalist on the* "*Challenger*," Londres, 1877.

Mais je m'expliquerai là-dessus ultérieurement.

‡ A. Risso, *Histoire naturelle des principales productions de l'Europe méridionale*, vol. iii., pl. vii., fig. 16. Paris, 1826.

§ G. B. Goode and T. H. Bean, *Oceanic Ichthyology, etc.*, pl. xxxviii., fig. 143.

Contrairement à ce que pensent les auteurs américains, l'espèce figurée n'est pas le *Paralepis borealis*, mais le *Paralepis Krφyeri.* C. Lütken, "Korte Bidrag til nordisk Ichthyographi. viii. Nogle nordiske Laxesild (Scopeliner)," *Videnskabelige Meddelelser fra den naturhistoriske Forening i Kjφbenhavn*, p. 230. 1891.

pas conservé sa situation primitive, immédiatement au devant de l'anale (définitive), étant placé plus ou moins loin en avant de celle-ci.

Or, pour passer de *Plagyodus* à *Prymnothonus*, il faut supposer une moindre migration de l'anus que pour passer de *Paralepis* (Krøyeri) à *Plagyodus.*

Maintenant, pareille migration est-elle possible, phylogéniquement et ontogéniquement ?

Oui, comme nous le verrons dans un instant.

Quant à la *disparition* des *ventrales* et de la *première dorsale* chez *Prymnothonus,*—il n'y a guère lieu de s'en étonner après la régression subie par ces nageoires chez certains *Paralepidæ,**—d'autant plus qu'un Téléostéen au corps très allongé, avec caudale rhipidicerque (homocerque) échancrée (mais déjà réduite), dorsale et anale refoulées au voisinage de la caudale, pectorales petites et suppression des ventrales (par atrophie, ou par changement de fonction) est une adaptation à la Vie Planctique Aiguilliforme, † que nous retrouvons, par convergence, chez les *Stomiatidæ.* ‡

Enfin, en ce qui concerne la *deuxième dorsale*, rudimentaire, mais courte et haute, chez les *Paralepidæ* connus,—tandis qu'elle est assez longue et basse chez *Prymnothonus,*—je décrirai, prochainement, sous le nom de *Notolepis Coatsi*, un Paralépide typique de la *Scotia* présentant cette dernière disposition.

III. *Conclusion.*—Je pense, comme M. Günther, qu'il y a lieu de mettre le genre *Prymnothonus* dans les *Paralepidæ.*

VI. Migrations de l'Anus.

I. *Position primitive de l'anus.*—1. Pour qui admet, comme je le fais, l'origine des *membres pairs* aux dépens de *plis latéraux continus,*—la formation de ces plis apparaît comme une conséquence directe de la nécessité d'éviter l'oblitération, ou le refoulement, de l'anus, par le prolongement de la nageoire impaire primordiale ininterrompue, le long de la face ventrale du corps.

En d'autres termes, c'est à l'existence de l'anus—et à la bifurcation du pli médian provoquée par lui—que nous devons les membres pairs.

* Par exemple, *Paralepis Rissoi.* C. Bellotti, "Appunti all' opera del dottor Emilio Moreau : Histoire naturelle des Poissons de la France e al relativo supplemento," *Atti della Società Italiana di Scienze naturali*, vol. xxxiii., fig. 3. Milan, 1890.

† L. Dollo, *Poissons de l'Expédition Antarctique Belge, etc.*, p. 106.

‡ Chez *Macrostomias longibarbatus*, où les ventrales ont cessé de fonctionner comme nageoires, pour devenir des organes tactiles. A. Brauer, "Diagnosen von neuen Tiefseefischen, welche von der Valdivia-Expedition gesammelt sind," *Zoologischer Anzeiger*, vol. xxv. p. 283, 1902 ; C. Chun, *Aus den Tiefen des Weltmeeres*, p. 537. Jéna, 1900.

Il en résulte que, *primitivement,* l'*anus* était placé *immédiatement devant l'anale,* qui venait buter contre lui.

2. Par conséquent, quand—comme chez les *Paralepidæ*—l'*anus* est *séparé* de l'*anale,* nous avons affaire à une disposition *secondaire*—soit que l'anale ait reculé, soit que l'anus ait avancé.

Par conséquent, aussi, la présence, chez *Prymnothonus,* d'une *longue frange embryonnaire* atteignant l'anus très antérieur—alors que l'anale définitive est très localisée et rejetée en arrière—ne peut être considérée que comme un cas de *Récapitulation.*

II. *Migrations de l'anus.*—Il y a donc eu des migrations de l'anus, chez les Téléostéens.

Phylogéniquement ? ontogéniquement ? ou les deux ? Cranio-caudalement ? caudo-cranialement ? ou les deux ? Quelle est l'amplitude maximum de ces déplacements ?

Le tableau suivant répond à ces questions :

1. *Phylogéniquement :*

Migration caudo-craniale :	*Rhamphosternarchus tamandua.** Anus mentonnier. Vie Benthique Limnique. *Gymnotidæ.* Ostariophysiens.
Migration cranio-caudale :	*Stylophthalmus paradoxus.* † Anus ultraterminal. Vie Planctique Abyssale. *Stomiatidæ.* Malacoptérygiens.

2. *Ontogéniquement :*

Migration caudo-craniale :	*Aphredoderus sayanus.* ‡ Anus jugulaire. Vie Nectique Limnique. *Aphredoderidæ.* Acanthoptérygiens perciformes.

* G. A. Boulenger, "On a Collection of Fishes from the Rio Jurua, Brazil," *Transactions of the Zoological Society of London*, vol. xiv. p. 427 et pl. xlii., 1898 ; "Teleostei (Systematic Part)," *Cambridge Natural History* (*Fishes, etc.*), vol. vii. p. 580. Londres, 1904.

† L'anus est rejeté au delà de l'extrémité postérieure du corps, au moyen d'une longue papille, qui s'insère au devant de la courte anale, refoulée elle-même contre la caudale. A. Brauer, *Valdivia, etc.*, p. 298 ; C. Chun, *Aus den Tiefen, etc.*, p. 535.

‡ L'anus, placé d'abord derrière les ventrales, émigre finalement en avant des pectorales. D. S. Jordan, *A Guide to the Study of Fishes*, vol. ii. p. 295. Westminster, 1905.

Migration cranio-caudale :	*Macrurus rudis.**
	Anus secondairement au devant de l'anale.
	Vie Benthique Abyssale.
	Macruridæ.
	Anacanthiniens.

III. *Conclusion.*—Ces migrations sont de nature à justifier les déplacements, phylogénique et ontogénique, invoqués pour faire rentrer *Prymnothonus* dans les *Paralepidæ.*

VII. Bionomie du Prymnothonus Hookeri.

I. "*Erebus*" *and* "*Terror.*"—On ne possède aucune donnée bionomique sur le *Prymnothonus Hookeri* de l'*Erebus* et de la *Terror.*

Vu l'habitat du *Prymnothonus D* du *Challenger,* nous avons fait remarquer que le premier aurait pu être capturé également au voisinage de la Terre de Wilkes, dont Ross s'est approché en Mars 1841.

Mais,—le numéro élevé (217) du dessin de Sir Joseph Dalton Hooker indiquant qu'il doit plutôt se rapporter à une observation de la fin de l'Expédition,—le *Prymnothonus Hookeri* de la *Scotia,* recueilli dans la Mer de Weddell, par 71° 50′ S. et 23° 30′ W.,—la pointe poussée par Ross, en Mars 1843, dans la même mer, par 71° 30′ S. et 15° W.,†—rendent, après tout, plus probable que le *Prymnothonus Hookeri* de l'*Erebus* et de la *Terror* provient de la *Mer de Weddell.*

II. "*Challenger*" *et* "*Scotia.*"—Comparons, maintenant, au point de vue bionomique, le *Prymnothonus D* dù *Challenger* et le *Prymnothonus Hookeri* de la *Scotia* :

	Prymnothonus D. ‡		Prymnothonus Hookeri.
	(I.) *Biogéographie.*		(I.) *Biogéographie.*
Habitat :	62° 26′ S. et 95° 44′ E.	*Habitat :*	71° 50′ S. et 23° 30′ W.
	S.E. Kerguelen.		Mer de Weddell.
	Océan Indien.		Océan Antarctique.
	Quadrant Australien.		Quadrant Américain.
	Station 156.		Station 414.
	Challenger.		*Scotia.*

* L'anus, placé d'abord entre les ventrales, émigre finalement au devant de l'anale. A. Günther, "Report on the Deep-Sea Fishes," *Voyage of H.M.S. Challenger during the years* 1873-76 : Zoology, vol. xxii. p. 132 et pl. xxvii. 1887.

† J. D. Hooker, *Summary, etc.*, p. x.

‡ J. Murray, "Summary of Results," *Voyage of H.M.S. Challenger during the years* 1873–76, vol. i. p. 501. 1895.

(II.) *Ethologie.*

1. *Profondeur.* — Entre 0 et 1975 fathoms.
2. *Nature du Fond.*—Vase à Diatomées.
3. *Température du Fond.*—31°·3 F.
4. *Température de la Surface.*—33° F.
5. *Mode de Capture.*—Chalut.
6. *Date de Capture.*—26 Février 1874.
7. *Heure de Capture.* — Entre 9.20 matin et 4.30 soir.
8. *Nombre d'Individus capturés.*—Un seul.

(II.) *Ethologie.*

1. *Profondeur.* — Entre 0 et 2378 mètres.
2. *Nature du Fond.*—Vase bleue (à 3800 mètres environ).
3. *Température du Fond.*—Inconnue.
4. *Température de la Surface.* — 29°·1 F.
5. *Mode de Capture.*—Filet vertical de $2^{m}\cdot50$ d'ouverture.
6. *Date de Capture.*—15 Mars 1904.
7. *Heure de Capture.*—3 heures du soir.
8. *Nombre d'Individus capturés.*—Trois, pris ensemble.

VIII. Conclusions.

1. La *Scotia* a retrouvé le *Prymnothonus Hookeri* de l'*Erebus* et de la *Terror* et en a rapporté des spécimens.

2. Le *Prymnothonus D* du *Challenger* paraît bien être un *Prymnothonus.*

3. Le genre *Prymnothonus* doit être classé dans les *Paralepidæ.*

(*Issued separately April* 5, 1907.)

VIII.—Craniometrical Observations on the Skull of Equus prjevalskii and other Horses. By O. Charnock Bradley, M.B., D.Sc., F.R.S.E.

(Read November 19, 1906. MS. received March 18, 1907.)

The peculiarities of the skull of the Prjevalsky horse (*Equus prjevalskii*) have been extensively studied by Salensky of St Petersburg, and compared by him with the skulls of various other members of the genus *Equus*. No special attempt, however, has been made to contrast the skull of the wild horse with that of different varieties of the domestic horse. There can be little doubt that given as rich a collection of Prjevalsky horse skulls as were at the disposal of Salensky, and given, at the same time, a representative collection of skulls of different types of the domestic horse, many points of great scientific interest would emerge from a comparison. In default of such a wealth of material, it is clearly desirable that opportunities afforded by even isolated specimens should be turned to as good an account as possible. In this conviction, Professor Cossar Ewart recently asked me to make measurements of the skull of a Prjevalsky horse, about three and a half years old, and compare them with similar measurements of the skulls of horses of the Celtic and Iceland or Forest types. The comparison proved of interest; and though the material was certainly not sufficiently abundant to justify any dogmatic conclusions, the differences in the three skulls are so striking as to merit record.

In the Prjevalsky horse the nasion was found to be placed relatively much farther forward than in the other skulls. This being so, it was decided to abandon this point as indicating the craniometric line of demarcation of cranium and face. On the suggestion of Professor Ewart, a line crossing the skull transversely on a level with the anterior border of the post-orbital process of the frontal bone was substituted, since the mesially sectioned skull showed that it fairly accurately indicated on the surface the level of the anterior limit of the actual cranial cavity. The mid-point of this transverse line may be referred to, for convenience, as the *inter-orbital point.*

The length of the cranium was measured from the opisthion to the inter-orbital point; and its breadth was estimated by two diameters. The transverse diameter of the cranium taken from the outermost part of the condyles on the squamous temporal bones (*condylar breadth*) is a measure-

ment of considerable value, because it lies between points which are superficial in the living animal. By comparing it with the length of the cranium, taken as 100, the *cephalic index A* of the subjoined table was computed. The condylar breadth, however, is not the breadth of the actual cranial box in the horse; nor was it altogether easy to determine upon a measurement which would accurately define the width of the cranium. Finally, however, a *maximum parietal breadth* between points on the parieto-squamosal suture was decided upon; and this was compared with the length of the cranium (=100) in a *cephalic index B.*

TABLE I.

Cranium.	*E. prjevalskii.*	Celtic.	Iceland.
Length, ...	170	171	184
Condylar breadth, ...	188	178	189
Maximum parietal breadth, . . .	87	90	93
Cephalic index A, ...	*110·58*	*104·09*	*102·71*
Cephalic index B, ...	*51·17*	*52·63*	*50·54*

From the above indices it is clear that the Prjevalsky cranium was much wider on a level with the temporo-mandibular articulation than either the Celtic or Iceland cranium (cephalic index A). The Iceland cranium was the narrowest; and the Celtic intermediate, but much nearer the Iceland than the Prjevalsky. Cephalic index B also demonstrates the relative narrowness of the Iceland cranium. It is of interest to note that, according to index B, the Prjevalsky cranium was slightly narrower, as compared with its length, than the Celtic, in spite of the fact that the width of the Prjevalsky skull measured between the temporo-mandibular articulations was much greater than that of the Celtic horse.

The measurements of the facial parts of the three skulls brought out certain distinctive features in a very definite manner, and corroborated conclusions which could be drawn from a mere inspection. The length of the face was measured from the inter-orbital point to the alveolar point, and its breadth between the most distant points on the sutures separating the malar and maxillary bones. A *facial index* was computed by comparing the breadth with the length taken as 100. In the living animal the width of the face, as determined by inspection, depends upon the transverse diameter between the two orbits. For this reason, it was decided to include a *frontal width* in the facial measurements, by determining the maximum distance between the post-orbital processes of the two frontal bones. A *frontal index* was then calculated in the same manner as the facial index.

A comparison of the figures in the following table with those in Table I. shows that the frontal breadth gives the greatest transverse measurement of the whole skull; or, in other words, that the skull of the horse is widest on a level with the orbits. At the same time it will be seen that the Prjevalsky skull differed from the other two in that the frontal breadth exceeded the condylar breadth by only 0·5 mm.: a point of some moment as demonstrating a peculiarity of the head behind the orbit.

TABLE II.

Face.	*E. prjevalskii.*	Celtic.	Iceland.
Length,	370	333	336
Breadth,	166	159	177
Facial index,	*44·86*	*47·74*	*52·67*
Frontal breadth,	188·5	192	206
Frontal index,	*50·94*	*57·65*	*61·30*
Cranio-facial length,	493	467	485
Cranio-facial index,	*75·05*	*71·30*	*69·27*

That the Iceland skull had a short, broad face is shown by the facial and frontal indices, as well as by the absolute measurements. The Prjevalsky horse, on the other hand, had a long, narrow face; the Celtic skull occupying an intermediate position.

In addition to determining the relationship between the length and breadth of the face, the proportion which the face formed of the length of the entire skull was also ascertained. This was done by measuring the distance from the opisthion to the alveolar point—*cranio-facial length*—and computing an index as follows:—

$$\textit{Cranio-facial index} = \frac{\text{Length of face} \times 100}{\text{Cranio-facial length}}.$$

In this way it was shown that the face of *Equus prjevalskii* was long in relation to the skull as a whole; that the Iceland horse had a short face; and that the Celtic horse stood between the two.

It might have been expected that the palate in the three horses would vary with the face. This, however, was not quite the case. The length-breadth or *palatine index* was certainly highest in the skull of the Iceland type, thus indicating a relatively broad palate in conformity with the broad face; but in the wild and Celtic horses the palatine indices are practically identical, in spite of the fact that the wild horse had relatively the longer face. The actual measurements are as follows:—

TABLE III.

Palate.	*E. prjevalskii.*	Celtic.	Iceland.
Length, ...	252	235	236
Breadth, ...	70	64	69
Palatine index, ...	*27·77*	*27·23*	*29·23*

The form and position of the orbit has notoriously much influence upon the appearance of the head of a living horse; even in a series of macerated skulls considerable differences can be detected on inspection. That there were differences in the orbits of the three skulls at present under consideration could be readily determined by the eye. The outline of the orbit of the Prjevalsky horse was much farther removed from a circle than was that of either the Celtic or Iceland skull; this is demonstrated by the indices in the following table, where the maximum diameter of the rim is held as being equal to 100. The peculiarity of the shape of the rim in *Equus prjevalskii* depended largely upon the position of the articulation between the post-orbital process of the frontal bone and the zygoma. That the articulation in this skull was farther back than in the Celtic and Iceland skulls may be shown in the following manner. The amount of the orbital rim formed by the squamous temporal bone is dependent on the position of the articulation. The farther forward the joint, the smaller the part taken by the squamous temporal in the formation of the orbit. In the wild horse the temporal formed 27 mm. of the rim; in the Celtic, only 11 mm.; and in the Iceland, 15 mm.

Not only did the shape of the orbit differ in the different skulls; its relative position was also different. The figures below show that in the wild horse the orbit was relatively farther back in the head than it was in the others.

TABLE IV.

Orbit.	*E. prjevalskii.*	Celtic.	Iceland.
Maximum diameter, ...	68	59	63
Minimum diameter, ...	55	56	61
Index, ...	*80·88*	*94·91*	*96·82*
1. Distance from the opisthion to the supra-orbital foramen, ...	182	180	189
2. Distance from the supra-orbital foramen to the alveolar point, . .	362	333	338
Index ((2)=100), ...	*50·27*	*54·05*	*55·91*

Some importance has been attached by Salensky to the relative width of the most anterior region of the face ("muzzle"). In order to compare the three skulls in this respect the maximum width of the premaxillæ (measured from the bases of the third incisor teeth) was determined, and, the length of the face being held as equal to 100, an index was calculated.

TABLE V.

	E. prjevalskii.	Celtic.	Iceland.
Width of premaxillæ, ...	66	64	70
Index (length of face = 100), . . .	*17·83*	*19·21*	*20·83*

It is clear that, in comparison with the length of the face as a whole, the wild horse had a narrow premaxillary region.

The chief points brought out by the above observations may be summarised as follows:—The actual width of the cranium of the wild horse was narrower than an examination of the animal during life would have led one to suppose. The face of the wild horse was long and narrow, and formed a considerable proportion of the total length of the skull. The Iceland face, on the contrary, was short and broad. The orbit of the wild horse had an elongated rim, and was placed relatively far back in the head. The premaxillary width in *Equus prjevalskii* was relatively less than in the other two skulls.

(*Issued separately April* 29, 1907.)

IX.—Notes on Aboriginals of the Northern Territory of South Australia.

By **W. Ramsay Smith**, D.Sc., M.B., C.M., Permanent Head of the Health Department, South Australia. (*Communicated by* Professor D. J. Cunningham, F.R.S.)

(MS. received January 7, 1907. Read same date.)

Whilst engaged recently in inquiring into diseases and sanitation in the Northern Territory of South Australia, I took occasion to investigate some points in the anthropology and ethnology of the aboriginals. This investigation broke some new ground, and produced results which I consider to be of sufficient interest to be recorded.

Physiognomy.

What impresses an observer on examining aboriginals of undoubtedly pure breed is the great variation to be observed in external facial character. This impression is all the stronger if one is familiar with Keane's statements regarding the uniformity in physical and mental characters of the inhabitants of the Australian continent, "in which a strong family likeness is at once detected between all the scattered groups of its primitive inhabitants." While there exists, no doubt, a large substratum of uniformity, too much must not be inferred from a casual examination; and one must be prepared to find a large amount of well-marked variation.

The examination of a number of Australian aboriginals, or even a number of photographs of aboriginals, will show that Australian heads will be found which show a facial resemblance to all the known recognised types, Ethiopian, Mongolian, Caucasian, American. It appears to me that in this respect there is some analogy between the Australian aboriginals and the Australian fauna. The primitive marsupials, distinguished by certain features, such as epipubic bones, inflected angle of the lower jaw, "aplacentation," double uterus, etc., having been cut off at an early period from competition with nearly all other classes of mammals, have developed along lines similar to those along which other forms have specialised, and now mimic other classes of animals, *e.g.*, carnivora, insectivora, rodents, etc. The Australian aboriginals have to some small degree undergone similar development, and now mimic in facial expression the four primary groups of Hominidæ as well as many intermediate forms. I speak generally, not in strictly scientific language nor in detail. Further, I would emphasise the

fact that no explanation is proffered as to how this evolution has occurred in either case. When Professor Klaatsch of Heidelberg was in Adelaide about

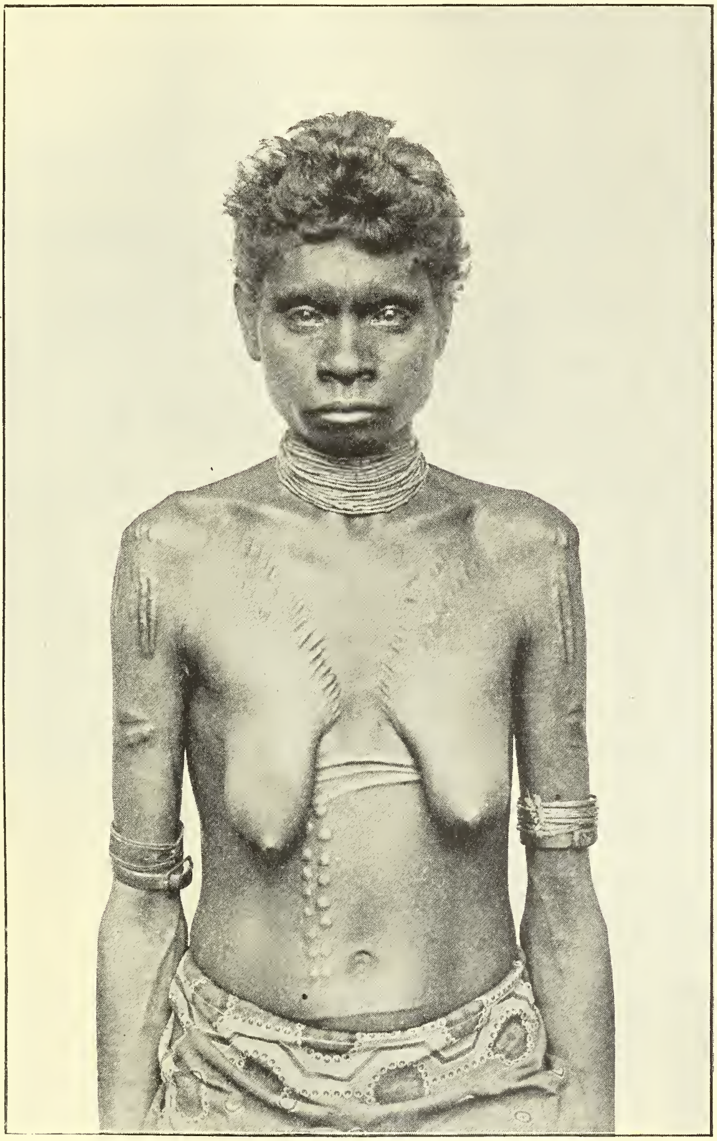
Alligator Tribe. (Photograph by Mr Foelsche.)

a year ago, Dr Rogers showed him a large number of photographs illustrating these variations of type, and presented him with some illustrative examples.

Body Markings.

Towards the end of 1903 an English gentleman, Captain Hayes, wrote to me regarding "recognition marks," which, he said, "our Society people

and costermongers have 'galore' and which the Hindoos of the Madras Presidency sport in the form of painted lines on their foreheads, so that



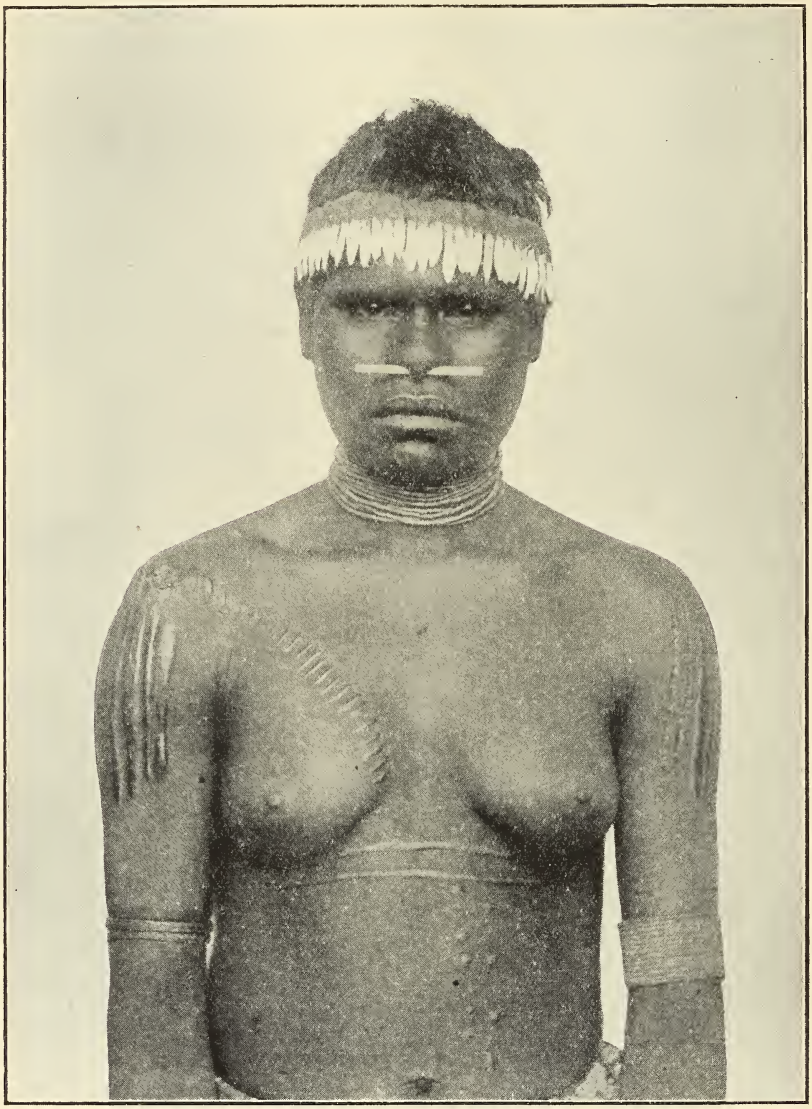
Alligator Tribe. (Photograph by Mr Foelsche.)

they can recognise the respective members of their own castes." He asked me if the Australian blacks had any recognition marks.

In connection with this subject of recognition markings I made many inquiries in various parts of the State, but I could obtain no satisfactory

information regarding it; nor could I find any key to the meaning of the scars on the bodies of aboriginals.

One day when talking with an old lubra and her daughter near Port



Alligator Tribe. (Photograph by Mr Foelsche.)

Darwin, I gathered that there was some connection between the marks across the upper part of the abdomen and the number of children they had borne—four and one respectively. I tested this information on other blacks and found confirmation of it. Mr Foelsche, who is one of the best informed and most exact of all observers of aboriginals, told me that he

knew that certain markings on a woman's back betokened widowhood; and when he showed me his magnificent set of photographs, on which he had spent many years of work, and said I might select what I pleased, I chose about a score of specimens illustrating markings on women. These accompany this paper. A few days afterwards an old blackfellow, "Ned,"

Larrakeyah Tribe. (Photograph by Mr Foelsche.)

called and brought me a message; and while talking with him I dropped the collection of photographs on the ground, and he helped to pick them up. He could not conceal his great interest in the pictures showing the women with all their charms as he had known them in his young days; and I rewarded him by exhibiting the photographs one after another, and he read off for me the significance of the marks, after this fashion: "Three fella picanniny, one fella boy, two fella girl: four fella picanniny, two fella boy

die, one fella girl die." On asking him the meaning of a line of short incisions from the shoulder inwards and downwards towards the lower end of the breast bone, he said, "Sister die." When we came to the widow's photograph, he shook his head and looked sorrowful. I said, "Where her blackfella sit down?" He said, "She no got blackfella." I said, "She no got Benjamin?" and he replied, "No more Benjamin."

In January, 1901, I sent a collection of photographs of aboriginals to Sir William Turner, which he considered to be of considerable interest. He said he had not previously seen a series to illustrate the raised scars, and added that it was interesting to see that the custom is not restricted to the male sex. There is no doubt that little is known regarding the significance of scarring.

Have the markings in men a similar significance? I cannot say: I had not sufficient opportunity of investigating this aspect of the question. I found, however, that in some tribes the first marks are made on the shoulders and thighs, and the last across the body at the level of the sternum. I was told the Larrakeyahs do not cut on the chest; that the Victorias do, and also the Borroloolas. I asked, "What for cut?" and was told, "Suppose it looks pretty: don't know." There is something in the "pretty" theory: I have noticed boys of from six to ten years old painted with the first markings. In some tribes, if a man dies, some blackfellow cuts the wife and the brother of the deceased (the latter across the abdomen and shoulder), and the brother cannot marry that lubra.

As regards the scars made, these are pathological curiosities, since many of the most outstanding are deeply pigmented. It is often stated that earth is rubbed into the fresh cuts, and that they are kept open from time to time by this process or by other means. I found no confirmation of this; on the contrary, I was informed that nothing is rubbed in, but that the incisions are covered with a leaf to keep away flies.

One cannot reason from analogy that similar markings in men and women have similar meanings. Even among white people, a ring may mean marriage or engagement in a woman, and in a man vanity or rheumatism.

Dentition.

Some years ago, Sir William Turner, when examining an aboriginal skull from South Australia, noticed that the incisor teeth were in contact by their cutting edges when the condyles of the lower jaw were articulated, and placed in contact with the ridge that bounds the back of the glenoid fossa, and the teeth clenched. He had also noticed this in a Malay, a Bushman,

and an Eskimo.* Sir John Lubbock had observed the same thing in prehistoric Danish skulls, indicating, as he said, a peculiar manner of eating. In his recent work, in which the Australian skulls in the Anthropological Museum at Cambridge are described, Duckworth discusses this question, like all other writers, from the point of view of dried skulls, and finds the evidence somewhat conflicting.

It does not seem to have struck any observer to make an examination, or a series of examinations, of the living subject, although one would have thought this essential to the proper investigation of the subject. Having read the latest contributions on this matter while in the territory, I made a point of examining a large number of aboriginals of various tribes and a number and variety of half-castes. The following is the result:—

In many tribes I examined, I found that nearly all the natives bite "flush." A few, however, overlap with the upper jaw, and a few with the lower—a native of Port Keats had the lower slightly overlapping. A native of Borroloola had the upper overlapping, but he was said to have a mixture of Macassar blood. A woman from Brock's Creek had the upper very much overlapping, but the lower was so small as to appear to be deformed. I examined three Port Essington "boys." The first had a European father and an aboriginal mother; the second a West Indian father and an aboriginal mother; the third a European father and an aboriginal mother. All three "bite flush." There is said to be a good deal of Malay blood among the natives there as elsewhere on the coast.

A half-caste woman whom I examined bit flush, and her child by a half-caste father also bit flush. I found a father (Larrakeyah tribe) with projecting upper jaw, while his son had the incisors flush.

The occurrence of supernumerary teeth calls for some remarks. Duckworth, in his work on *Morphology and Anthropology* (1904), says: "Completely-formed accessory molar teeth are not common in the Hominidæ, although the palate and alveolar arcade in many crania of the aboriginals of Australia seem to be spacious enough to accommodate them. It is, however, in the cranium of such an aboriginal native that Sir William Turner records the occurrence of no less than three accessory molar teeth, and such anomalies are more frequent in the Melanesian and Australian aborigines than in other Hominidæ."

The skull referred to, which was described by Sir William Turner in the *Journal of Anatomy and Physiology*, vol. xxxiv., was a skull from Morambro Station in this State, which I was fortunate to obtain and to present to the Edinburgh University Museum. The other day, on looking

* *Journal of Anat. and Phys.*, vol. xxv. p. 461, 1891.

over Dr Rogers's collection, I saw a broken skull of an aboriginal which he was good enough to give me. It was found by him about eight miles from Adelaide. It shows an extra molar on the left side which, though of full size, has not erupted. On the right side, about the level of the third molar, there is a supernumerary molar of diminished size, possessing four cusps, and projecting through the facial surface of the superior maxilla.

These are not the only abnormalities of aboriginal dentition that I have lately observed. Several skulls show very marked examples of more or less complete division of the root of the first bicuspid tooth; and one skull from the Northern Territory shows that this tooth on the left side had a complete inner or lingual root and a grooved outer or labial root, while the right one had two separate roots. The general fact to be noted is that subdivision of the roots of the bicuspids is almost invariably more marked in the first tooth than in the second; but books on anatomy, with a very few exceptions, state the contrary.

In connection with this subject of dentition among the lower races, I may refer to a skull which I recently received from New Caledonia. In the upper jaw the "bicuspids" have the characters of permanent molars as regards size, roots, and wearing of the enamel. They show, however, the bulging above the neck which is characteristic of milk molars; and their true temporary nature is proved by finding the permanent molars exposed on trephining the jaw in the region above them. In the lower jaw the first bicuspids are normal; the second have the characters of permanent molars as in the upper jaw, but their true temporary nature is also determined by the discovery of the imprisoned permanent molars after trephining. From extensive inquiries made I believe the conditions exhibited by this skull are altogether unique. I hope that Professor Cunningham will be able to describe these and other specimens in greater detail by and by.

Speaking of teeth leads naturally to some observations on the mouth. Professor Cunningham, when writing to me about a specimen I had sent from Professor Watson, said, "I notice in the tongue that patches of the mucous membrane are deeply pigmented in the lymphoid region. Is this common among natives?" This question of pigmentation also arises in connection with malaria. Lofton has described black patches in the tongue which he considers to be diagnostic of malaria. I took an opportunity of examining several blacks, and found that pigmentation of the tongue and mouth is not uncommon. One good-looking girl, "Rosie," the belle and the flirt of the tribe, showed large black patches on each side of the tongue,

their long axis being transverse to the margin; and she had numerous large black spots inside the lips and cheek.

A male aboriginal from Brock's Creek presented a pigmented appearance of the gums near the incisor teeth. This was found to be due to varicose veins; and a very peculiar appearance of the nose was found to be due to the same cause.

Subincision.

Circumcision is practised by some tribes but not by others though living in close proximity. The mode of circumcision and the details associated with it differ in the various tribes that practise the rite. Among the Port Keats natives, the king performs the ceremony; the prepuce is nicked all round and removed. It is then put into a shell, which is closed up by gum, and this is hung alongside the penis until the wound is healed.

For several years I had been puzzled regarding some points connected with the operation of subincision. About eight years ago I saw an Adelaide specimen subincised to the extent of an inch. This I sent to Sir William Turner. Recently Professor Watson gave me another specimen, from Central Australia, which I presented to the Anthropological Museum of Edinburgh University in his name. Both of these, so far as appearances go, might have been specimens of natural partial hypospadias. I saw, however, in a deeply scarred man from Borroloola in the Northern Territory, a partial incision which terminated posteriorly to the right of the urethra, and presented an appearance quite different from that of a natural hypospadias. An old blackfellow gave me the explanation of this, so far as that tribe was concerned. The first operation performed on a young man is circumcision. A year or so after, a partial subincision is made; and the next year the subincision is made complete.

Mutilation of the Finger.

The custom of mutilating the finger of girls in some tribes is well known. I have never seen reference to any but the index finger of the right hand. When speaking with Mr Foelsche on this matter, I remarked that I had seen the left finger mutilated. He could not recollect ever having seen or heard of this, and thought I must be mistaken or that the occurrence was accidental. In order to make sure I retraced my steps, and in a short time found three instances, which I photographed. An old native called William gave me some information about this peculiarity. He said that when the cutting is done at once, it is the right hand finger that is cut; but when

one week or more is allowed to pass, then they cut the left. Cutting in the young is done by means of a mussel shell. In the grown up, amputation is performed by means of a spider's web ligature. This is put round the finger and tightened at intervals so as to eat into the joint, which it does in the course of five or six days. After a time the piece of the finger drops off. If it does not drop off, the finger sometimes remains bent.

I had been informed by whites that the finger was bent strongly and the ligature applied to it in that position, and that ulceration began in the skin over the projecting part; but I could obtain no confirmation of this from the blacks. I have also heard it stated that the mutilation is inflicted only at marriage. Of this I found no confirmation from the aboriginals whom I questioned.

At Brock's Creek I saw a black youth who had lost the terminal phalanx of his right forefinger, and who had a deep constriction on the middle phalanx. On inquiry I found that this was due to the lad having put on an iron ring so tightly that he could not get it off, and the result was as stated.

The Foot of the Aboriginal.

In 1893 I had occasion to make a series of investigations into the structure and functions of the lower extremity of the human subject. The results I embodied in a thesis which was not published at that time. Some statements in it were suggested by certain strictures made by Mr Ellis of Gloucester upon artistic work, or, as he would probably have preferred to term it, artists' work.

In dealing with the action of the toes I remarked that the use of the great toe is taken advantage of in many actions, involving special muscular effort or special dexterity in maintaining equilibrium, or in certain actions of the foot that have a resemblance to grasping. This is often illustrated in art, as in Leighton's "Athlete struggling with a Python," and Myron's "Discobolus." The same thing is noticed in literature: "He sat with his foot bent down, and the nails set into the ground to give him foothold, even as a bird turns its claws inwards as it sits on a branch" (S. R. Crockett: *Mad Sir Uchtred of the Hills*, chap. ix.). I went on to say: "In making these statements I am fully aware of Ellis's criticism that such positions, if not 'impossible,' are 'unnatural' and 'bad art.' But such criticism is the outcome of a particular way of looking at the study of function. Some writers, studying the structure of the foot, cannot free themselves quite from the trammels of long habit, and yield to the temptation to deduce function from structure instead of from action. So confident

are they of the correctness of their method and the truth of their results that they do not hesitate to lay down the law regarding what careful observers ought to see. Artists, who follow the established yet ever re-tested representations of the human body as it appears at rest or in action, are taken to task for representing the unnatural and impossible, and are excused by those writers only on the ground that they have formed bad ideals and employed bad models. I confess I have more faith in the results of a few thousand years of careful observation by artists of what *is* than in any deduction of what *ought to be*."

On my recent journeyings amongst the Filipinos, my attention was directed to the dexterity of some of the native races with their feet. Afterwards when examining the blacks in the Northern Territory I was struck with the great width of the fore part of the foot in many natives, the looseness of the great toe, and the power of grasping which was exhibited. Most natives are as ready with a foot as with a hand. A man or woman when talking will drop a pipe; immediately the toes close on it, and with a quick movement it is passed up behind the other leg into the hand which is hanging down at the outside of the opposite thigh.

No more complete refutation of Ellis's views could well be afforded than by this condition of the Northern Territory blackfellow's foot, which shows only a difference in degree in a structure that is present, and of action that is possible, in the foot of the white subject.

Since my return, and since lecturing on the subject to the Field Naturalists' Society at the University here, I see a report that Professor Klaatsch has been studying in detail what he regards as an extreme instance in an aboriginal among the prisoners from Port Keats in the gaol at Port Darwin whom I had examined and photographed.

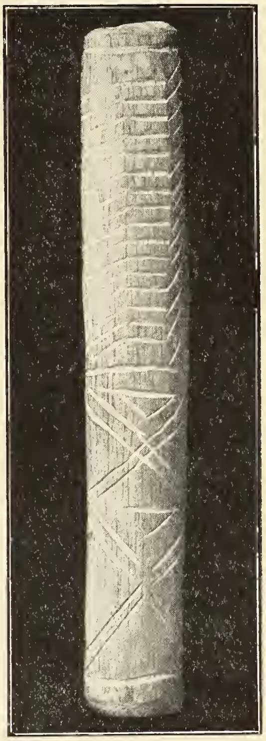
Too much stress must not be laid on the occurrence of a single case of this sort when discussing the affinities of the Hominidæ; but should large numbers or whole tribes be found similarly constituted, then the case is different.

Writing.

In the Palmerston Hospital during my visit there was a black boy, named Spider, suffering from "ulcerating granuloma pudendi." One day the nurse found him carving a small piece of wood, and asked him what he was doing. He said he was writing a letter to send to his father. She asked what he had said, and he replied that he had written that he was sorry to say he was no better. She told him she thought he was somewhat better; so he took another stick and began to carve a message that "Missie"

thought he was a little better. The nurse was good enough to procure the first letter for me (see accompanying figures). One day I happened to be talking with a very intelligent blackfellow, and I asked him, "You savee writing?" He said, "Yes: me savee writing." I handed him the stick and said, "You savee this one writing?" He took it in his hand, looked at it carefully on both sides, seemed disappointed, and said at length, "Me no savee Borroloola writing." He was right. I found that Spider, the writer of the letter, was a Borroloola boy. I then asked him about the kind of

Message-stick—Aboriginal "Yabber-stick," flat side.



"Yabber-stick," rounded side.

writing to be found in his tribe; and he told me the only sort they recognised was a message to the effect to come along because there was to be a big fight. He said this communication consisted of a small piece of bamboo with a thread tied round and then passed through it. There would appear to be considerable differences, even among very nearly adjacent tribes, in literary advancement, so far as writing is concerned.

Was this letter from Spider, like some of the smoke signals the aboriginals use, merely a preconcerted sign, or can some of these tribes arrange and combine elementary characters or signs so as to convey complex messages?

"Singing Dead."

The custom of pointing the death-bone is well known. The thing pointed is not always a bone: in some places a piece of wood is used. In parts of the Territory, if the victim does not die within a reasonable time, his tribe fellows gather together, sit round him, and "sing him dead." This peculiar custom seems to be little known.

What strikes one forcibly when moving among these blacks is their restlessness. They shift camp on the least provocation, or for no cause that one can discern. When you ask where a certain blackfellow is, you are told, "Him wuk alonga Mr So-and-so's." You don't know whether the word is "walk" or "work," but it amounts to pretty much the same thing. The blacks will work for a day, a week, a month, so steadily that you think they have become domesticated, or at least domiciled; then one day they are missing, and nothing will induce them to return, unless they are allowed to take a "spell" of rest or wandering when the symptoms of restlessness begin to manifest themselves. This restlessness is characteristic of all Australian blacks, and stands out in strong contrast to most South Sea Islanders, who are examples of extreme stayers at home. This is probably the one universal character among Australian aboriginals. As regards local characters and customs, the student cannot be too careful. Anthropology has suffered much from assuming that what is true of one tribe is true of another, especially if it has lived in close proximity. It is a safe rule in dealing with aboriginals never to reason from analogy, but to write down every fact, with the informant's name and the exact locality or tribe to which it refers, and then to verify and reverify it. One will generally be right in affirming and wrong in denying.

The blackfellow is difficult to understand. You think you know him, but you don't. You believe you have got the better of a blackfellow when you have forced him to work and hear him going about it, singing in his native tongue. But the song is composed of all the curse-words he knows, and a history, accurate or hypothetical, but in any case not creditable, of you and your ancestors. The more we know of the blackfellow the more we are convinced that there are whole subterranean rivers of anthropology unmapped and untapped.

(*Issued separately April* 29, 1907.)

X.—The Functions of the Rolandic Cortex in Monkeys. By W. A. Jolly, M.B., and Sutherland Simpson, M.D., D.Sc. (*From the Physiological Laboratory, University of Edinburgh.*) *Communicated by* Professor E. A. Schäfer, F.R.S.

(MS. received March 1, 1907. Read March 18, 1907.)

The Rolandic cortex is discussed in the following paper, and an account is given of experiments on monkeys, performed with the object of mapping out this region more accurately and of delimiting in it the areas concerned with movements of different parts of the body. The method employed is a new one, and will be described in detail.

The theory that the functions of the brain reside in distinct areas in its substance was advanced in early times. Proof that this theory is correct has only been obtained comparatively recently.

It was known to the Greek physicians that each hemisphere of the brain presides functionally over the opposite side of the body. This was evident to them from the situation of paralysis and the nature of convulsions which follow upon injuries to a hemisphere. Further, they believed, from the study of monoplegias, that the movements of different groups of muscles have their source in different regions of the brain.

The sensibility of the cerebral cortex to excitation was not known to the Greeks, and remained unknown until our own time.

During mediæval times localisation of function in distinct parts of the brain was accepted as a fact on the authority of the Greeks, and support was lent to the belief by clinical observations on the effects of injuries in producing loss of speech or of memory.

Willis, in the year 1682, writing in terms of the theory current at that time, that namely of the production and distribution of animal spirits, located in the cortex of the brain the function of initiating voluntary muscular movements, and regarded the central part of the cerebrum, the spinal cord, and the nerves as the pathway through which the cortex exercises its function upon the muscles.

Attempts which were made in the dawn of the era of physiological experiment to elucidate the facts were unsuccessful.

Haller (1776) came to the conclusion that the function of producing

muscular movements does not lie in the cortex cerebri but in the deeper parts of the brain.*

Rolando,† the pioneer in electrical stimulation of the brain, although he obtained violent muscular contractions by the passage of a galvanic current, where one electrode was introduced into the substance of the hemisphere, was unable, owing to the imperfection of his methods, to reach definite conclusions.

Among more modern experimenters, Bouillaud ‡ described experiments which rendered it probable, in his opinion, that cerebral functions are located in distinct areas. Flourens,§ on the other hand, whose method of investigation consisted in removing portions of the cerebral lobes, was unable to obtain any evidence of localisation of function, and concluded that the cerebral lobes co-operate as a whole in the full and complete exercise of their functions.

The experiments of Flourens and others were held generally to have disproved the doctrine of localisation of function, and with regard to motor localisation in the cortex, Broca expressed the accepted opinion prior to 1870 in the words: "Nul n'ignore que les circonvolutions cérébrales ne sont pas des organes moteurs." || The many attempts ¶ which had been made to obtain muscular movements in response to irritation of the cerebral hemispheres, had been unsuccessful.

Little better success attended the efforts made before 1870 to establish the theory of cortical localisation by means of pathological evidence. Hughlings Jackson ** and Bastian, †† from a consideration of cases of epilepsy and paralysis, adduced evidence in its favour. Hughlings Jackson found in epilepsy a definite succession or "march" of movements of the limb muscles. Thus he determined that when a movement begins in the digits of the hand, it affects successively the wrist,

* "Non ergo videtur plenam perfectamque causam motus musculosi in cortice cerebri habitare: cum præterea plurima experimenta demonstrent, profundo demum loco et a cortice cerebri valde remoto medullam lædi oportere, ut convulsio superveniat."—*Elementa Physiologiæ*, tom. iv. lib. x. sec. viii. § 23, Naples, 1776.

† *Saggio sopra la vera struttura del cervello e sopra le funzioni del sistema nervoso*, 1809.

‡ *Journal de Physiol. expérim.*, 1830, t. x. p. 91.

§ *Recherches expérimentales sur les propriétés et les fonctions du système nerveux dans les animaux vertébrés*, Paris, 1842.

|| *Bulletins de la Soc. anatomique de Paris*, 1861, 2e série, tome vi. p. 355.

¶ Budge, *Untersuchungen über das Nervensystem*, Frankfurt, 1842.
Longet, *Anat. et Physiol. du système nerveux*, Paris, 1842.
Matteucci, *Traité des phénomènes électrophysiologiques*, Paris, 1843.
Weber, *Wagner's Handwörterbuch der Physiologie*, Bd. iii.
Schiff, *Lehrbuch d. Physiologie*, 1858.

** *London Hosp. Rep.*, 1864, vol. i. p. 459.

†† *Journ. Ment. Sc.*, London, 1869.

elbow, and shoulder, while on the other hand, when the shoulder movement takes place first, the march proceeds downwards towards the digits.

Broca,* by means of clinical and pathological observations in cases of aphasia, localised the function of producing articulate speech in the posterior part of the third frontal convolution of the left hemisphere. Doubt has recently been cast upon the correctness of this localisation by the observations of Marie,† and the question may, in view of his work, be regarded as still open.

To Fritsch and Hitzig ‡ is due the establishment, upon a basis of experimental evidence, of the doctrine of localisation of function within the cerebral cortex. To their work also we owe our knowledge that the cortex is excitable by electrical stimuli. These observers, who worked on the dog, proved that an area of the cortex in the neighbourhood of the crucial sulcus might be mapped out by electrical stimulation from the rest of the brain. They proved that this area in one hemisphere governs the muscular movements of the opposite side of the body, and that the movements of different groups of muscles have their source in motor centres lying separate from each other within the excitable area. Further, they found that when the centre whose excitation elicited definite movements is extirpated, the voluntary production of these movements is impaired.

Fritsch and Hitzig employed the galvanic current, finding that they obtained less constant results by the use of the faradic current. The latter method of stimulation has, however, been shown to be preferable for accurate work, and was the form used by Ferrier. Ferrier § confirmed the results of Fritsch and Hitzig on the dog, and applied the method of electrical stimulation to the brain of the monkey and other animals.

According to Ferrier, movements of the limbs and face can be obtained by stimulation of an area on the convexity of the hemisphere comprising the ascending frontal and parietal gyri and part of the angular gyrus. Within this region lay centres which he represented as areas roughly circular in outline, stimulation of which produces definite movements of groups of muscles. In front of this region Ferrier described an area bounded anteriorly by a line drawn at right angles to the anterior extremity of the precentral sulcus and posteriorly by a continuation of the precentral sulcus upwards to the longitudinal fissure, stimulation of which causes

* *Loc. cit.*, p. 330. † *La Semaine médicale*, 1906.

‡ "Ueber die elektrische Erregbarkeit des Grosshirns," *Arch. f. Anat.*, *Physiol.*, *u. wissensch. Med.*, 1870, S. 300.

§ *Proc. Roy. Soc.*, London, 1874, vol. xxii. p. 229; 1875, vol. xxiii. p. 409.

opening of the eyes, dilation of the pupils, and movements of the head and eyes to the opposite side.

Subsequent to Ferrier's work, Hitzig* also investigated the monkey's brain by electrical stimulation. He differed from Ferrier in that he restricted the area governing the movements of limbs and face to the ascending frontal convolution. Within this gyrus Hitzig described motor centres for the lower and upper extremities, the face and the mouth, tongue and jaws. The position of these centres was only approximately correct. Thus Hitzig placed a centre concerned with movements of the ear and closure of the eyes above the inferior genu of the Rolandic fissure, and the highest point of the precentral fissure in an area of the cortex which was afterwards clearly shown by Horsley and Schäfer † to be embraced in the centre for the upper extremity.

Ferrier's results were confirmed by, among others, Luciani and Tambourini,‡ who came to the conclusion that the excitable areas for the limbs and face in monkeys are not limited to the ascending frontal convolution, as maintained by Hitzig, but that Ferrier was right in considering that they extend also to the ascending parietal and angular gyri. This view of Ferrier, that the motor areas extend behind the fissure of Rolando, which, as we now know, does not hold good, was generally accepted and greatly influenced subsequent work.

The next advance in motor localisation in the monkey was made by Horsley and Schäfer,§ who established the fact that the excitable area extends to the mesial face of the hemisphere. Ferrier had found in one case that irritation of the fronto-parietal portion of the marginal gyrus gave rise to muscular movements. Horsley and Schäfer, using interrupted induction shocks with the coil fitted with Helmholtz's side wire, showed that minimal stimuli elicited from part of the convolution "contraction of perfectly definite groups of muscles, or in some cases of single muscles, producing more or less co-ordinated movements," thus proving that the marginal gyrus contributes to the true motor area. They supported their conclusion by extirpating the excitable area in this convolution. The outstanding result of this experiment was distinct paralysis of the leg. The animal was able to assume and maintain a nearly normal attitude and to employ its arm for the purpose of progression.

The motor areas in the ascending frontal convolution were also for the first time correctly delimited by the same observers. They showed that

* *Untersuchungen über das Gehirn*, Berlin, 1874. † *Phil. Trans.*, 1887.

‡ *Ricerche sperimentali sui centri psico-motori corticali*, 1878. See also *Brain*, London, 1879, vol. i. p. 529.

§ *Proc. Roy. Soc.*, London, 1884, vol. xxxvi. p. 437. *Phil. Trans.*, 1887.

the boundaries between these areas could be represented by lines drawn antero-posteriorly across the ascending frontal convolution, the leg area having its lower boundary on the convex surface of the hemisphere in a line drawn from the posterior termination of a small sulcus which lies on the upper portion of the frontal lobe in the monkey, and which had been described and designated X by Schäfer,* to the fissure of Rolando. Within the arm area Horsley and Schäfer showed that the order of representation from above downwards was shoulder, forearm, wrist, and fingers. Within the face area which they showed comprised centres not only for facial muscles, but also for the whole of the upper part of the alimentary tube, they demonstrated that stimulation at its upper extremity produced movements of the upper face muscles, the eyelids and alæ nasi moving when the electrodes were applied in this position. Ferrier had taken the view that retraction of the angle of the mouth was the characteristic response of the upper extremity of the face area. Close to the margin above the sulcus X a small area was delimited from which, and from the adjacent cortex on the mesial aspect, movements of the tail and trunk could be obtained. This centre, which the authors denominated the trunk area, will be more particularly referred to later.

In the frontal lobe the authors described a region which they termed the head area or area for visual direction. It extended from the margin of the hemisphere, round which it dipped for a short distance, outwards and somewhat backwards to the upper and anterior limit of the face area. Posteriorly it was bounded by the arm area and in front by the non-excitable portion of the lobe. Excitation of this area caused movements of the head, and from a part of it, as Ferrier had shown, opening of the eyes, dilation of the pupils, and turning of the head to the opposite side with conjugate deviation of the eyes to that side.

The localisation of definite movements within the areas was further studied by Beevor and Horsley,† who found that the succession of movements in the joints was similar to that described by Hughlings Jackson as occurring in epilepsy. In the earlier part of their work these authors assumed a very extensive motor representation of the thumb, considering that thumb movements could be obtained by stimulation over a large portion of the face area. Later they represented this centre as restricted to the lower part of the arm area.

* *Journal of Physiology*, vol. iv. p. 316. This sulcus is considered by Cunningham to be in all probability a representative of the sulcus precentralis superior or of the basal part of the first frontal sulcus (*Cunningham Memoirs*, Royal Irish Academy, 1892, p. 281).

† *Phil. Trans.*, 1887, 1888, and 1894.

Beevor and Horsley * extended their investigations to the brain of the anthropoid. They came to the conclusion that the excitable area in the cortex of the orang-utan—the animal employed by them—was less in extent than had been believed to be the case in the lower ape, and that it was much interrupted by spaces from which no effect could be obtained even by the application of strong stimuli. The upper third of the ascending parietal convolution, and the part of the paracentral lobule behind a continuation over the margin of the upper end of the Rolandic fissure, they found entirely or almost entirely inexcitable. The lower portion of the ascending parietal convolution they considered to form part of the motor area. In the frontal lobe the only excitable area found by them lay immediately in front of the precentral sulcus, and its stimulation caused turning of the eyes to the opposite side.

Grünbaum and Sherrington † investigated the cerebral cortex in the orang, gorilla, and two species of chimpanzee (*Troglodytes niger* and *Troglodytes calvus*). The method adopted was that which Sherrington had used ‡ in his work on the cortex of the lower apes, viz., unipolar faradisation. These observers found in these animals that the motor area is continuous, that it embraces the whole length of the ascending frontal convolution, extending above over the margin to include part of the mesial surface of the hemisphere, stopping short, however, of the calloso-marginal fissure, and that its posterior boundary lies within the Rolandic fissure. In no case did the motor area extend into the ascending parietal convolution, although occasionally the posterior wall of the fissure of Rolando was found to be excitable. It was found impossible to obtain muscular movements by stimulation of the ascending parietal convolution when the ascending frontal convolution had been destroyed.§

Within the motor area it was found possible to localise, among others, movements of the nostril, palate, chest wall, anal and vaginal orifices. Inhibition effects were also obtained. The anterior boundary of the motor area could not be represented as a definite sharp line, but the excitable area was found to fade off gradually in an anterior direction. Conjugate deviation of the eyeballs was caused by stimulation of a region in the frontal lobe separated from the Rolandic motor area by a field of inexcitable cortex. The reactions of this region differed markedly from those of the Rolandic motor cortex, and it was placed by the authors in a different category.

* *Phil. Trans.*, 1890.
† *Proc. Roy. Soc.*, London, 1901, vol. lxix. p. 206.
‡ *Ibid.*, 1893, vol. lii.
§ *Ibid.*, 1903, vol. lxxii. p. 152.

Their work on the anthropoid brain led Grünbaum and Sherrington to emphasise what had been pointed out by Schäfer,* that the fissures in the neighbourhood of the Rolandic area, from the inconstancy of their position, form unreliable guides to functional regions. They excepted from this rule, however, the two genua of the Rolandic fissure. The upper genu, termed by Sherrington the cruro-brachial,† was found to lie opposite the space intervening between leg and arm areas and the lower or brachio-facial between arm and face areas.

Subsequent to these observations upon anthropoids, Rothmann ‡ described experiments which led him to the conclusion that in Macacus the motor area extends to the ascending parietal convolution. He found that limb movements followed stimulation thereof with weak currents, and that in some cases movements of the thumb were produced more readily by stimulation behind the fissure of Rolando than in front. He further found that, after destruction of the excitable precentral cortex, stimulation of the parietal convolution still produces movements of the thumb, fingers, and forearm.

The views of this author were opposed by Brodmann,§ and are not supported by the recent work of Vogt nor by our own results.

In view of the results obtained by Grünbaum and Sherrington in the anthropoid, it appeared to us advisable to attempt more accurately to delimit the cortical motor centres in monkeys. We desired to determine whether the movements which can be obtained in these animals by stimulation of the ascending parietal and other convolutions found to be non-motor in the anthropoid, warrant the inclusion of these convolutions in the motor area, or whether they are to be explained as due to a spread of current to the adjacent excitable cortex. With this object we made a number of experiments on monkeys, using species of Macacus and Callithrix. The method adopted, which was suggested to us by Professor Schäfer, was as follows:—The animals were anæsthetised throughout the experiment with ether. A skin flap was turned down and the skull trephined over the fissure of Rolando on the left side. The opening was then enlarged with bone forceps, the dura incised, and the convolutions exposed. After exploration of the Rolandic area by means of unipolar faradisation, in which we employed an electrode kindly given to us by Professor Sherrington, we determined the minimum strength of stimulus which produced muscular movements on application of the electrode to either side of the fissure of

* *Festschrift zu Karl Ludwig*, Leipzig, 1887.

† *The Integrative Action of the Nervous System*, London, 1906.

‡ *Neurologisches Centralblatt*, 1904, No. 14, p. 668.

§ *Ibid.*, p. 669.

Rolando. A thin plate of vulcanite, one edge of which had been sharpened, was then inserted into the cortical substance at the bottom of the fissure of Rolando. Stimulation was applied to the posterior wall of this fissure and to the ascending parietal convolution. No response was now obtained from this part of the cortex, while application of the electrode to the part of the ascending frontal convolution in front of the vulcanite plate was followed as before by movements of muscles. The isolating plate was inserted to a depth which would permit of its dividing the grey matter without penetrating the underlying white substance. In some of these experiments we

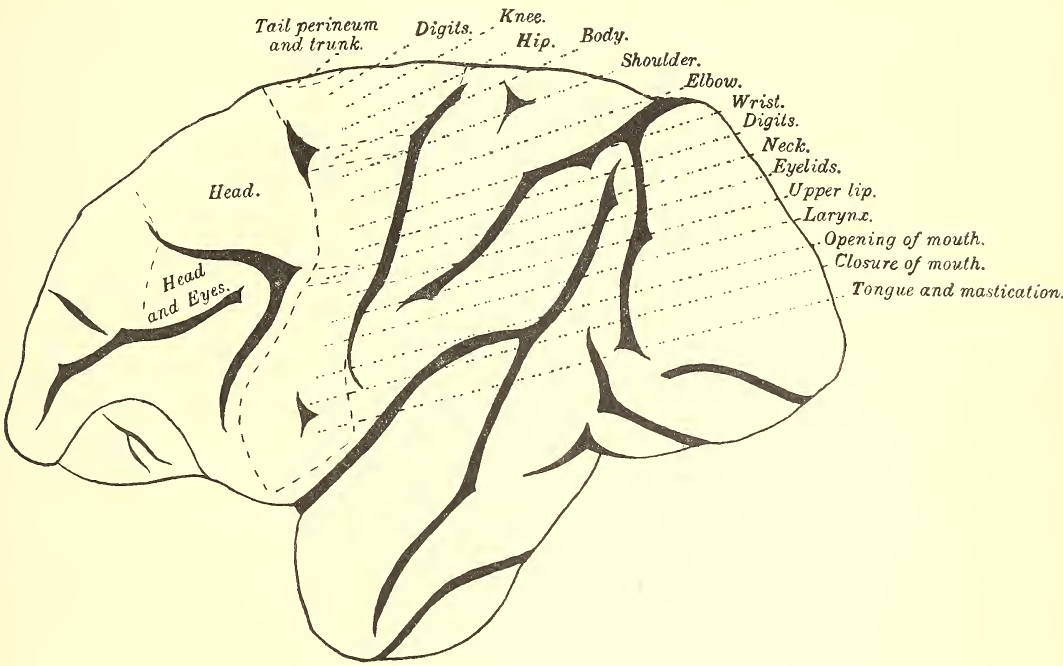


FIG. 1.

also employed the bipolar method of stimulation, using platinum points ·5 mm. apart.

The same method of isolation was applied to the determination of the boundaries of the centres in front of the Rolandic fissure.

We also, with the object of limiting the spread of current, made use of brass plates whose edges were placed on the surface of the cortex and which were connected to earth by wires.

We found, as previous investigators have done, that implicit reliance cannot be placed on the sulci as guides in cerebral topography. We accordingly measured and sketched the sulci rapidly after exposure of the

Rolandic cortex of one hemisphere, marking upon the sketch the position of points stimulated. In order to obtain a composite picture of the relation of the areas to the convolutions, we transferred our markings of the results of each experiment to the hardened brain of a Macacus and photographed them.

Our results are shown in figs. 1 and 2.

During the course of our investigation, Sherrington has published a diagram of the convexity of the hemisphere of *Cercopithecus callithrix* * which illustrates the changes produced by tetanus toxin in the functional

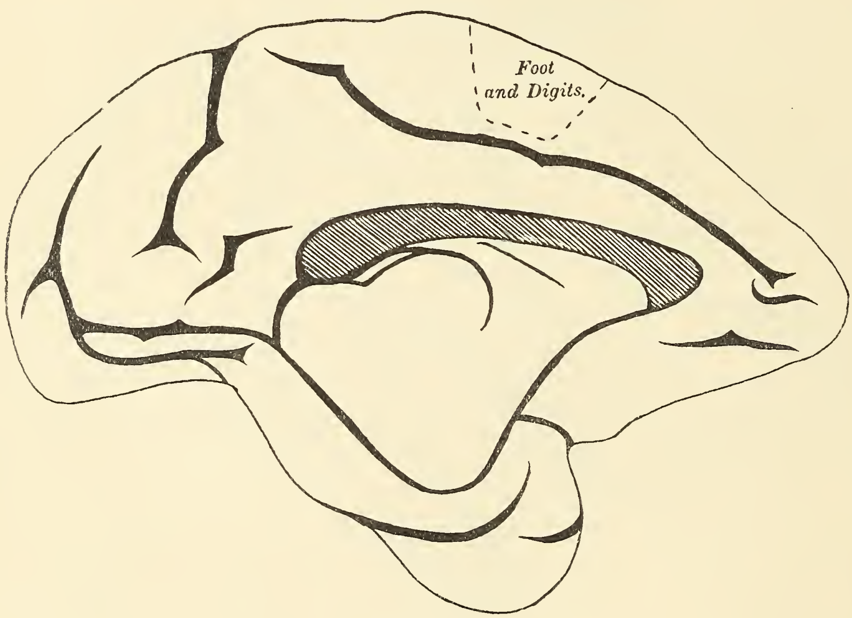


Fig. 2.

topography of the motor cortex. The outlines of the various centres depicted in this diagram show that this observer had, by the use of unipolar faradisation, arrived at conclusions with regard to the areas figured by him similar to those which we have reached by our method of isolation.

Vogt † also, in a recent paper, states that he has confirmed Sherrington's results by unipolar faradisation in the orang, and records stimulation experiments on the lower apes with results which accord with those of Sherrington and are similar to those which we have obtained.

* *The Integrative Action of the Nervous System*, p. 297.

† *Anatomischer Anzeiger*, Ergänzungsheft zum xxix. Bd., 1906, p. 74.

The excitable areas and their boundaries, as determined by us, are briefly as follows:—

On the convexity of the hemisphere close to the margin of the longitudinal fissure lies a small area superior to sulcus X, and separated from it by part of the leg area, from which movements of the tail and perineum are obtained, and in association with these, movements of the muscles of the trunk.

The area for the leg is situated partly on the mesial aspect and partly on the convexity of the hemisphere. On the mesial aspect it occupies part of the marginal convolution, extending posteriorly from a continuation over the margin of the Rolandic fissure to a point in front corresponding to the anterior extremity of sulcus X. Below, it extends almost to the calloso-marginal fissure. From this part of the marginal gyrus movements of the lower extremity are obtained, chiefly of the foot and digits. On the convex surface the centre for the leg covers an area extending from the margin to a line drawn antero-posteriorly from the posterior extremity of sulcus X to the upper border of the superior genu of the Rolandic fissure. This area, which is wider above than below, extends in front to the anterior extremity of sulcus X.

Below the leg centre, and interposed between it and the arm centre, is a small area lying at the level of the superior genu of the Rolandic fissure and extending forwards almost to the posterior extremity of sulcus X, from which can be elicited movements of the body muscles.

Movements of the arm are obtained by stimulating over an area stretching from the lower boundary of the body centre to the level of an oblique line drawn from a little below the sagittal part of the precentral fissure to the Rolandic fissure at the level of the inferior genu.

The face centre extends from this line downwards almost to the Sylvian fissure. The posterior boundary of the face area is in its upper part the fissure of Rolando. In its lower part the excitable area does not quite reach this fissure posteriorly, but leaves it as the latter inclines backwards, and finds its limit behind in a line drawn directly downwards.

From the upper portion of the face area a small part can be cut off stretching from its upper boundary downwards for a distance of about 1 mm. in the average brain. Movements of the neck result from stimulation of this part of the cortex in its anterior half and movements of the eyelids in its posterior half. In the remaining part of the face area from above downwards we obtain movements of the upper lip, larynx, opening of mouth, closure of mouth, tongue movements, and those of mastication.

The anterior boundary of the excitable area is that which we found

most difficult to determine accurately. In its lower part it corresponds nearly with that figured by Horsley and Schäfer. In its upper part it lies more posteriorly.

In the frontal lobe there is an area lying below the horizontal part of the precentral sulcus and in the angle of that sulcus, stimulation of which is followed by opening of the eyes and deviation of head and eyes to the opposite side. Above the horizontal part of the precentral sulcus, between it and the margin, there is an ill-defined region within the frontal lobe from which movements of the head can be caused. In our experience this region gives less definite and constant response to stimulation than the area lying in the angle of the precentral sulcus. We were unable to study the eye-movements in detail, as we were not fortunate in obtaining monkeys with exceptionally large and well-developed brains, a condition which has been shown by Mott and Schäfer * to be necessary for their precise differentiation.

The area which we have described as lying close to the margin, and from which movements of tail, anus, vagina, and of the trunk are obtained, requires further mention. It corresponds with the area on the convexity of the hemisphere described by Schäfer † as responding with movements of the trunk and tail, and which is named by him the trunk area. The centre for movements of the trunk in monkeys was placed in the prefrontal lobe by Munk,‡ who stated that he obtained contractions of the muscles of the back, abdomen, and diaphragm by stimulation of this part of the brain. He further stated that extirpations in this region are followed by paresis of the trunk muscles. These results were not confirmed by Horsley and Schäfer,§ who found no permanent paralysis to result from ablations in the anterior part of the frontal lobes. They pointed out that the movements of the trunk observed by Munk are to be attributed to the spreading of the strong currents which he employed to other regions of the brain.

Vogt || applies the term trunk centre to the area which we have described between the areas for the lower and upper extremities at the level of the superior genu of the Rolandic fissure. This area is indicated in Sherrington's figure ¶ of the monkey's brain by the word "Body."

Our experiments confirm what is stated by Horsley and Schäfer, that movements of the trunk can be elicited from an area close to the margin of

* *Brain*, vol. xiii. p. 165. † *Text-book of Physiology*, vol. ii. p. 741.

‡ *Ueber die Functionen der Grosshirnrinde*, Berlin, 1890, p. 51.

§ *Phil. Trans.*, 1888, p. 4. || *Loc. cit.*

¶ *The Integrative Action of the Nervous System*, p. 297.

the longitudinal fissure. These movements take place in association with those of the tail and perineum, and chiefly affect the muscles of the back.

It is necessary to consider how far the results here detailed are in accordance with those obtained by other methods of research.

With regard to the posterior boundary of the Rolandic motor area, it is to be noted that impairment of voluntary movements of groups of muscles follows upon extirpation of the corresponding motor centres in the cortex.

Grünbaum and Sherrington have found,* on extirpating areas in the ascending frontal convolution in the anthropoid, that severe but diminishing paralyses result. Thus after destruction of the hand area the hand remains paralysed for several days. This paralysis passes off, until after a week or two the ape is able to make use of the hand for voluntary movement. No paralysis, not even of a passing nature, succeeds ablations of large portions of the ascending parietal convolution.

We have performed a number of experiments on monkeys, in which we have destroyed one or other of the motor areas as delimited by us in the ascending frontal convolution and obtained paralysis of the corresponding part of the body, complete at first but passing off as in the experiments of Grünbaum and Sherrington.

Vogt † records eleven extirpation experiments on monkeys. In five of these the lesions involved the cortex anterior to the fissure of Rolando, and in six the lesions were made posterior to that fissure. Motor paralysis followed only upon lesions in front of the fissure. In the other cases, while movements of the affected limb could be carried out, the author states that these movements were ataxic.

It is known that lesions of the motor cortex are followed by degeneration of nerve fibres in the pyramidal tract. Various experimenters have described degeneration resulting from lesions in front of, and also behind, the fissure of Rolando.

Mellus ‡ describes experiments on *Macacus sinicus* in which he removed a portion of cortex from the ascending parietal convolution between the lower extremity of the intraparietal sulcus and the fissure of Rolando and a little above the inferior genu of the fissure of Rolando. He found degenerated fibres both coarse and fine passing from the lesion downward through the centrum semiovale and present in the posterior limb of the internal capsule.

* *Proc. Roy. Soc.*, London, 1901, vol. lxix. p. 207.

† *Loc. cit.*

‡ *Proc. Roy. Soc.*, London, 1895, vol. lviii. p. 206.

Sherrington * states that he is unable to find any unequivocal degeneration as a result of large lesions of the post-central gyrus in the anthropoid.

Vogt,† working on lower apes, arrives at results differing from those of the last named author. He finds degenerated fibres in the internal capsule as a sequel to lesions whether made anterior or posterior to the fissure of Rolando, but the position of the degeneration is different in the two cases. The degeneration in the internal capsule, resulting from ablations in the ascending frontal convolution, finds its posterior limit in horizontal section, according to this observer, at the level of the oral end of the nucleus ventralis lateralis posterior, while ablations of cortex in the ascending parietal convolution give rise to degeneration posterior to this level. The author brings forward evidence in support of his view that the line separating the areas in the internal capsule presenting degeneration in the two cases forms an important boundary between two distinct systems of fibres.

Such divergence in results, as regards paralysis and degeneration, produced by lesions in front of and behind the fissure of Rolando, are in favour of our ascribing different functions to the ascending frontal and ascending parietal gyri.

We have next to consider what support, if any, for our views of the extent of the motor area can be drawn from a study of the myelination of the nerve fibres.

It has been shown that the ascending frontal and parietal convolutions are alike characterised by early development of myelination. Flechsig ‡ depicts the ascending frontal and the anterior part of the ascending parietal convolutions in man as one area, whose posterior boundary thus lies near the middle of the ascending parietal convolution. The anterior boundary of this area corresponds nearly with the anterior limit of the Rolandic excitable cortex, but the posterior boundary does not coincide with any dividing line arrived at by the method of electrical excitation. Further, the posterior boundary does not correspond, as we shall presently see, with any change in the histological structure of the cortex, but lies near the middle of an area which has been delimited by histological methods and named by Brodmann § the area culminis. This area culminis corresponds to the caudal part of the post-central area, together with the oral part of the intermediate post-central area of Campbell. ||

* *The Integrative Action of the Nervous System*, p. 277. † *Loc. cit.*

‡ *Gehirn und Seele*, Leipzig, 1896, Taf. iv.

§ "Beiträge zur histologischen Lokalisation der Grosshirnrinde," *Journ. f. Psych. u. Neurol.*, 1903–1906. "Diskussion," *Neurol. Centralbl.*, 1904, p. 669. See also Vogt, *loc. cit.*

|| *Histological Studies on the Localisation of Cerebral Function*, Cambridge, 1905.

No indication appears to be afforded by the study of myelination of the boundary between areas of the cortex, which is shown by the method of electrical excitation to lie within the fissure of Rolando.

Considerable advances have been made in recent years in the study of the histological structure of the cortex cerebri, and areas have been mapped out upon it by Brodmann, Campbell, and others characterised by differences in the arrangement of fibres and in the size, shape, and position of the cells.

An area in the ascending frontal convolution and adjacent portion of the marginal convolution is found to differ with regard both to fibre and cell arrangement from the cortex anterior and posterior to it. Its characteristic feature is the presence of very large cells—giant cells—described by Betz and Bevan Lewis. The anterior and posterior boundaries of the area containing these giant cells, as mapped out by Brodmann * in the brain of Cercopithecus, correspond closely with the limits of the region from the excitation of which we have obtained movements of the lower and upper extremities, body, neck, and upper part of face. It is interesting to observe that the posterior boundary of this area in its lower part leaves the fissure of Rolando and passes downwards in a manner similar to the posterior limit of excitability as determined by us. This region is termed by Brodmann the area giganto-pyramidalis, and corresponds with the precentral area of Campbell. Anteriorly it lies contiguous to the area agranularis of the former author, while behind it joins the area paradoxa or post-central area of Campbell.

The lower part of the face area, viz., that from which movements chiefly of mastication and deglutition are obtained, is considered by Brodmann to form an area of cortex differing in histological structure from the area giganto-pyramidalis but identical with the region in the frontal lobe, from stimulation of which we have described movements of the head as resulting. The region in the angle of the precentral sulcus, from which lateral movements of the head and eyes are obtained, is characterised also, according to the same author, by a separate and distinct histological structure.

We have examined the microscopic structure of the excitable area in the species of monkeys employed by us with the object especially of determining the posterior limit. This we find, taking the presence of giant cells as the most reliable guide, to lie in the lower part of the anterior lip of the Rolandic fissure in the greater part of its length.

Within the motor area further histological differentiation is possible. It was shown by Bevan Lewis † that the giant cells, termed by him "motor

* See fig. 62 in Vogt's paper, *loc. cit.*

† *Brain*, 1878, vol. i. p. 79.

cells," are largest in the upper part of the Rolandic area and diminish in size from above downwards. This observer also showed that the motor cells are arranged in groups. These observations have recently been confirmed by Campbell,* working on the human and anthropoid brains, and he states further that the grouped arrangement of cells is found only in the areas forming motor centres for the limbs, while in the part of the ascending frontal convolution at the level of the superior genu of the Rolandic fissure, from which movements of the body muscles are obtained, he finds the cells arranged in a solitary manner. According to Campbell, the size, configuration, and grouping of the giant cells vary in relation to the motor centres in which they are found, and the conclusions which he reaches by a consideration of their variations are in accord, *mutatis mutandis,* with the results which we have derived from the method of electrical excitation in monkeys when care is taken to isolate the areas in the manner before described.†

* *Loc. cit.*

† The expenses of this investigation were defrayed by grants from the Moray Fund (University of Edinburgh) and the Carnegie Trust for the Universities of Scotland.

(*Issued separately May* 6, 1907.)

XI.—The Minors of a Product-Determinant. By Thomas Muir, LL.D.

(MS. received November 12, 1906. Read January 21, 1907.)

1. If for $a\alpha + b\beta + c\gamma + \ . \ . \ .$ we put

$$\frac{a,\, b,\, c,\ \ldots\ .}{\alpha,\, \beta,\, \gamma,\ \ldots\ .} \quad \text{or} \quad \frac{a\ b\ c\ \ldots}{\alpha\ \beta\ \gamma\ \ldots},$$

then the product of $|a_1 b_2 c_3|$ and $|f_1 g_2 h_3|$ may be written

$$\begin{vmatrix} \dfrac{a_1\ a_2\ a_3}{f_1\ g_1\ h_1} & \dfrac{a_1\ a_2\ a_3}{f_2\ g_2\ h_2} & \dfrac{a_1\ a_2\ a_3}{f_3\ g_3\ h_3} \\ \dfrac{b_1\ b_2\ b_3}{f_1\ g_1\ h_1} & \dfrac{b_1\ b_2\ b_3}{f_2\ g_2\ h_2} & \dfrac{b_1\ b_2\ b_3}{f_3\ g_3\ h_3} \\ \dfrac{c_1\ c_2\ c_3}{f_1\ g_1\ h_1} & \dfrac{c_1\ c_2\ c_3}{f_2\ g_2\ h_2} & \dfrac{c_1\ c_2\ c_3}{f_3\ g_3\ h_3} \end{vmatrix}$$

and the minor which is the cofactor of the element in the place 1, 1 is *

$$\begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \cdot \begin{vmatrix} f_2 & f_3 \\ g_2 & g_3 \\ h_2 & h_3 \end{vmatrix}$$

i.e.

$$|b_1\, c_2| \cdot |f_2\, g_3| + |b_1\, c_3| \cdot |f_2\, h_3| + |b_2\, c_3| \cdot |g_2\, h_3|;$$

so that if, as usual, we denote the cofactors of the elements in $|a_1\, b_2\, c_3|$, $|f_1 g_2 h_3|$ by the corresponding capital letters, we may assert that *In the determinant which is the product of* $|a_1\, b_2\, c_3|$ *and* $|f_1\, g_2\, h_3|$ *the cofactor of* $\dfrac{a_1\, a_2\, a_3}{f_1\, g_1\, h_1}$ is $\dfrac{A_1\, A_2\, A_3}{F_1\, G_1\, H_1}$.

2. Proceeding to the case where the number of determinants to be multiplied together is three, namely

$$|a_1\, b_2\, c_3|,\ |f_1\, g_2\, h_3|,\ |m_1\, n_2\, r_3|,$$

let us denote

$$\frac{a_1\ a_2\ a_3}{f_1\ g_1\ h_1} m_1 + \frac{a_1\ a_2\ a_3}{f_2\ g_2\ h_2} n_1 + \frac{a_1\ a_2\ a_3}{f_3\ g_3\ h_3} r_1$$

* This is usually written

$$\begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \cdot \begin{vmatrix} f_2 & g_2 & h_2 \\ f_3 & g_3 & h_3 \end{vmatrix},$$

but throughout the present paper row-by-column multiplication is used, so as to have the results immediately applicable to *matrices*.

by

$$\begin{array}{ccc|c} a_1 & a_2 & a_3 & \\ \hline f_1 & g_1 & h_1 & m_1 \\ f_2 & g_2 & h_2 & n_1 \\ f_3 & g_3 & h_3 & r_1 \end{array} .$$

The product then is

$$\left| \begin{array}{ccc} \begin{array}{ccc|c} a_1 & a_2 & a_3 & \\ \hline f_1 & g_1 & h_1 & m_1 \\ f_2 & g_2 & h_2 & n_1 \\ f_3 & g_3 & h_3 & r_1 \end{array} & \begin{array}{ccc|c} a_1 & a_2 & a_3 & \\ \hline f_1 & g_1 & h_1 & m_2 \\ f_2 & g_2 & h_2 & n_2 \\ f_3 & g_3 & h_3 & r_2 \end{array} & \begin{array}{ccc|c} a_1 & a_2 & a_3 & \\ \hline f_1 & g_1 & h_1 & m_3 \\ f_2 & g_2 & h_2 & n_3 \\ f_3 & g_3 & h_3 & r_3 \end{array} \\ \\ \begin{array}{ccc|c} b_1 & b_2 & b_3 & \\ \hline f_1 & g_1 & h_1 & m_1 \\ f_2 & g_2 & h_2 & n_1 \\ f_3 & g_3 & h_3 & r_1 \end{array} & \begin{array}{ccc|c} b_1 & b_2 & b_3 & \\ \hline f_1 & g_1 & h_1 & m_2 \\ f_2 & g_2 & h_2 & n_2 \\ f_3 & g_3 & h_3 & r_2 \end{array} & \begin{array}{ccc|c} b_1 & b_2 & b_3 & \\ \hline f_1 & g_1 & h_1 & m_3 \\ f_2 & g_2 & h_2 & n_3 \\ f_3 & g_3 & h_3 & r_3 \end{array} \\ \\ \begin{array}{ccc|c} c_1 & c_2 & c_3 & \\ \hline f_1 & g_1 & h_1 & m_1 \\ f_2 & g_2 & h_2 & n_1 \\ f_3 & g_3 & h_3 & r_1 \end{array} & \begin{array}{ccc|c} c_1 & c_2 & c_3 & \\ \hline f_1 & g_1 & h_1 & m_2 \\ f_2 & g_2 & h_2 & n_2 \\ f_3 & g_3 & h_3 & r_2 \end{array} & \begin{array}{ccc|c} c_1 & c_2 & c_3 & \\ \hline f_1 & g_1 & h_1 & m_3 \\ f_2 & g_2 & h_2 & n_3 \\ f_3 & g_3 & h_3 & r_3 \end{array} \end{array} \right| ,$$

and the minor which is the cofactor of the element in the place 1, 1, consisting as it does of terms in b_1, terms in b_2, and terms in b_3, may be dealt with in three portions. The aggregate of the terms in b_1 is evidently

$$b_1 \cdot \frac{f_1\, f_2\, f_3}{m_2\, n_2\, r_2} \cdot \begin{array}{ccc|c} c_1 & c_2 & c_3 & \\ \hline f_1 & g_1 & h_1 & m_3 \\ f_2 & g_2 & h_2 & n_3 \\ f_3 & g_3 & h_3 & r_3 \end{array} \quad - \quad b_1 \cdot \frac{f_1\, f_2\, f_3}{m_3\, n_3\, r_3} \cdot \begin{array}{ccc|c} c_1 & c_2 & c_3 & \\ \hline f_1 & g_1 & h_1 & m_2 \\ f_2 & g_2 & h_2 & n_2 \\ f_3 & g_3 & h_3 & r_2 \end{array}$$

i.e.

$$b_1\, c_2 \left\{ \frac{f_1\, f_2\, f_3}{m_2\, n_2\, r_2} \cdot \frac{g_1\, g_2\, g_3}{m_3\, n_3\, r_3} - \frac{f_1\, f_2\, f_3}{m_3\, n_3\, r_3} \cdot \frac{g_1\, g_2\, g_3}{m_2\, n_2\, r_2} \right\}$$

$$+ b_1\, c_3 \left\{ \frac{f_1\, f_2\, f_3}{m_2\, n_2\, r_2} \cdot \frac{h_1\, h_2\, h_3}{m_3\, n_3\, r_3} - \frac{f_1\, f_2\, f_3}{m_3\, n_3\, r_3} \cdot \frac{h_1\, h_2\, h_3}{m_2\, n_2\, r_2} \right\},$$

i.e.

$$b_1 c_2 \cdot \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix} + b_1 c_3 \cdot \begin{vmatrix} f_1 & f_2 & f_3 \\ h_1 & h_2 & h_3 \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix} .$$

Similarly, the aggregate of terms in b_2 is found to be

$$b_2 c_1 \cdot \begin{vmatrix} g_1 & g_2 & g_3 \\ f_1 & f_2 & f_3 \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix} + b_2 c_3 \cdot \begin{vmatrix} g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix} \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix} ,$$

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THOMAS MUIR.
Proc. Roy. Soc. Edin., vol. xxvii., 1907, pp. 135–166.

and the aggregate of terms in b_3 to be

$$b_3c_1 \cdot \begin{vmatrix} h_1 & h_2 & h_3 \\ f_1 & f_2 & f_3 \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix} + b_3c_2 \cdot \begin{vmatrix} h_1 & h_2 & h_3 \\ g_1 & g_2 & g_3 \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix}.$$

The minor in full is therefore

$$|b_1c_2| \cdot \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix} + |b_1c_3| \cdot \begin{vmatrix} f_1 & f_2 & f_3 \\ h_1 & h_2 & h_3 \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix} + |b_2c_3| \cdot \begin{vmatrix} g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix},$$

i.e.

$$\begin{aligned} & A_3(H_3R_1 + H_2N_1 + H_1M_1) \\ & + A_2(G_3R_1 + G_2N_1 + G_1M_1) \\ & + A_1(F_3R_1 + F_2N_1 + F_1M_1)\,; \end{aligned}$$

so that, formulating as before, we have the result—*In the determinant which is the product of* $|\mathrm{a}_1\mathrm{b}_2\mathrm{c}_3|$, $|\mathrm{f}_1\mathrm{g}_2\mathrm{h}_3|$, $|\mathrm{m}_1\mathrm{n}_2\mathrm{r}_3|$ *the cofactor of*

$$\begin{array}{ccc|c} \mathrm{a}_1 & \mathrm{a}_2 & \mathrm{a}_3 & \\ \hline \mathrm{f}_1 & \mathrm{g}_1 & \mathrm{h}_1 & \mathrm{m}_1 \\ \mathrm{f}_2 & \mathrm{g}_2 & \mathrm{h}_2 & \mathrm{n}_1 \\ \mathrm{f}_3 & \mathrm{g}_3 & \mathrm{h}_3 & \mathrm{r}_1 \end{array} \quad \textit{is} \quad \begin{array}{ccc|c} \mathrm{A}_1 & \mathrm{A}_2 & \mathrm{A}_3 & \\ \hline \mathrm{F}_1 & \mathrm{G}_1 & \mathrm{H}_1 & \mathrm{M}_1 \\ \mathrm{F}_2 & \mathrm{G}_2 & \mathrm{H}_2 & \mathrm{N}_1 \\ \mathrm{F}_3 & \mathrm{G}_3 & \mathrm{H}_3 & \mathrm{R}_1 \end{array}.$$

3. Just as the minor in § 1 is the result of the multiplication

$$\begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \cdot \begin{vmatrix} f_2 & f_3 \\ g_2 & g_3 \\ h_2 & h_3 \end{vmatrix},$$

so the minor in § 2 is the result of the multiplication

$$\begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \cdot \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix};$$

and, the former being used as a symbol for the minor to which it gives rise, the latter may with convenience be similarly employed. Further, we may utilise the first result in establishing the second: for, the first being

$$A_3H_1 + A_2G_1 + A_1F_1,$$

the second

$$= \begin{vmatrix} \dfrac{b_1\ b_2\ b_3}{f_1\ g_1\ h_1} & \dfrac{b_1\ b_2\ b_3}{f_2\ g_2\ h_2} & \dfrac{b_1\ b_2\ b_3}{f_3\ g_3\ h_3} \\ \dfrac{c_1\ c_2\ c_3}{f_1\ g_1\ h_1} & \dfrac{c_1\ c_2\ c_3}{f_2\ g_2\ h_2} & \dfrac{c_1\ c_2\ c_3}{f_3\ g_3\ h_3} \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \end{vmatrix}$$

$$= \begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \cdot \begin{vmatrix} f_1 & f_2 \\ g_1 & g_2 \\ h_1 & h_2 \end{vmatrix} \cdot |m_2n_3| + \begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \cdot \begin{vmatrix} f_1 & f_3 \\ g_1 & g_3 \\ h_1 & h_3 \end{vmatrix} \cdot |m_2r_3| + \begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \cdot \begin{vmatrix} f_2 & f_3 \\ g_2 & g_3 \\ h_2 & h_3 \end{vmatrix} \cdot |n_2r_3|$$

$$
\begin{aligned}
&= (A_3H_3 + A_2G_3 + A_1F_3) \cdot R_1 \\
&\quad + (A_3H_2 + A_2G_2 + A_1F_2) \cdot N_1 \\
&\quad + (A_3H_1 + A_2G_1 + A_1F_1) \cdot M_1
\end{aligned}
\quad = \quad
\begin{array}{ccc|c}
A_1 & A_2 & A_3 & \\
\hline
F_1 & G_1 & H_1 & M_1 \\
F_2 & G_2 & H_2 & N_1 \\
F_3 & G_3 & H_3 & R_1 ,
\end{array}
$$

as before. Similarly, having reached this we can show that the cofactor of

$$
\begin{array}{ccc|ccc}
a_1 & a_2 & a_3 & x_1 & y_1 & z_1 \\
\hline
f_1 & g_1 & h_1 & m_1 & m_2 & m_3 \\
f_2 & g_2 & h_2 & n_1 & n_2 & n_3 \\
f_3 & g_3 & h_3 & r_1 & r_2 & r_3
\end{array}
$$

in the determinant which is the product of

$$|a_1\, b_2\, c_3|,\ |f_1\, g_2\, h_3|,\ |m_1\, n_2\, r_3|,\ |x_1\, y_2\, z_3|$$

is

$$
\begin{array}{ccc|ccc}
A_1 & A_2 & A_3 & X_1 & Y_1 & Z_1 \\
\hline
F_1 & G_1 & H_1 & M_1 & M_2 & M_3 \\
F_2 & G_2 & H_2 & N_1 & N_2 & N_3 \\
F_3 & G_3 & H_3 & R_1 & R_2 & R_3 ,
\end{array}
$$

and so on. Consequently, as the restriction in the foregoing to three-line determinants is readily seen to detract in no way from the validity of the procedure, we have the following general theorem—*The cofactor of any element in the determinants*

$$|a_{11}\, a_{22} \cdots a_{nn}|,\ |b_{11}\, b_{22} \cdots b_{nn}|,\ \cdots\cdots\cdots$$

being denoted by the corresponding capital letter similarly suffixed, the cofactor of any element in the determinant which is the product of those determinants is got by changing every letter in the expression for the said element into the corresponding capital letter.

From this we have the corollary that *the adjugate of the product of any number of determinants is not only equal to but is identical in form with the product of their adjugates.*

4. The theorem of § 3, it will be seen, may be described as asserting that the minors of highest order (the principal minors) and the minors of lowest order (the elements) in the product-determinant are expressible in one and the same form. Let us now consider the minors of intermediate order; but, in so doing, let us forsake the classification of minors into principal, secondary, etc., and group them instead according to the number of their rows and columns.

5. Beginning then with minors of *two* rows we note first that one case has already been considered, the principal minors of a determinant of the

third order belonging to this class. Taking next the determinant which is the product of two determinants of the fourth order $|a_1b_2c_3d_4|$, $|f_1g_2h_3k_4|$, we see that the two-line minor belonging, say, to the 3rd and 4th rows and to the 1st and 4th columns is

$$\begin{vmatrix} c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} \cdot \begin{vmatrix} f_1 & f_4 \\ g_1 & g_4 \\ h_1 & h_4 \\ k_1 & k_4 \end{vmatrix},$$

and that this is equal to

$$\frac{|c_1\,d_2|, \quad |c_1\,d_3|, \quad |c_1\,d_4|, \quad |c_2\,d_3|, \quad |c_2\,d_4|, \quad |c_3\,d_4|}{|f_1\,g_4|, \quad |f_1\,h_4|, \quad |f_1\,k_4|, \quad |g_1\,h_4|, \quad |g_1\,k_4|, \quad |h_1\,k_4|},$$

a result exactly similar to that of § 1 but not expressible in the same final form, on account of the two-line determinants occurring in it being no longer principal minors. Taking, in the third place, the determinant which is the product of *three* four-line determinants

$$|a_1\,b_2\,c_3\,d_4|, \quad |f_1\,g_2\,h_3\,k_4|, \quad |m_1\,n_2\,r_3\,s_4|,$$

we have for the two-line minor belonging, say, to the 1st and 3rd rows and to the 2nd and 3rd columns the expression

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ c_1 & c_2 & c_3 & c_4 \end{vmatrix} \cdot \begin{vmatrix} f_1 & f_2 & f_3 & f_4 \\ g_1 & g_2 & g_3 & g_4 \\ h_1 & h_2 & h_3 & h_4 \\ k_1 & k_2 & k_3 & k_4 \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \\ s_2 & s_3 \end{vmatrix}$$

which is equal to

$$\begin{vmatrix} \dfrac{a_1\,a_2\,a_3\,a_4}{f_1\,g_1\,h_1\,k_1} & \dfrac{a_1\,a_2\,a_3\,a_4}{f_2\,g_2\,h_2\,k_2} & \dfrac{a_1\,a_2\,a_3\,a_4}{f_3\,g_3\,h_3\,k_3} & \dfrac{a_1\,a_2\,a_3\,a_4}{f_4\,g_4\,h_4\,k_4} \\[2ex] \dfrac{c_1\,c_2\,c_3\,c_4}{f_1\,g_1\,h_1\,k_1} & \dfrac{c_1\,c_2\,c_3\,c_4}{f_2\,g_2\,h_2\,k_2} & \dfrac{c_1\,c_2\,c_3\,c_4}{f_3\,g_3\,h_3\,k_3} & \dfrac{c_1\,c_2\,c_3\,c_4}{f_4\,g_4\,h_4\,k_4} \end{vmatrix} \cdot \begin{vmatrix} m_2 & m_3 \\ n_2 & n_3 \\ r_2 & r_3 \\ s_2 & s_3 \end{vmatrix},$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ c_1 & c_2 & c_3 & c_4 \end{vmatrix} \cdot \begin{vmatrix} f_1 & f_2 \\ g_1 & g_2 \\ h_1 & h_2 \\ k_1 & k_2 \end{vmatrix} \cdot |m_2\,n_3| + \begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ c_1 & c_2 & c_3 & c_4 \end{vmatrix} \cdot \begin{vmatrix} f_1 & f_3 \\ g_1 & g_3 \\ h_1 & h_3 \\ k_1 & k_3 \end{vmatrix} \cdot |m_2\,r_3|$$

$$+ \cdots\cdots + \begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ c_1 & c_2 & c_3 & c_4 \end{vmatrix} \cdot \begin{vmatrix} f_3 & f_4 \\ g_3 & g_4 \\ h_3 & h_4 \\ k_3 & k_4 \end{vmatrix} \cdot |r_2\,s_3|$$

$$
= \begin{array}{cccccc|c}
|a_1\, c_2| & |a_1\, c_3| & |a_1\, c_4| & |a_2\, c_3| & |a_2\, c_4| & |a_3\, c_4| & \\
\hline
|f_1\, g_2| & |f_1\, h_2| & |f_1\, k_2| & |g_1\, h_2| & |g_1\, k_2| & |h_1\, k_2| & |m_2\, n_3| \\
|f_1\, g_3| & |f_1\, h_3| & \cdots & \cdots & \cdots & |h_1\, k_3| & |m_2\, r_3| \\
|f_1\, g_4| & |f_1\, h_4| & \cdots & \cdots & \cdots & |h_1\, k_4| & |m_2\, s_3| \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
|f_3\, g_4| & |f_3\, h_4| & \cdots & \cdots & \cdots & |h_3\, k_4| & |r_2\, s_3|
\end{array},
$$

where the $6+36+6$ two-line determinants are those formable from the three arrays of the expression with which we started.

Similarly, if the given determinant be the product of *four* determinants $\Delta_1, \Delta_2, \Delta_3, \Delta_4$, it is seen that the two-line minor belonging, say, to the 2nd and 4th rows and to the 1st and 2nd columns of the product is the bipartite function * representable briefly by

$$
\begin{array}{c|c}
\text{Two-line minors formable} & \text{Two-line minors formable} \\
\text{from rows 2 and 4 of } \Delta_1 & \text{from columns 1 and 2 of } \Delta_4 \\
\hline
\text{Two-line minors formable} & \text{Two-line minors formable} \\
\text{from conjugate of } \Delta_2 & \text{from } \Delta_3
\end{array}
$$

and so on generally.

6. In at least one case the two-line minor of a product-determinant is of very special interest. This is when the second determinant of the product $\Delta_1\Delta_2\Delta_3$ is the adjugate of another determinant, the reason being that important changes are then possible on both sides of the identity. Thus, taking the product

$$|a_1\, b_2\, c_3\, d_4| \cdot |F_1\, G_2\, H_3\, K_4| \cdot |m_1\, n_2\, r_3\, s_4|$$

we find, in the first place, that each element of the product-determinant is itself expressible as a determinant; for example, for the element in the place 1,1

$$
\begin{array}{cccc|c}
a_1 & a_2 & a_3 & a_4 & \\
\hline
F_1 & G_1 & H_1 & K_1 & m_1 \\
F_2 & G_2 & H_2 & K_2 & n_1 \\
F_3 & G_3 & H_3 & K_3 & r_1 \\
F_4 & G_4 & H_4 & K_4 & s_1
\end{array}
$$

* In the original memoir (*Trans. Roy. Soc. Edin.*, xxxii. pp. 461–482), in which the properties of the functions which represent the elements of product-determinants are investigated, it is stated (p. 481) that they were given the name '*bipartite*' from Cayley's use of the word for a special set of the functions, namely, those of the *third degree.* It has often since appeared to me that it would have been preferable to have extended the meaning of another word, namely, the word '*cumulant*,' this being the name given by Sylvester to a special set of the *second order.*

we can substitute the determinant

$$-\begin{vmatrix} \cdot & a_1 & a_2 & a_3 & a_4 \\ m_1 & f_1 & g_1 & h_1 & k_1 \\ n_1 & f_2 & g_2 & h_2 & k_2 \\ r_1 & f_3 & g_3 & h_3 & k_3 \\ s_1 & f_4 & g_4 & h_4 & k_4 \end{vmatrix};$$

and, in the second place, the bipartite which is the equivalent of the minor in question having the square array

$$\begin{matrix} |F_1 \ G_2| & |F_1 \ H_2| & \cdots\cdots & |H_1 \ K_2| \\ |F_1 \ G_3| & |F_1 \ H_3| & \cdots\cdots & |H_1 \ K_3| \\ \cdot\cdot\cdot & \cdot\cdot\cdot & \cdots\cdots & \cdot\cdot\cdot \\ |F_3 \ G_4| & |F_3 \ H_4| & \cdots\cdots & |H_3 \ K_4|, \end{matrix}$$

where every element contains $|f_1 \ g_2 \ h_3 \ k_4|$ as a factor, may be simplified by the removal of this factor throughout. The following interesting result is thus obtained:

$$\left\|\begin{matrix} \begin{vmatrix} \cdot & a_1 & a_2 & a_3 & a_4 \\ m_2 & f_1 & g_1 & h_1 & k_1 \\ n_2 & f_2 & g_2 & h_2 & k_2 \\ r_2 & f_3 & g_3 & h_3 & k_3 \\ s_2 & f_4 & g_4 & h_3 & k_4 \end{vmatrix} & \begin{vmatrix} \cdot & a_1 & a_2 & a_3 & a_4 \\ m_3 & f_1 & g_1 & h_1 & k_1 \\ n_3 & f_2 & g_2 & h_2 & k_2 \\ r_3 & f_3 & g_3 & h_3 & k_3 \\ s_3 & f_4 & g_4 & h_4 & k_4 \end{vmatrix} \\ \begin{vmatrix} \cdot & c_1 & c_2 & c_3 & c_4 \\ m_2 & f_1 & g_1 & h_1 & k_1 \\ n_2 & f_2 & g_2 & h_2 & k_2 \\ r_2 & f_3 & g_3 & h_3 & k_3 \\ s_2 & f_4 & g_4 & h_4 & k_4 \end{vmatrix} & \begin{vmatrix} \cdot & c_1 & c_2 & c_3 & c_4 \\ m_3 & f_1 & g_1 & h_1 & k_1 \\ n_3 & f_2 & g_2 & h_2 & k_2 \\ r_3 & f_3 & g_3 & h_3 & k_3 \\ s_3 & f_4 & g_4 & h_4 & k_4 \end{vmatrix} \end{matrix}\right\|$$

$$= \ |f_1 \ g_2 \ h_3 \ k_4| \cdot \begin{array}{rccccr|c} & |a_1 \ c_2| & |a_1 \ c_3| & \cdots\cdots & & |a_3 \ c_4| & \\ \hline & |h_3 \ k_4| & -|g_3 \ k_4| & \cdots\cdots & & |f_3 \ g_4| & |m_2 \ n_3| \\ - & |h_2 \ k_4| & |g_2 \ k_4| & \cdots\cdots & - & |f_2 \ g_4| & |m_2 \ r_3| \\ & \cdot\cdot\cdot & \cdot\cdot\cdot & \cdots\cdots & & \cdot\cdot\cdot & \cdots \\ & |h_1 \ k_2| & -|g_1 \ k_2| & \cdots\cdots & & |f_1 \ g_2| & |r_2 \ s_3|. \end{array}$$

This includes a notable theorem of Hesse's regarding axisymmetric determinants, to which he devotes six pages (pp. 246–251) of his well-known paper "Ueber Determinanten und ihre Anwendung in der Geometrie, ... " (*Crelle's Journ.*, xlix. pp. 243–264), the last sentence of which is "Auf dem angegebenen Wege lässt sich auch, unter der Voraussetzung dass $u_{k\lambda} = u_{\lambda k}$ sei, die allgemeine Gleichung:

$$
\begin{vmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ u_{21} & u_{22} & \cdots & u_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ u_{n1} & u_{n2} & \cdots & u_{nn} \end{vmatrix} \cdot \mathrm{U}
$$

$$
= \begin{vmatrix} u_{11} & u_{12} & \cdots & u_{1n} & \alpha_1 \\ u_{21} & u_{22} & \cdots & u_{2n} & \alpha_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{n1} & u_{n2} & \cdots & u_{nn} & \alpha_n \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n & 0 \end{vmatrix} \cdot \begin{vmatrix} u_{11} & u_{12} & \cdots & u_{1n} & \gamma_1 \\ u_{21} & u_{22} & \cdots & u_{2n} & \gamma_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{n1} & u_{n2} & \cdots & u_{nn} & \gamma_n \\ \gamma_1 & \gamma_2 & \cdots & \gamma_n & 0 \end{vmatrix} - \begin{vmatrix} u_{11} & u_{12} & \cdots & u_{1n} & \alpha_1 \\ u_{21} & u_{22} & \cdots & u_{2n} & \alpha_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{n1} & u_{n2} & \cdots & u_{nn} & \alpha_n \\ \gamma_1 & \gamma_2 & \cdots & \gamma_n & 0 \end{vmatrix}
$$

ableiten, wo U eine ganze homogene Function 2ten Grades sowohl in Rücksicht auf die Grössen α als auf γ und vom $n-2$ten Grade in Rücksicht auf die Grössen u ist." It will be seen that in our extension the exact form of Hesse's U is specified.

7. It is evident that so far as the right-hand side of the identity in § 5 is concerned analogous specialisation is possible when any number of factors are adjugate determinants.

By making *all* the factors in the theorem of § 3 adjugate determinants, we see that *the cofactor of any element in the product-determinant of the adjugates of the* n-*line determinants* $\Delta_1, \Delta_2, \Delta_3, \ldots$ *is equal to the corresponding element in the product of* $\Delta_1, \Delta_2, \Delta_3, \ldots$ *multiplied by* $(\Delta_1 \Delta_2 \Delta_3 \ldots)^{n-2}$.

8. Returning now to the point reached at the end of § 5, we take up the question of the *three*-line minors of a product-determinant, and note that as the principal minors of a four-line determinant are three-line minors, one case has already been considered, and that the result is quite in keeping with that represented diagrammatically at the close of § 5, the single point of difference being that instead of the word 'two-line' we have to substitute 'three-line' throughout. Taking the next case where three-line minors are possible, namely, when the factor-determinants are of the fifth order, we find the three-line minor which occupies rows 1, 2, 3 and columns 3, 4, 5 of the products

$$
|a_1b_2c_3d_4e_5| \cdot |f_1g_2h_3k_4l_5|,
$$
$$
|a_1b_2c_3d_4e_5| \cdot |f_1g_2h_3k_4l_5| \cdot |m_1n_2r_3s_4t_5|,
$$
$$
\cdots\cdots\cdots\cdots
$$

and which, as we know, are representable by

$$
\begin{vmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ b_1 & b_2 & b_3 & b_4 & b_5 \\ c_1 & c_2 & c_3 & c_4 & c_5 \end{vmatrix} \cdot \begin{vmatrix} f_3 & f_4 & f_5 \\ g_3 & g_4 & g_5 \\ h_3 & h_4 & h_5 \\ k_3 & k_4 & k_5 \\ l_3 & l_4 & l_5 \end{vmatrix},
$$

$$\cdot \quad \begin{vmatrix} a_1 & a_2 & \dots & a_5 \\ b_1 & b_2 & \dots & b_5 \\ c_1 & c_2 & \dots & c_5 \end{vmatrix} \cdot \begin{vmatrix} f_1 & f_2 & \dots & f_5 \\ g_1 & g_2 & \dots & g_5 \\ h_1 & h_2 & \dots & h_5 \\ k_1 & k_2 & \dots & k_5 \\ l_1 & l_2 & \dots & l_5 \end{vmatrix} \cdot \begin{vmatrix} m_3 & m_4 & m_5 \\ n_3 & n_4 & n_5 \\ r_3 & r_4 & r_5 \\ s_3 & s_4 & s_5 \\ t_3 & t_4 & t_5 \end{vmatrix},$$

. . . .

to be expressible as bipartites whose law of formation agrees, and is seen to agree of necessity, with that specified in the simpler case. Hence we conclude that *every minor of the product-determinant equal to* $\Delta_1\Delta_2 \dots \Delta_z$, *say the* r-*line minor situated in rows* $\alpha, \beta, \gamma, \dots$ *and in columns* $\alpha', \beta', \gamma', \dots$ *is expressible in the same form as the elements of that determinant, that is to say, in the form of a bipartite, the elements of which are in order the* r-*line minors formable from rows* $\alpha, \beta, \gamma, \dots$ *of* Δ_1, *from* $\Delta_2, \Delta_3, \dots$, *and from columns* $\alpha', \beta', \gamma', \dots$ *of* Δ_7.

(*Issued separately May* 29, 1907.)

XII.—Heusler's Magnetic Alloy. By Alexander D. Ross, M.A., B.Sc., Houldsworth Research Scholar, University of Glasgow. *Communicated by* Professor A. Gray, F.R.S.

(MS. received March 26, 1907. Read May 6, 1907.)

Since the discovery by Heusler in 1903 of a magnetic alloy of copper, manganese, and aluminium, considerable interest has been aroused by the peculiarities of its properties and the difficulty of their explanation. In the summer of 1905 a preliminary investigation of a sample of the alloy was made in the Physical Laboratory of Glasgow University, and the results obtained have been published in a paper by Professor A. Gray.*

The tests now described were carried out on an alloy with a somewhat lower percentage of copper, the composition being:—25 per cent. manganese, 12·5 per cent. aluminium, a trace of lead, and the remainder copper. The specimens were cast in the form of rings and rods by Messrs Steven & Struthers of Glasgow. So far only the rods have been used, the tests being made by the magnetometric method. In order to avoid any difficulty regarding the effective lengths of the specimens, elongated ellipsoids of revolution have been used instead of cylindrical rods. The factors investigated by Ewing have been employed to give the demagnetising force due to the specimen and the true magnetising field (H).

With a view to testing the apparatus, preliminary I-H cycles were taken with a rod of the alloy about 60 cms. long and 1·30 cms. in diameter. It was tested in the condition as supplied by the makers, the casting being simply " dressed," and the ends cut square. A portion about one-third the length of the original rod was cut off and retested. On applying the corrections investigated by Du Bois, the two curves were in good agreement. The alloy exhibited very little hysteresis, and the saturation point was well marked.

A specimen, which had been ground on emery to an ellipsoidal form, was subjected to thermal treatment. After being taken through a cycle (fig. 1, curve 1) it was heated for ten minutes to 50° C., slowly cooled, and retested. No change in magnetic quality was observed. Successive heatings were then made to 100° for a few minutes, to 100° for two and a half hours, and to 145° for a few minutes. In each case the specimen was slowly cooled

* *Proc. Roy. Soc.*, Sect. A, 77, p. 256.

and tested at the room temperature before proceeding to the next heating. So far the magnetic quality was little affected. The specimen was now baked at a temperature of 160° for three days. This produced a distinct improvement in quality (fig. 1, curve 2). The process of heating for short and long periods was continued step by step, and the magnetic quality showed gradual deterioration with a marked increase in hysteresis. Fig. 1, curve 3 shows the final state which was obtained after heating for two and a half days to 220° C. In the heating up to and including 160° a gas furnace

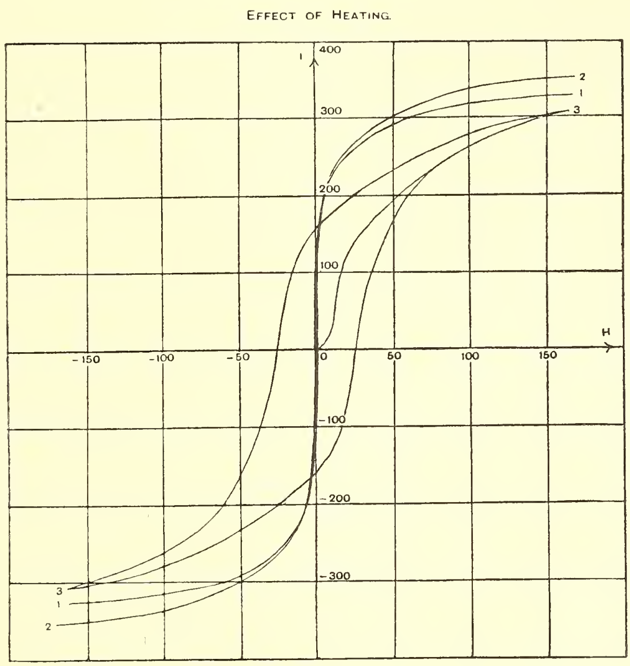

Fig. 1.

was employed, but for prolonged heating at higher temperatures an electric oven was constructed and found to give very steady temperatures. In all the experiments the temperature was measured with a platinum and platinum-iridium thermometer. The series of heatings proved that the time effect is very large, especially in regard to the increase in hysteresis, and it was decided to reserve its further investigation until a special furnace could be constructed for this purpose.

Another ellipsoidal specimen was employed in an investigation of the permanent effects produced by bringing the alloy for a short time to a series of increasing temperatures and quenching thereat. But before this treatment was carried out the magnetic condition of the specimen was tested

both while the temperature was that of liquid air and after the normal temperature had been resumed. The specimen was enclosed in a glass tube which fitted into a cylinder of asbestos placed inside the magnetising solenoid. One end of the glass tube was closed; the other was open and bent up so that the tube could be kept filled with liquid air. The curve (fig. 2, curve 2) obtained when the specimen was thus cooled to 190° C. showed an increase of about 25 per cent. in the saturation value of I, but otherwise did not differ materially from that obtained at the room tempera-

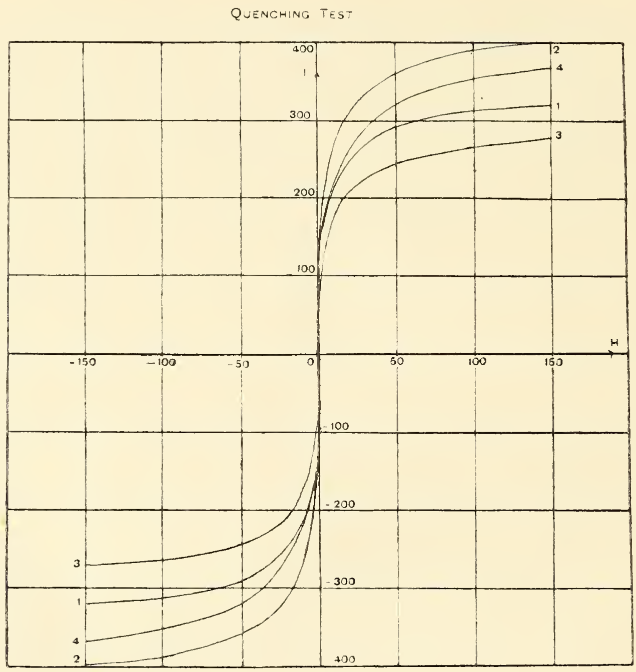

Fig. 2.

ture (fig. 2, curve 1).* On retesting the specimen when it had warmed up to normal temperature it was found that the improvement had been only temporary.

The specimen was now heated to 400° C. and quenched by plunging vertically into cold water. The magnetic quality was somewhat destroyed by this treatment, and there was a decided diminution in hysteresis (fig. 2, curve 3). [With steel, nickel, and cobalt similar effects have been obtained by the author, but in a diminishing ratio.] Immersion of the quenched alloy in liquid air produced a temporary increase of over 30 per cent. in the

* The effect on the magnetometer due to the liquid air being magnetic was quite negligible.

saturation value of I, with a slight augmentation of the hysteresis (fig. 2, curve 4). The specimen was laid aside for nearly ten months, and in this interval almost recovered its original susceptibility, while the dimunition in the hysteresis which had been effected by the quenching was only reversed to a very slight extent.

The quenching tests were again resumed and cycles obtained for the following series of temperatures: 400°, 450°, 500°, 555°, 610°, 660°, 712°, and 745°. At each stage three tests were made: (1) with the quenched specimen, (2) with the specimen immersed in liquid air, and (3) with the specimen back to atmospheric temperature. Fig. 3 exhibits the nature of the results

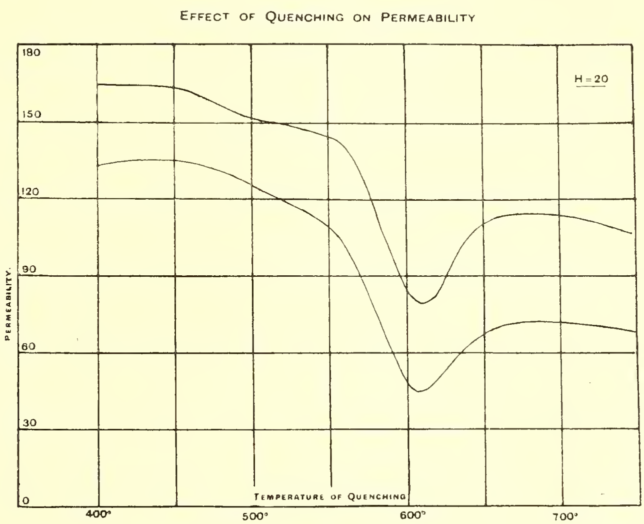

Fig. 3.

obtained in this investigation. The abscissæ represent the temperatures of quenching, and the ordinates give the permeability corresponding to a field of 20 C.G.S. units for the specimen in the quenched condition. The lower curve shows the values obtained in the cycles at ordinary temperature; those obtained with the specimen in liquid air are shown in the upper curve. It will be noticed that there is a very marked diminution in permeability on quenching at 610°, and on quenching at higher temperatures the quality is to a considerable degree restored.

Throughout the investigation there were traces of cracking in the specimen due to the quenching, and this would probably affect the magnetic tests to a slight extent. While being heated in the furnace preparatory to quenching at 800°, the specimen broke in two. An examination of the sections of fracture showed that a decided change in constitution had taken

place. Originally fairly homogeneous, if we except the presence of a few small "blow-holes," the alloy now exhibited at least two constituents—large, irregular, and lustrous nodules appearing imbedded in the more uniform matrix. To such changes in structure we may probably ascribe the peculiar behaviour on quenching at 610°.

The above research has been carried out in the Physical Laboratory of the University of Glasgow, and the author desires to take this opportunity of thanking Professor Gray and Dr Muir for suggestions received during the progress of the work. The preliminary experiments were carried out in collaboration with Mr Robert Jack, and the author proposes to continue the research on the lines indicated above.

(*Issued separately May* 29, 1907.)

XIII.—The Relation between Normal Take-up or Contraction and Yarn Number for the same degree of Twist in Twisted Threads. By Thomas Oliver, B.Sc. (London and Edinburgh), Carnegie Research Fellow. *Communicated by* Dr C. G. Knott.

(MS. received December 17, 1906. Read same date.)

In a former paper * communicated to this Society in February, the author worked out experimentally and analytically a relation between normal take-up and degree of twist in the same thread.

This relation for the take-up in the second twisting of a two-ply yarn is

$$y = \frac{\pi^2 d^2}{8L} x(5x + 2n) \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

where d = diameter of single thread.
L = length of single thread under experiment.
x = turns of twist impressed in the second twisting.
n = turns of twist in each single.

The present investigation deals with the problem in which x is a constant in different sizes of threads. Therefore n and d are the only variables on the right-hand side of the equation.

Single threads are approximately cylinders. But as the sections in any one thread are not uniform, and are difficult to measure, it is found totally impracticable to number yarns in the same way as wires are gauged, *i.e.* proportional, directly or indirectly, to their diameters.

The only practical basis of numbering is the relation of length to weight. All systems of numbering yarns in use in the textile industries may be classified under one or other of two categories, viz., "inverse" or "direct." In the "inverse" systems, the yarn number is *inversely* proportional to the weight of unit length. In the "direct" systems, on the other hand, the yarn number is *directly* proportional to the weight of unit length. It is probable that more than 90% of the yarns throughout the world are numbered inversely. The only notable exceptions are jute, reeled and thrown silk; these are numbered on direct systems. Cotton, spun silk, linen, worsted, and practically all woollen yarns, are numbered inversely.

* *Proc. Roy. Soc. Edin.*, 1906, vol. xxvi., pt. iii. p. 189.

Let . w = weight of unit length of thread.
c = " inverse " yarn number.
k = weight of unit length of No. 1's, a constant.
A = average area of cross-section of thread.
d = average diameter of thread.
ρ = volume density of a thread in its commercial state;

$$\text{then} \quad C=\frac{k}{w} \qquad \text{but} \quad A=\frac{\pi}{4}d^2$$
$$\text{and} \quad w=A\rho$$
$$=\frac{\pi}{4}d^2\rho$$
$$\therefore \quad d^2=\frac{4k}{\pi\rho}\cdot\frac{1}{C}.$$

Substituting in the expression for the take-up, y:

$$y=\frac{\pi k}{2\rho L}x(5x+2n)\frac{1}{C} \quad . \quad . \quad . \quad . \quad (2)$$

n and C are the only variables on the right-hand side of the equation, if we confine the comparison to threads of similar nature in which ρ may be assumed constant.

If n is a constant throughout the range of threads of different sizes in the same quality, then equation (2) may be written

$$y=cA. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where y = take-up,
$c=\frac{\pi k}{2\rho}x(5x+2n)$, a constant.
A = 1/C, the yarn number reciprocal.

For experimental purposes, n, the twist in the singles may be kept constant or varied at pleasure. In commercial yarns, n is never constant throughout a range of different yarn numbers. The degree of twist must increase as the thread becomes smaller.

The precise relation which should hold between the degree of twist and size of thread has long been a debatable question. The late Mr T. R. Ashenhurst of Bradford Technical College, founding his argument on geometrical considerations, enunciated the theory that the twist in yarn should be directly proportional to the square root of the yarn number, or inversely proportional to the diameter of the thread. This theory has been styled by later writers, " the theory of relative twist."

The late Mr M. M. Buckley of Halifax, from the results of experiments, contended that twist in yarn should be simply proportional to the yarn number.

From the cloth designer's point of view, there is doubtless much to be said for Ashenhurst's theory. The problem is, however, far more a dynamical than a geometrical one, and it is probable that Buckley's theory is much nearer the truth for crossbred worsted yarns.

In commerce, ranges of yarns are found with all kinds of relations holding between the degrees of twist in their members, according to the fancy of the spinner or the demands of his customers. Frequently the relation cannot be construed into any simple algebraic form; as a rule, however, the twists in a range of yarns are approximately proportional to some power of the yarn number which lies between $\frac{1}{2}$ and 1. The author's experiments and analyses on this subject are not yet complete, and therefore it is not intended to discuss the matter further than is necessary for the working out of the "take-up" problem.

The author has found that between the limits of crossbred numbers usually spun, the degree of twist is approximately a linear function of the yarn number;

i.e. $n = a\mathrm{C} + b$,

where n = degree of twist in singles.

C = yarn number.

a and b are constants.

In one range of yarns, the degree of twist in turns per inch,

$$n = \cdot 3\mathrm{C} + 2 \quad . \quad . \quad . \quad . \quad (4)$$

between the limits 10's and 40's.

From this range, three threads numbered 12's, 16's, and 24's were selected for experiment. Their diameters were measured by means of a microscope fitted with an eyepiece micrometer.

Table I. shows various data necessary to effect a comparison between the experimental and analytical results.

TABLE I.

Yarn number. C	Average diameter of singles. d	Turns per inch in singles. n	$\frac{\pi^2 d^2}{8\mathrm{L}} \cdot \mathrm{L}^2$ * or $\frac{k}{4}$	$\frac{k}{5}$	$\frac{2k}{15}$
12	·0145″	5·5	·0260	·0208	·0139
16	·0124″	7·0	·0190	·0152	·0101
24	·0101″	9·0	·0126	·0101	·0067

* If x and n in formula (1) be taken as turns per inch, then $y = \frac{\pi^2 d^2}{8\mathrm{L}} \cdot \mathrm{L}^2 x(5x + 2n)$. L is taken as 100, so as to give percentage results.

In Part I. (p. 205, vol. xxvi., pt. iii.) it was set forth that, owing to the friction amongst the individual fibres in the thread, the analytical expression obtained for the "take-up" did not give the same result as obtained by experiment. It was also shown that for the direct or open-band second twisting,

$$y=\frac{k}{5}x(5x+2n) \qquad . \qquad . \qquad . \qquad . \quad (5)$$

and for the inverse or crossband second twisting,

$$y=\frac{2}{15}kx(5x-3n) \qquad . \qquad . \qquad . \qquad . \quad (6)$$

gave results which did not differ materially from those obtained by experiment. In the above equations $k=\frac{\pi^2 d^2 L}{2}$, when x and n are expressed as turns per inch.

In equation (6), for convenience x is to be taken positive although measured in the negative direction.

Table II. shows the "take-up" per cent. on the hypothesis that equations (5) and (6) are true.

TABLE II.

PERCENTAGE CONTRACTION.

Turns per inch. x	"Direct" or "openband" twist.			"Inverse" or "crossband" twist.		
	2/12's.	2/16's.	2/24's.	2/12's.	2/16's.	2/24's.
1	·33	·29	·23	−·16	−·16	−·15
2	·88	·73	·56	−·18	−·22	−·23
3	1·63	1·32	1·00	−·06	−·18	−·24
4	2·59	2·07	1·53	+·19	−·04	−·19
5	3·77	2·96	2·17	·59	+·20	−·07
6	5·14	3·96	2·91	1·13	·54	+·12
7	6·72	5·15	3·75	1·80	·99	·37
8	8·52	6·48	4·69	2·62	1·53	·70
9	10·54	7·97	5·73	3·57	2·18	1·08
10	...	...	...	4·66	2·93	1·54
11	...	...	...	5·89	3·78	2·06
12	...	...	...	7·53	4·73	2·65

Table III. shows the "take-up" per cent. if

$$y=x(5x+2n)\div 4C \text{ for the direct second twisting;} \qquad . \qquad (7)$$

$$\text{and } y=x(5x-3n)\div 6C \text{ for the inverse second twisting.} \qquad . \qquad . \quad (8)$$

C = single yarn number, as before.

TABLE III.
PERCENTAGE CONTRACTION.

Turns per inch. x	"Direct" or "openband" twist.			"Inverse" or "crossband" twist.		
	2/12's.	2/16's.	2/24's.	2/12's.	2/16's.	2/24's.
1	·33	·30	·24	−·16	−·17	−·15
2	·87	·75	·58	−·18	−·23	−·24
3	1·62	1·36	1·03	−·06	−·19	−·25
4	2·58	2·13	1·58	+·19	−·04	−·19
5	3·75	3·05	2·24	·59	+·21	−·07
6	5·12	4·13	3·00	1·12	·55	+·12
7	6·73	5·36	3·86	1·80	1·02	·39
8	8·50	6·75	4·83	2·61	1·58	·72
9	10·50	8·29	5·98	3·56	2·27	1·12
10	...	...	...	4·65	3·02	1·59
11	...	...	...	5·88	3·90	2·14
12	...	...	...	7·53	4·88	2·75

For the experimental investigation the same apparatus was used as that described on p. 197, vol. xxvi., pt. iii. The thread was stretched between two clamps. One of these formed the end of the shorter arm of a bell-crank lever, the other arm of which carried a weight. This weight could be varied so that the thread might be twisted under any tension desired. The other clamp B formed the end of a bar which could be rotated by a wheel, the turns being indicated on a dial. The bar could be moved in the direction of the thread by a special form of rack and pinion, so that the longer arm of the bell-crank lever always remained horizontal. The motion of the bar could be measured on a scale, while the thread contractions were obtained by taking the difference of the readings on the scale.

Table IV. shows the results of experiments.

TABLE IV.
PERCENTAGE CONTRACTION.

Turns per inch. x	"Direct" or "openband" twist.			"Inverse" or "crossband" twist.		
	2/12's.	2/16's.	2/24's.	2/12's.	2/16's.	2/24's.
1	·46	·40	·30	−·14	−·20	−·11
2	1·02	·91	·65	−·15	−·22	−·17
3	1·86	1·59	1·17	−·09	−·19	−·11
4	2·90	2·41	1·85	+·21	+·02	−·02
5	3·92	3·26	2·49	·55	·30	+·12
6	5·20	4·25	3·29	1·08	·62	·26
7	6·63	5·38	4·10	1·76	1·10	·53
8	8·32	6·65	5·05	2·55	1·67	·85
9	...	...	...	3·44	2·32	1·20
10	...	...	...	4·55	3·05	1·62
11	...	...	...	5·88	3·92	2·07
12	...	...	...	7·30	4·95	2·65

The results of Table III. are shown graphically in fig. 1, while the graphical interpretation of Table IV. appears in fig. 2. Percentage contractions are plotted as ordinates, and yarn number reciprocals as abscissæ:

The reciprocal of 12 = ·0833
" " 16 = ·0625
" " 24 = ·0417

Each graph has its corresponding degree of two-ply twist affixed; *e.g.*

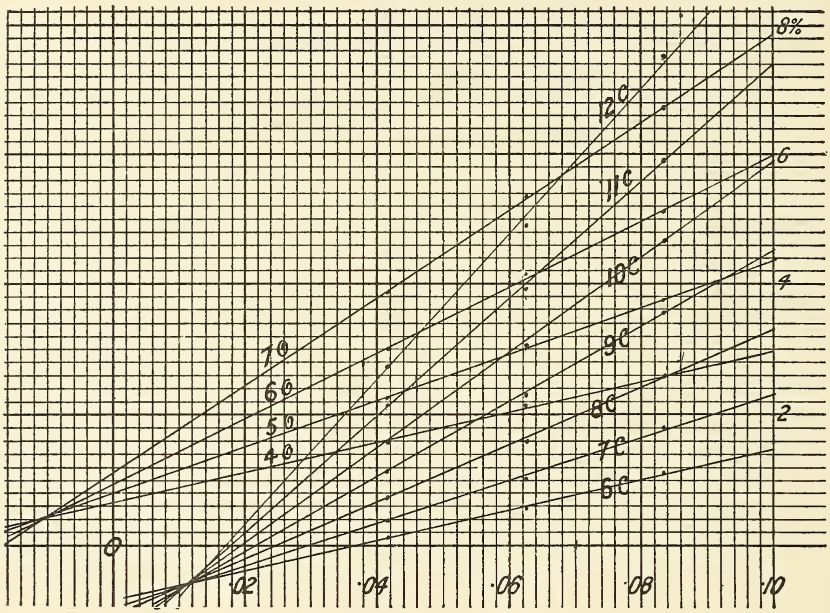

Fig. 1.

11C denotes the graph for 11 turns of crossband twist, while 6O denotes the graph for 6 turns of openband twist.

The graph which lies most evenly among the plotted points is a straight line in each case. Further, the openband lines all converge to a point in the second quadrant, while the crossband lines converge to a point in the fourth quadrant. That the graph may be a straight line is dependent on the degree of twist in the singles being a linear function of the yarn number.

Equations (7) $y = x(5x + 2n) \div 4C$
and (8) $y = x(5x - 3n) \div 6C$

are empirical formulæ which give results of practically the same value as the fundamental formulæ (5) and (6), and also approximate closely to

the experimental results. They have also the advantage of ready reference; the yarn number is always available, while the diameter could only be obtained after a tedious micrometer measurement.

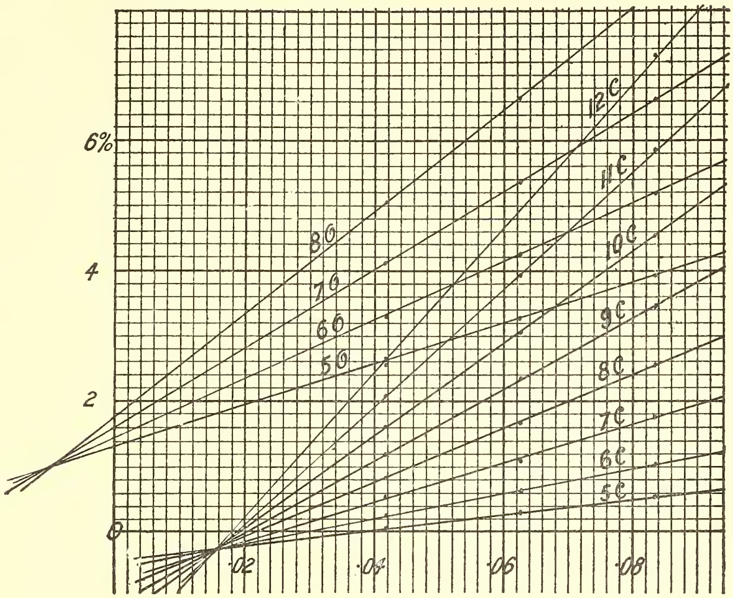

Fig. 2.

Formula (4) shows that

$$n = \cdot 3C + 2 \text{ approximately.}$$

Substituting this value for n in formulæ (7) and (8),

$$y = \frac{x(5x + 2n)}{4C}$$

$$= \frac{x(5x + 4 + \cdot 6C)}{4C}$$

$$= \frac{x}{4}(5x + 4)\frac{1}{C} + \cdot 15x$$

$$= \frac{x}{4}(5x + 4)A + \cdot 15x \qquad \text{where } A = \frac{1}{C}$$

$$= \alpha A + \beta \qquad \text{when } x \text{ is constant.}$$

Similarly for crossband twist,

$$y = \frac{x(5x - 3n)}{4C}$$

$$= \frac{x}{6}(5x - 6)A - \cdot 15x$$

$$= \alpha' A - \beta \qquad \text{when } x \text{ is constant.}$$

These are equations of the first degree in A, and therefore represent straight line graphs, which in the first instance cut the y-axis above the origin, and in the second instance cut the y-axis below the origin. The crossband line should cut the axis as much below the origin as the open-band line for the same value of x cuts the axis above the origin. Any divergence from this condition noticeable in fig. 1 is due to the fact that in the actual yarns experimented with, the degrees of twist in the singles were not exactly linear functions of the yarn number. This fact may be seen from fig. 3, in which the ordinates represent degrees of twist in turns per inch, n, and the abscissæ represent yarn numbers, C. The straight line does not pass exactly through the points.

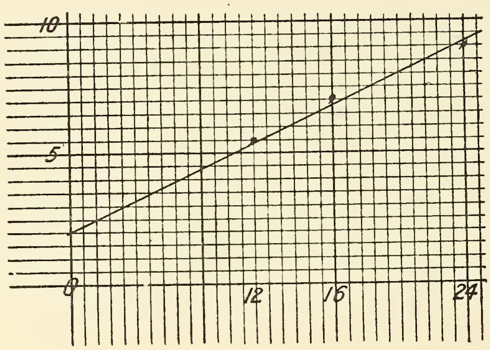

Fig. 3.

To test the theory further, the same yarns were taken, but the twists in the singles were altered to seven turns per inch in each case.

Since the "direct" twist gives a two-ply thread, which is too hard for any useful purpose, and the results obtained are therefore of no practical value, the remainder of the investigation was confined to the "inverse" twist.

Table V. shows the experimental results for the case when all the single threads had seven turns per inch each.

Fig. 4 shows the results of Table V. As before, ordinates represent percentage contractions; and abscissæ, yarn number reciprocals.

The point of convergence of the lines is practically the origin. This is quite in accordance with the theory. Since n is constant in all the threads,

$$y = \frac{x(5x - 3n)}{6C} \quad \text{may be written}$$

$$= kA \quad \text{where } A = \frac{1}{C}, \text{ the yarn number reciprocal, and } k \text{ is a constant.}$$

Now in this equation y vanishes simultaneously with A; *i.e.*, the graphical picture of the equation is a straight line passing through the origin.

TABLE V.

Percentage Contraction.

Turns per inch. x	"Inverse" or "crossband" twist.		
	2/12's.	2/16's.	2/24's.
1	− ·21	− ·17	− ·12
2	− ·29	− ·24	− ·16
3	− ·23	− ·18	− ·12
4	− ·05	− ·04	− ·03
5	+ ·28	+ ·23	+ ·14
6	·74	·59	·39
7	1·35	1·08	·72
8	2·10	1·69	1·11
9	2·98	2·35	1·57
10	4·00	3·14	2·10
11	5·14	4·03	2·70
12	6·43	5·04	3·35

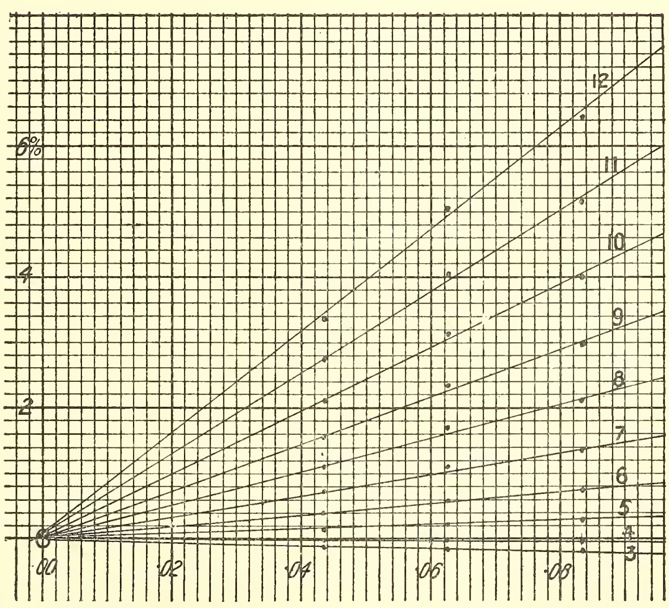

Fig. 4.

It will now be instructive to consider an extreme case in which the degrees of twist in the singles have no simple relation to the yarn numbers.

A range of crossbred worsted yarns made from a different quality of "top" ("top" is the term applied to the long straight fibres of wool which pass subsequently into the worsted thread; the short curly fibres are combed out and rejected as "noil") was selected.

Table VI. shows the yarn numbers and their respective degrees of twist which formed the range.

TABLE VI.

Yarn No.	12's.	24's.	32's.	40's.
Turns per inch in singles.	5·6	8·0	16·0	23·0

Fig. 5 shows the graphical relationship between the yarn numbers as abscissæ and the turns of twist as ordinates.

Table VII. shows the percentage "take-up" for the "inverse" twisting of the various members of this range of threads.

Fig. 6 contains the corresponding graphs connecting percentage "take-up" as ordinates and yarn number reciprocals as abscissæ.

TABLE VII.

Percentage Contraction.

Turns per inch.	"Inverse" or "crossband" twist.			
	2/12's.	2/24's.	2/32's.	2/40's.
1	−·09	−·08	−·10	−·25
2	0	−·09	−·13	−·40
3	+·25	+·02	−·08	−·43
4	·70	·21	+·03	−·45
5	1·25	·48	·19	−·40
6	1·91	·85	·39	−·31
7	2·70	1·25	·59	−·20
8	3·56	1·69	·85	−·04
9	4·46	2·22	1·13	+·12
10	5·73	2·80	1·42	·30
11	7·01	3·56	1·78	·50
12	8·40	4·45	2·10	·72
13	...	5·35	2·52	·95
14	...	6·35	2·90	1·20
15	...	7·36	3·35	1·42

Reciprocal of 12 = ·0833
" 24 = ·0417
" 32 = ·0312
" 40 = ·025

It will be seen from fig. 6 that in this case the graphs are not straight lines throughout the range. But as the yarn number increases or its

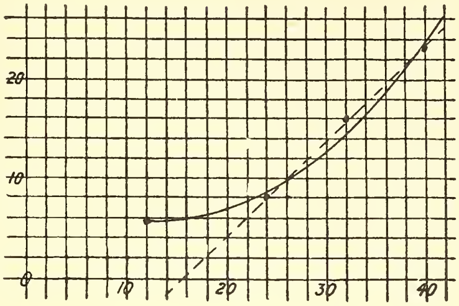

Fig. 5.

reciprocal decreases, the graphs become more nearly straight. (The approximately straight parts of the curve are produced tangentially to

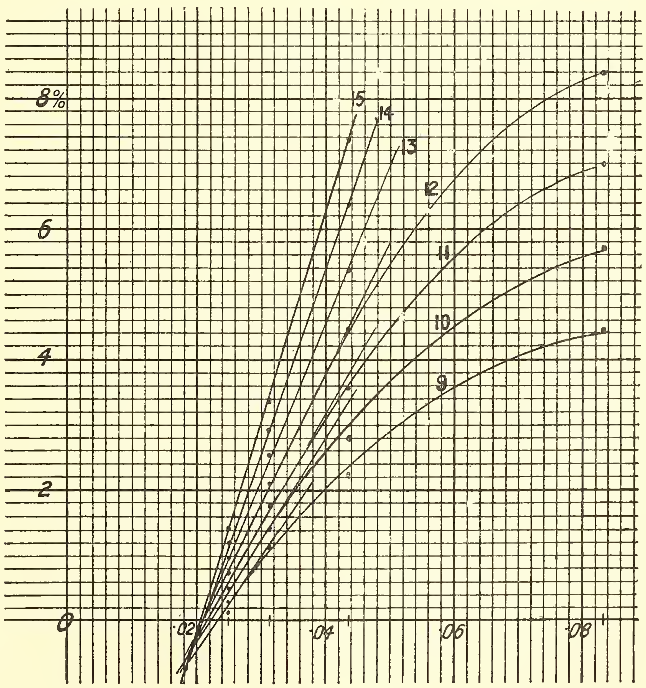

Fig. 6.

the curve in each case.) That this is in accordance with the theory is evident if we refer to fig. 5. A straight line can be drawn, as in the diagram, through the plotted points corresponding to 24's, 32's, and 40's.

We have seen that when such a linear relationship holds, the graph for the "take-up" is also rectilinear. The equation to the flowing curve, which lies most evenly amongst the points of fig. 5, is:

$$\begin{aligned} n &= \cdot 022(C-12)^2+5\cdot 6 \\ &= \cdot 022C^2-\cdot 53C+8\cdot 8 \quad . \quad . \quad . \quad . \quad (9) \end{aligned}$$

This equation does not hold beyond the limits 12's and 40's, and it will be seen from fig. 5 that even between these limits the graph does not represent closely the relationship between the plotted points. Substituting this value of n in the expression for the "take-up,"

$$y=(\cdot 83x^2-2\cdot 93x)A+\cdot 177x-\frac{\cdot 0073x}{A} \qquad \text{where } A=\frac{1}{C}.$$

Let us consider the cases when $x=10$ and $x=12$;

$$\text{then,} \qquad y=53\cdot 7A+1\cdot 77-\frac{\cdot 073}{A} \qquad \text{when } x=10\,;$$

$$y=84\cdot 0A+2\cdot 12-\frac{\cdot 087}{A} \qquad \text{when } x=12.$$

On examining the above expressions for y we see that they can be broken into two parts, y_1 and y_2:

$$\text{where} \qquad \left.\begin{aligned} y_1 &= 53\cdot 7A+1\cdot 77 \\ y_2 &= \frac{\cdot 073}{A} \end{aligned}\right\} \text{when } x=10$$

$$\text{and} \qquad y=y_1-y_2\,;$$

y_1 is an equation to a straight line,

and y_2 is an equation to a rectangular hyperbola.

Also, we are tempted to say that when A is very large, the graph for the "take-up" approximates to a straight line, and similarly when A is small, the "take-up" graph approaches hyperbolic form. This assumption would be quite erroneous, because the equation is only known to represent the actual conditions between the limits, ·025 and ·0833 for A. Therefore A can neither be very large nor very small without violating the conditions on which the equation was deduced.

From equation (9) we calculate that for 50's yarn (*i.e.* for $A=\cdot 02$), n, the twist should be 37·3 turns per inch—a proposal which no spinner could entertain.

Fig. 7 shows the synthesis of the graph for $x=10$ and its relation to the graph exhibiting the experimental results for ten turns of twist. The latter is lettered B in the diagram.

It is always possible to obtain two straight lines which approximate closely to a curve between limits. From fig. 8 we see that for the case

under consideration these limits are wide, and may be taken so as to comprise the range in question.

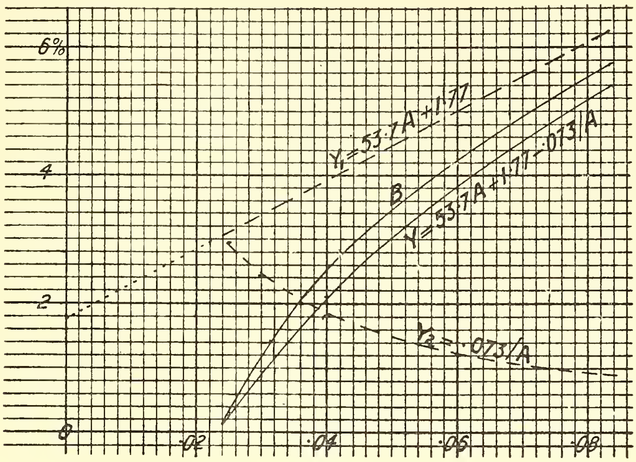

Fig. 7.

Fig. 8 shows the graphs for $x = 10$ and $x = 12$.

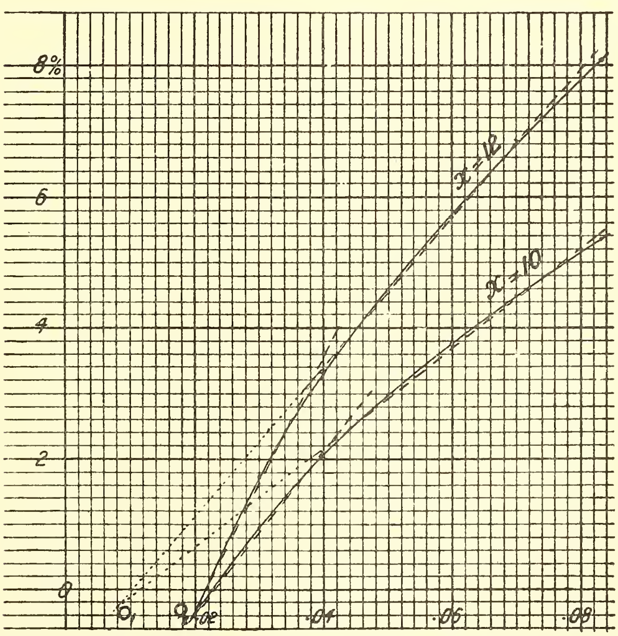

Fig. 8.

The straight lines which approximate to the curves between the limits $A = {\cdot}04$ and $A = {\cdot}08$ intersect when produced at point O_1; similarly, the

straight lines which nearly coincide with the curves between the limits $A = \cdot 025$ and $A = \cdot 04$ intersect at O_2.

In summarising this subject, we find the following main conclusions have been arrived at:—

(1) When the twist in the single thread is a constant throughout a range of crossbred worsted numbers, the contraction is simply proportional to the yarn number reciprocal for any fixed degree of two-ply twist.

(2) When the twists in the single threads can be expressed as a linear function of the yarn number throughout a range of crossbred worsted numbers, the contraction is a linear function of the yarn number reciprocal for any fixed degree of two-ply twist throughout the same range.

In conclusion, the author has pleasure in acknowledging his indebtedness to the Carnegie Trust for the Universities of Scotland for the financial assistance which has enabled him to prosecute this research.

(*Issued separately May* 30, 1907.)

XIV.—The Relation between Normal Take-up or Contraction and Degree of Twist in Twisted Threads when the Singles are of Unequal Sizes. By Thomas Oliver, B.Sc. (London and Edinburgh), Carnegie Research Fellow. *Communicated by* Dr C. G. Knott.

(MS. received December 17, 1906. Read same date.)

In a former paper (vol. xxvi., pt. iii., p. 186) the author proved that the contraction due to forming a single thread,

$$=\frac{\pi^2 d^2}{8l}n^2,$$

and the contraction due to the two-ply twisting,

$$=\frac{\pi^2 D^2}{2L}x^2,$$

where D = diameter of circle in which the axes of singles revolve.
d = diameter of single.
l = length of untwisted single.
L = length of twisted single.
n = turns of twist in single.
x = turns of twist in 2-ply.

Also, if the fibres have perfect freedom of rotation the total contraction will be equal to that due to $n+x$ turns in the single state plus that due to x turns in the double, *i.e.* the total contraction from the single to the two-ply condition

$$=\frac{\pi^2 d^2}{2l}\left\{\frac{(n+x)^2}{4}-\frac{n^2}{4}\right\}+\frac{\pi^2 D^2}{2L}\cdot x^2$$

$$=\frac{\pi^2 d^2}{8l}\left(2nx+x^2\right)+\frac{\pi^2 D^2}{2L}x^2$$

L may be substituted for l, as they differ very little from each other; and writing k for $\frac{\pi^2}{8L}$, the total contraction

$$=kd^2x(2n+x)+4kD^2x^2$$

Let the two unequal singles A and B have diameters d_1 and d_2, and degrees of twist n_1 and n_2 turns respectively. Let them receive a further torsion of x turns in forming into a two-ply thread. Then the contraction on A will be

$$y_1=kd_1{}^2x(2n_1+x)+4kD^2x^2$$

and the contraction on B will be

$$y_2 = kd_2^2x(2n_2 + x) + 4kD^2x^2$$

Assuming that corresponding points on the axes of the singles move in a circle, then $D = \frac{d_1 + d_2}{2}$.

Then
$$y_1 = kd_1^2x(2n_1 + x) + 4k\frac{(d_1 + d_2)^2}{4}x^2$$

$$y_2 = kd_2^2x(2n_2 + x) + 4k\frac{(d_1 + d_2)^2}{4}x^2$$

Writing the ratio $d_1 \div d_2 = f$,

$$y_1 = kd_2^2x\{(2f^2 + 2f + 1)x + 2f^2n_1\} \quad . \quad . \quad . \quad (1)$$

$$y_2 = kd_2^2x\{(f^2 + 2f + 2)x + 2n_2\} \quad . \quad . \quad . \quad (2)$$

When the single threads are of different sizes, corresponding points on their axes may not move in the same circle. Assuming that the centre

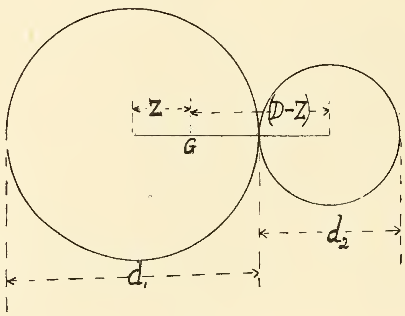

Fig. 1.

of area of cross-section is also the centre of revolution, corresponding points on the axes will revolve in circles of different sizes.

Let z be the distance of the centre of area from the axis of the thick thread A; then, by equating moments about G, the centroid of area,

$$\frac{\pi}{4}d_1^2z = \frac{\pi}{4}d_2^2(D - z)$$

$$= \frac{\pi}{4}d_2^2\left(\frac{d_1 + d_2}{2} - z\right)$$

$$\therefore \quad z = \frac{d_2^2(d_1 + d_2)}{2(d_1^2 + d_2^2)}$$

$$= \frac{f + 1}{f^2 + 1} \cdot \frac{d_2}{2} \qquad \text{where } f = d_1/d_2 \text{ as before.}$$

$$\therefore \quad y_1 = kd_1^2x(2n_1 + x) + 4k(2z)^2x^2$$

$$= kf^2d_2^2x(2n_1 + x) + 4k\left(\frac{f+1}{f^2+1}\right)^2 d_2^2x^2$$

$$= kd_2^2x\left\{f^2(2n_1 + x) + 4\left(\frac{f+1}{f^2+1}\right)^2 x\right\}$$

$$= kd_2^2x\left\{\frac{f^6 + 2f^4 + 5f^2 + 8f + 4}{(f^2+1)^2}x + 2f^2n_1\right\} \quad . \quad . \quad (3)$$

Similarly,

$$y_2 = kd_2^2x(2n_2 + x) + 4k(d_1 + d_2 - 2z)^2x^2$$

$$= kd_2^2x\left\{(2n_2 + x) + 4\left(f + 1 - \frac{f+1}{f^2+1}\right)^2 x\right\}$$

$$= kd_2^2x\left\{(2n_2 + x) + 4f^4\left(\frac{f+1}{f^2+1}\right)^2 x\right\}$$

$$= kd_2^2x\left\{\frac{4f^6 + 8f^5 + 5f^4 + 2f^2 + 1}{(f^2+1)^2}x + 2n^2\right\} \quad . \quad . \quad (4)$$

By substituting suitable numbers in the formulæ, it will be found that y_1 is in general greater than y_2 taking the first assumption as correct, viz., that corresponding points in the axes revolve in the same circle. But if we assume that the corresponding axial points revolve about the centroid of section, then numerical substitution shows that y_2 is vastly greater than y_1. Obviously, between these limits there can be some centre of revolution about which the torsion will give equal values for the contractions in the singles.

Let s = the distance of this point from the axis of the thick thread A;

then

$$y_1 = kd_1^2x(2n_1 + x) + 4k(2s)^2x^2$$

$$y_2 = kd_2^2x(2n_2 + x) + 4k(d_1 + d_2 - 2s)^2x^2$$

Let $y_1 = y_2$ and $f = d_1/d_2$ as before;

then

$$kd_1^2x(2n_1 + x) = kd_2^2x(2n_2 + x) + 4kx^2\{(d_1 + d_2)^2 - 4s(d_1 + d_2)\}$$

$$f^2(2n_1 + x) = 2n_2 + x + 4x\left\{(f+1)^2 - 4(f+1)\frac{s}{d_2}\right\}$$

$$\therefore \quad \frac{s}{d_2} = \frac{3f+5}{16} + \frac{n_2 - f^2n_1}{8(f+1)x}$$

or

$$s = \frac{3f+5}{16}d_2 + \frac{n_2 - f^2n_1}{8(f+1)x}d_2$$

From this result, it is evident that in general there is no single point which can serve as a centre of revolution throughout the second twisting so as to give equal contractions on both singles. Because s will be dependent on x except in the special case when $n_2 = f^2n_1$,

$$\text{or} \quad \frac{n_2}{n_1} = f^2 = \left(\frac{d_1}{d_2}\right)^2$$

Considering threads as cylinders of uniform density, this is equivalent to the case: the degree of twist proportional to the "yarn number" or

"grist" on an inverse system of numbering. This case is considered extreme in usual practice, but is quite within the range of practicability especially in worsted yarns.

In this case s would $= \frac{3f+5}{16} d_2$.

The experimental investigation of this problem requires the apparatus shown diagrammatically in fig. 2. Each single thread, carrying a weight W at one end, passes over an aluminium pulley, P, and is secured at the other end by a clamp. The pulleys are mounted side by side on ball bearings. S is a scale on which contractions may be read by different readings; N is a needle which determines the length of the two-ply thread.

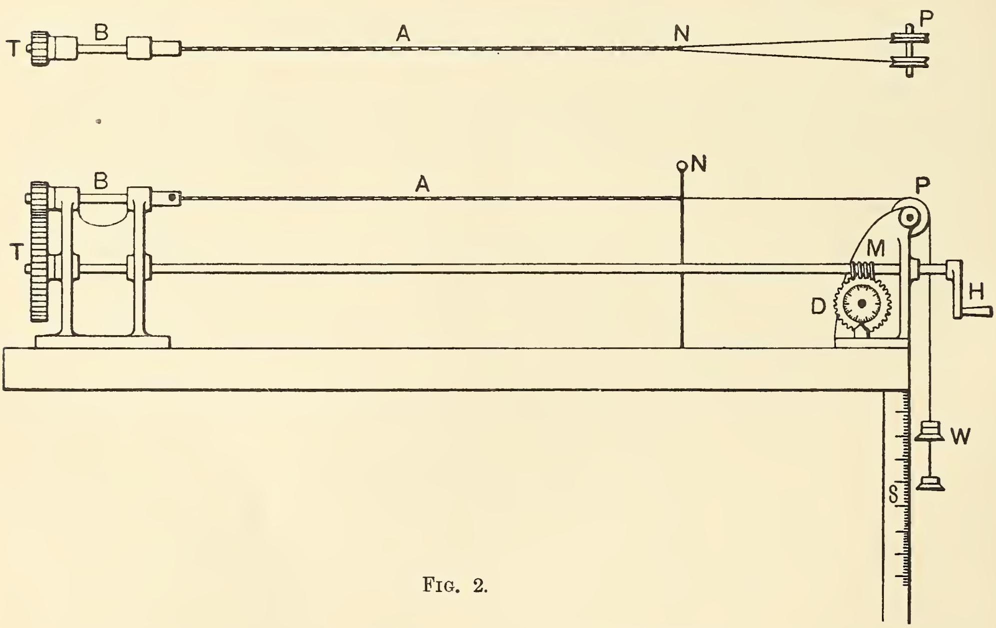

Fig. 2.

Twist is put into the thread A by turning handle H, which communicates rotary motion to shaft B through toothed wheels T. Worm M turns dial D. The number of revolutions is indicated on the dial. In each twist, the tensile load W on the single thread was inversely proportional to the yarn number or "grist," *i.e.* directly proportional to the weight per unit length of yarn.

Table I. shows the percentage contraction for two-ply twists in crossbred worsted: (*a*) 6's and 20's yarns, carrying loads of 13·3 grams and 4 grams respectively; (*b*) 12's and 24's yarns, carrying loads of 4 grams and 2 grams respectively. The results of Table I. are pictured graphically in fig. 3. The ordinates of the diagram represent percentage take-up, while the

abscissæ represent degrees of twist expressed in numbers of turns per inch length of twist. The positive abscissæ represent direct twist, *i.e.* in the same direction as the twist in the single threads; while the negative abscissæ

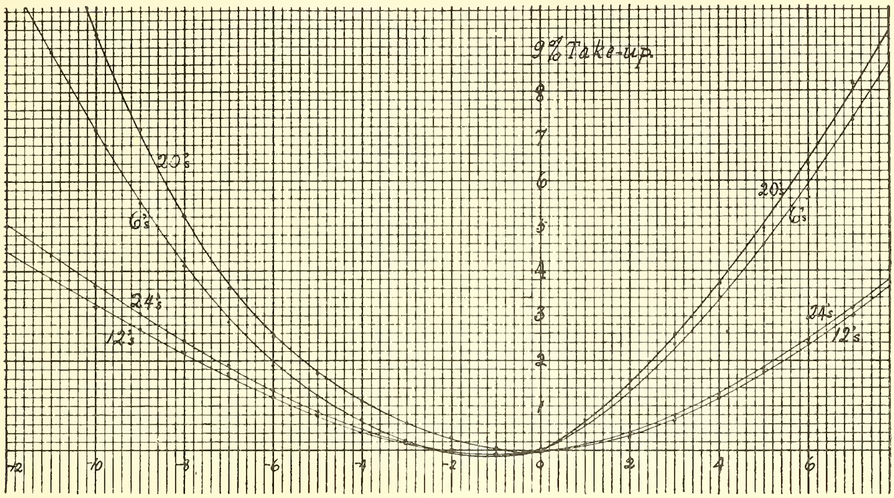

Fig. 3.

represent inverse twist, *i.e.* in the opposite direction to the twist in the singles. Table II. similarly shows the percentage contraction for two-ply twists in Cheviot woollen yarn: (*a*) 16-cut and 28-cut yarns, carrying

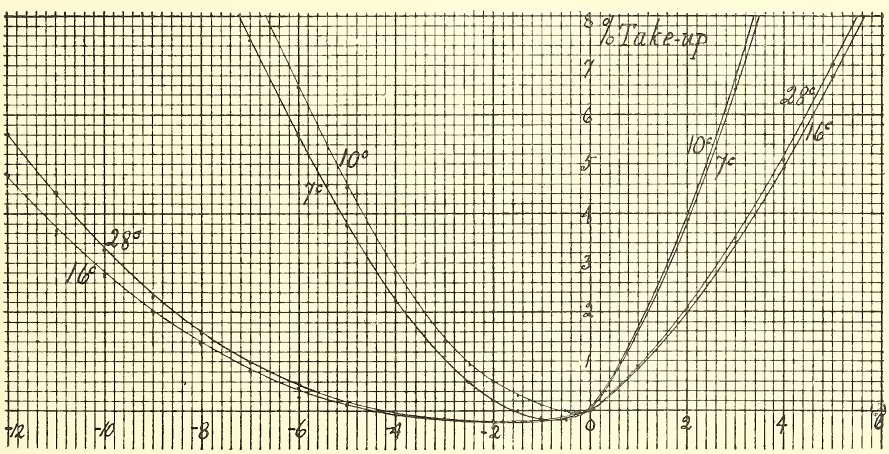

Fig. 4.

17·5 grams and 10 grams respectively; (*b*) 7-cut and 10-cut yarns, carrying 18 grams and 12·6 grams respectively. Fig. 4. shows graphically the results of Table II.

TABLE I.

Percentage Contraction for Worsted Yarns.

Turns per inch.	6's and 20's twist.				12's and 24's twist.			
	Direct.		Inverse.		Direct.		Inverse.	
	6's.	20's.	6's.	20's.	12's.	24's.	12's.	24's.
1	·61	·70	−·16	+·08	·10	·14	−·09	−·06
2	1·39	1·58	−·04	·30	·32	·39	0	+·02
3	2·35	2·58	+·18	·62	·68	·76	+·16	·20
4	3·40	3·72	·69	1·12	1·20	1·29	·40	·49
5	4·58	4·96	1 21	1·72	1·76	1·84	·79	·89
6	5·81	6·37	1·92	2·53	2·36	2·48	1·23	1·35
7	7·33	8·15	2·87	3·62	3·01	3·16	1·70	1·90
8	8·91	10·17	4·10	5·22	3·68	3·88	2·18	2·46
9	...	...	5·51	7·16	4·41	4·64	2·70	3·07
10	...	...	7·16	9·31	5·16	5·44	3·21	3·68
11	...	...	8·92	11·36	...	...	3·83	4·32
12	...	...	10·89	14·12	...	...	4·51	5·01

TABLE II.

Percentage Contraction for Woollen Yarns.

Turns per inch.	16-cut and 28-cut twist.				7-cut and 10-cut twist.			
	Direct.		Inverse.		Direct.		Inverse.	
	16 cut.	28 cut.	16 cut.	28 cut.	7 cut.	10 cut.	7 cut.	10 cut.
1	·88	·92	−·20	−·19	1·61	1·65	−·16	+·09
2	1·98	2·08	−·26	−·24	3·89	3·98	+·20	·59
3	3·39	3·50	−·19	−·18	6·50	6·71	1·10	1·42
4	4·92	5·08	−·12	−·08	10·11	10·50	2·22	2·79
5	6·76	7·01	+·10	+·19	13·69	15·51	3·76	4·52
6	8·46	8·79	·48	·55	...	...	5·59	6·56
7	10·58	11·22	·80	·96	...	...	7·51	8·75
8	12·91	13·88	1·35	1·56	...	...	...	...
9	...	...	2·01	2·33	...	...	...	...
10	...	...	2·76	3·27	...	...	...	...
11	...	...	3·66	4·44	...	...	...	...
12	...	...	4·78	5 64	...	...	...	...

The average diameter and turns of twist per inch in each single of Tables I. and II. are shown in Table III.; the former measured with a microscope fitted with an eyepiece micrometer, the latter by a yarn torsion-meter. The fourth column of Table III. shows the ratios of the diameters of singles in each two-ply twist; *e.g.* the ratio of the diameters of 6's and 20's worsted is $\frac{\cdot 021}{\cdot 0112} = 1\cdot 89$. This column furnishes data to compare the experimental and analytical results.

TABLE III.

Description of Yarn.	Average diameter of singles.	Turns per inch in singles.	f or $\frac{d_1}{d_2}$.
	d	n	
6's worsted	·021″	4·6	1·89
20's „	·0112″	14·0	
12's „	·0145″	5·6	1·45
24's „	·010″	8·0	
16-cut woollen	·023″	8	1·44
28 „ „	·016″	11·5	
7 „ „	·036″	4	1·2
10 „ „	·030″	7	

When the twist in the singles is expressed in turns per inch,

$$k = \frac{\pi^2}{8L} \cdot L^2 = \frac{\pi^2 L}{8} = 123$$

The numerical results obtained by experiment and mathematical analysis for 6's and 20's worsted are shown in Table IV. The body of the table shows percentage contractions for varying degrees of twist.

TABLE IV.

Percentage Contraction for 6's and 20's Worsted.

Degree of twist in turns per inch.	Experimental data.		Calculated from the mathematical analyses.				
	6's.	20's.	A_1 6's.	A_2 20's.	B_1 6's.	B_2 20's.	C 6's or 20's.
+ 8	8·91	10·17	15·37	12·42	9·00	23·92	13·97
+ 7	7·33	8·15	12·22	9·89	7·32	18·68	11·12
+ 6	5·81	6·37	9·47	7·63	5·80	14·07	8·57
+ 5	4·58	4·96	6·93	5·64	4·44	10·13	6·35
+ 4	3·40	3·72	4·84	4·00	3·23	6·81	4·43
+ 3	2·35	2·58	3·14	2·55	2·19	4·15	2·84
+ 2	1·39	1·58	1·70	1·40	1·30	2·12	1·57
+ 1	·61	·70	·67	·48	·57	·75	·63
0	...	...	...	...	...	...	...
− 1	− ·16	+ ·08	− ·31	− ·28	− ·41	− ·10	− ·30
− 2	− ·04	+ ·30	− ·27	− ·28	− ·67	+ ·42	− ·28
− 3	+ ·18	+ ·62	+ ·14	·00	− ·77	1·59	+ ·05
− 4	·69	1·12	+ ·90	+ ·56	− ·70	3·15	·72
− 5	1·21	1·72	2·02	1·42	− ·49	5·85	1·71
− 6	1·92	2·53	3·50	2·58	− ·11	9·00	3·00
− 7	2·87	3·62	5·32	4·00	+ ·43	12·72	4·63
− 8	4·10	5·22	7·54	5·82	1·13	17·11	6·58
− 9	5·51	7·16	10·20	7·74	1·97	22·17	8·85
− 10	7·16	9·31	12·95	9·83	2·98	27·78	11·43

"Direct" twist is reckoned positive and "inverse" twist negative.

The last five columns of Table IV. are headed by the letter which distinguishes the corresponding graph in fig. 5. A_1 is obtained by calculating from equation (1), A_2 from equation (2), B_1 from equation (3), B_2 from equation (4); C is a mean of the analytical equations, obtained in the conclusion of the mathematical analysis. The value of s, the distance from the axis of the thick thread of that centre of revolution which makes $y_1 = y_2$, is calculated and substituted in either equation.

The values of d_2, f, n_1 and n_2 are taken from Table III., and substituted in the equations. The latter thus reduce to the following simpler forms for the particular case under consideration.

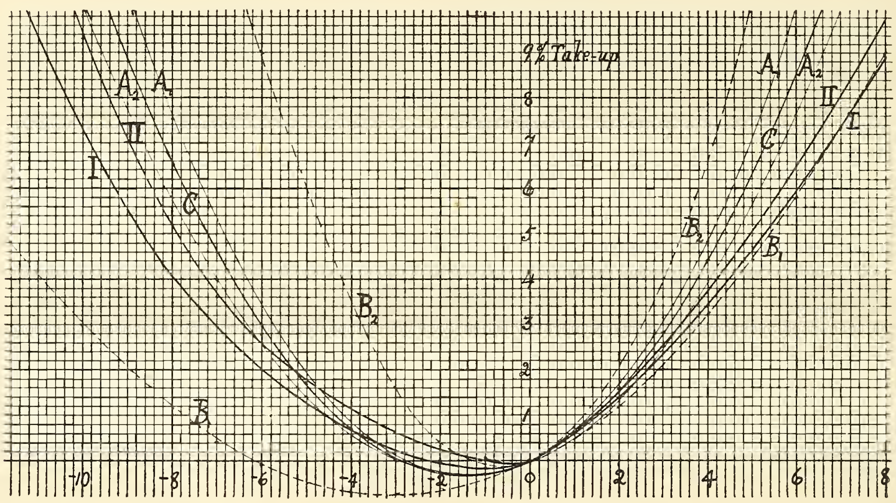

Fig. 5.

Corresponding to

Equation 1, $y_1 = \frac{x}{65 \cdot 8}(11 \cdot 78x + 32 \cdot 4)$ Graph A_1

" 2, $y_2 = \frac{x}{65 \cdot 8}(9 \cdot 26x + 28)$ " A_2

" 3, $y_1 = \frac{x}{65 \cdot 8}(5 \cdot 2\, x + 32 \cdot 4)$ " B_1

" 4, $y_2 = \frac{x}{65 \cdot 8}(21 \cdot 1\, x + 28)$ " B_2

Mean equation, y_1 or $y_2 = \frac{x}{65 \cdot 8}(10 \cdot 57x + 30 \cdot 5)$ " C

Fig. 5 shows graphically the results of Table IV. The ordinates represent percentage contraction; the abscissæ, turns per inch of twist.

Experimental.	Graph I. represents the actual results for the				6's yarn.		
	„ II.	„	„	„	20's	„	
From mathematical analysis.	„ A_1	„	„	„	6's	„	if corresponding points on the axes of the singles revolved in the same circle.
	„ A_2	„	„	„	20's	„	
	„ B_1	„	„	„	6's	„	if corresponding points on the axes of the singles revolved about the centroid of the cross-section.
	„ B_2	„	„	„	20's	„	
	„ C	„	„	„ for either yarn, if the conditions of revolution were such that the take-up for each single were equal.			

The elbow at the origin on each curve of fig. 4 is noteworthy; it is more pronounced than the corresponding change at the origin in the curves of fig. 3. If we consider the great difference in structure of woollen and worsted threads, we may expect the latter to give more uniform results than the former. The worsted thread contains only the long fibres of the wool staple arranged parallel to the axes of the thread before twist is put on in the process of spinning; the short and curly fibres of the staple are all combed out and rejected as "noil." The woollen thread, on the other hand, contains every fibre of the wool staple, long and short, straight and curly; further, the fibres are mixed in every conceivable direction in the carding operation.

To summarise the main conclusions which may be drawn from a study of the analytical and experimental data shown in the tables and diagrams, let us consider first the direct twist or positive side of fig. 5. The graphs I. and II. plotted from the experimental data are lower than A_1 and A_2 plotted on the assumptions (1) that corresponding points on the axes of the singles revolve in the same circle, (2) that the individual fibres in the singles have perfect freedom of rotation. Further, the relative positions of the curves are inverted; Graph II., with the results for the smaller thread, is higher than Graph I. with the results for the thicker thread. The above hypothesis gives the graph A_1 for the thicker thread higher than the graph A_2 for the smaller thread. The numbers in Tables I. and II. and the graphs in figs. 3 and 4 all show that the contraction is invariably greater in the smaller thread. The graphs B_1 and B_2 obtained on the hypothesis (1) that corresponding axial points revolve about the centroid of the cross-section, (2) that individual fibres in the singles rotate freely, show vastly greater contraction on the smaller thread. Graphs I. and B_1 for the thick thread practically coincide throughout the "direct" twisting.

On passing over to consider the negative side of fig. 5, we at once notice (1) that the elongation in the initial stages of the "inverse" twisting is much more pronounced in the hypothetical conditions than in the actual; (2) that as twisting proceeds, the hypothetical contractions increase much faster than the contractions actually observed in the threads. The graphs

of the former cut the corresponding graphs of the latter at different points (B_1 would cut I. outside the limits of the diagram).

The condition represented by graphs B_1 and B_2 is evidently extreme, but it must be borne in mind that the analytical results have all been deduced on the hypothesis that the fibres have perfect freedom of rotation. This is not true; indeed, it is very far from the truth except in the initial stages of the second twisting. In the actual thread, the individual fibres are neither absolutely free to rotate in the singles nor absolutely constrained to move as a whole. The fibres, practically free to rotate for the first one or two turns, gradually become more constrained in their relative movements, though never becoming absolutely so. On account of this constraint, it is therefore reasonable to expect that we should find, as we do, that the hypothetical curves shoot out above the experimental curves in the contraction parts of the diagram, and also that the former dip down beneath the latter in the elongation part of the diagram. The friction of the fibres rubbing on each other will oppose motion in either direction. Another fact which must be taken into account in explaining any divergence which occurs, is that the fibres projecting from the surface of the singles seriously interfere with the free motion of the latter as a whole about the centre of revolution in the second twisting. The ratio of the contractions would be very much greater but for this interference. We should naturally expect to find that the ratios for the woollen threads would be smaller than for the worsted threads, because the typical woollen thread possesses a large number of surface fibres projecting from the body of the thread. The aim in making a worsted thread, on the other hand, is to develop parallelism so far as possible amongst the individual fibres. That this inference is just, is borne out by a consideration of fig. 3 and fig. 4. The individual members in the pairs of graphs in fig. 4, representing the results of twisting woollen threads, are closer together than those in fig. 3, which represent the results of twisting worsted threads. This is especially noticeable on the positive sides of the diagrams, which represent the "harder" twist.

In conclusion, the author has pleasure in acknowledging his indebtedness to the Carnegie Trust for the Universities of Scotland for the financial assistance which has enabled him to prosecute this research.

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XV.—On the Influence of Temperature on the Photo-Electric Discharge from Platinum.

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(MS. received March 18, 1907. Read same date.)

Introductory.

Zeleny (*Physical Review*, xii., p. 321, 1901) has investigated the variation with temperature of the photo-electric discharge from a platinum wire in air at ordinary atmospheric pressure.

He found that the photo-electric current decreased some 40 per cent. as the temperature was raised to about 200° C.; after this a continuous increase in the current occurred up to 600° C., when its value was twice as great as at ordinary atmospheric temperatures. Zeleny further noticed a curious hysteresis effect, the photo-electric currents for corresponding temperatures being far greater during the cooling of the wire than during the heating.

These experiments, as already stated, were carried out in air at ordinary pressure. The presence of so much gas, especially if the potential gradient between the electrodes is not great enough to give the saturation current, will complicate the phenomena enormously, and it is really impossible to judge, from experiments made under these conditions alone, as to the influence of temperature on the actual photo-electric discharge, that is, on the rate at which negative corpuscles are emitted per unit area of the illuminated surface.

In order to eliminate any influence of temperature on the passage of the current through the gas, and especially on the secondary ionisation in the gas, due to "ionisation by collision," it is essential to work in high vacua, when the mean free path of the ions is large compared with the distance between the electrodes, and ionisation by collision is in consequence negligible.

The authors have carried out many series of observations under these conditions, viz., at pressures low enough to ensure the practical absence of any secondary ionisation in the gas itself, and under potential gradients

giving saturation currents, or currents which are practically independent of the potential difference betweeen the electrodes. (A series of experiments, hitherto unpublished, made by one of us, has shown that even in the highest attainable vacua an increase in the potential difference between parallel plane electrodes will produce a slight increase in current under all conditions, although this increase is very small in high vacua, and could not possibly affect the results of any experiments described below.)

Series of observations were also taken in gases at atmospheric pressure, and at a pressure of about 50 millimetres of mercury, the latter pressure being chosen for the reason that saturation (or the nearest approach to it possible using the photo-electric effect as the ionising agent) could be easily obtained with the voltage at our disposal. These series, although, as explained above, scarcely showing the effect of temperature on the actual photo-electric discharge, gave some interesting results, and incidentally led us to discover the important effect of absorbed gases on the discharge.

The experiments were made with temperatures ranging from 5° C. to 500° C. It was found impracticable, with the form of apparatus used, to employ higher temperatures than 500°. When the platinum was made hotter than this, the sealing-wax joints closing the experimental vessel soon became overheated and gave way. The pressure, too, could not be maintained as low as $\frac{1}{300}$ millimetre (at which pressure the experiments in vacuo were carried out) if higher temperatures were employed, whilst at about 500° C. the platinum commenced to give off negative corpuscles due to heat alone. This effect increases rapidly with further increase in temperature, and soon completely masks the photo-electric discharge.

Source of Ultra-violet Light.

The source of ultra-violet light used throughout the series of experiments was a discharge between iron terminals in an atmosphere of pure hydrogen, as described by one of us in a previous article (Varley, *Phil. Trans. of the R.S. London*, p. 439, 1903). The spark apparatus is shown diagrammatically in fig. 1, in which T T are the iron wire terminals between which the sparks pass, the length of the gap being about 6 millimetres. The wires T T were enclosed in glass tubes G G, enclosed in turn in wider glass tubes, and finally passing through brass side tubes S S, attached to the main brass vessel B. At the front of this vessel a quartz window, Q, was sealing-waxed on (all joints are made with sealing wax), while two narrow tubes, A and E, served to pass a continuous stream of pure hydrogen, prepared electrolytically and dried by bubbling very slowly through con-

centrated sulphuric acid, through the apparatus during the whole period of the research. The spark terminals T T were connected to the secondary of an induction coil, I (see fig. 3), used as a simple transformer, alternating current of periodicity 100 per second, supplied from an alternator on the premises, being used to excite the primary of the coil after transforming down to give a root mean square voltage of 45. Four large Leyden jars, L, of total capacity 0·012 microfarad, were placed in parallel with the spark gap.

The voltage on the primary of the induction coil and the periodicity of

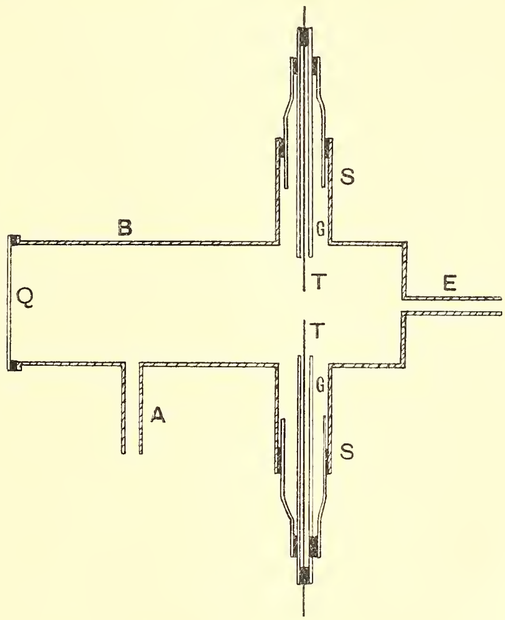

Fig. 1.—Spark Apparatus (half actual size).

the current being kept always constant to within 1 or 2 per cent., the intensity of the ultra-violet light obtained was always the same to within the same degree of accuracy, and remained so for weeks without the slightest change in the intensity being detectable. A control "leak apparatus," specially set up for the purpose of testing the constancy of the illumination during the course of the various sets of observations, proved to be quite unnecessary, and was only used, therefore, as an occasional check. This constancy had the great advantage of enabling readings taken at any time to be compared with confidence with those taken days or even weeks before.

Owing to the difficulty of obtaining the regular use of the alternator, we tried various other sources of ultra-violet light, including a powerful Nernst lantern lamp, recommended by Mr H. S. Allen, but all proved quite

unsatisfactory for our purpose. With the Nernst lamp, for example, the intensity of the ultra-violet available proved to be only a very small fraction (less than a hundredth) of that obtained from our spark in hydrogen, and quite unsuitable for a research of this character.

A quartz convex lens of about five inches focal length was attached to the same stand as the spark apparatus, so that the spark gap, T T, was at a

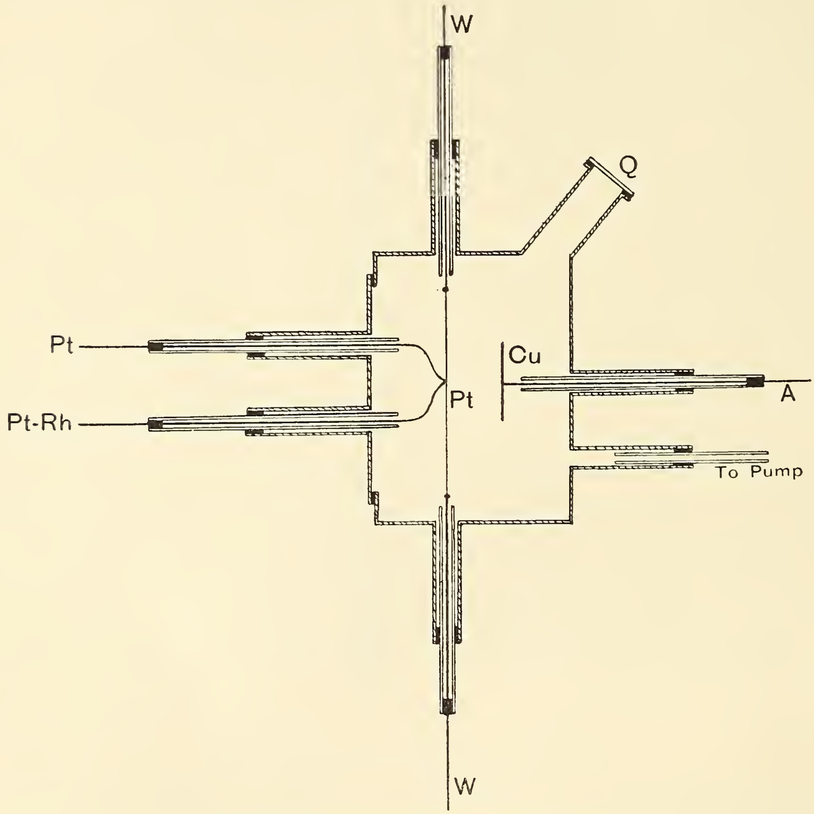

Fig. 2.—Leak Apparatus (half actual size).

distance from the lens equal to its focal length, thus giving a perfectly parallel beam of ultra-violet light.

Apparatus and Method of Experiment.

The photo-electric currents were measured from a piece of platinum foil to a copper disc placed some 1·2 centimetres in front of it: the apparatus used for the purpose is shown diagrammatically in fig. 2.

One of us (Varley, *Phil. Trans.*, p. 439, 1903; note on p. 455) had previously found that for measurements at very low pressures, say below

one-fiftieth of a millimetre of mercury, satisfactory results could not be obtained by using wire or gauze electrodes, owing to the ions, shot off from the sensitive electrode normally to the illuminated surface, passing in part through the meshes of the gauze, just as would be the case with a stream of cathode rays. We decided, therefore, to use only plane electrodes, the ultra-violet light being admitted into the leak apparatus, as we may name it, through a quartz window Q, and falling obliquely on to the central portion of the platinum foil Pt.

The foil, about 5 cms. long by 2·5 cms. wide, was held in position by being riveted to copper strips at the ends, whilst these were screwed on to two copper rods, W W, which latter passed through glass tubes, sealing-waxed into brass tubes attached to the main apparatus. These and all

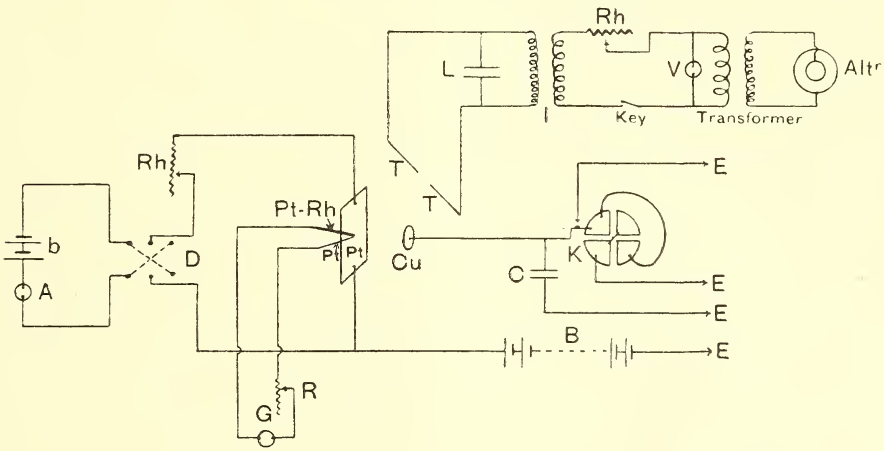

Fig. 3.—Diagram of Electrical Connections.

other joints were made air-tight by sealing wax. The rods, W W, served as conductors for the heating current, whilst the copper strips served to ensure the equal distribution of this current throughout the breadth of the foil. Only the central portion of the foil, about 1·5 sq. cms. in area, was illuminated by the ultra-violet light, so that the field between this and the copper electrode, Cu, in front of it would be fairly uniform.

The temperature of the foil at the centre was determined by means of a platinum-rhodoplatinum junction welded on to the foil, the two wires being led out of the apparatus through two brass tubes at the back, narrower glass tubing around the wires serving to insulate them. The connections of these wires with copper leads were made in a beaker containing an insulating oil, the temperature of which could be read by a suitable thermometer. The thermo-electric E.M.F. generated when the foil was heated was read by means of a Sullivan galvanometer, G (fig. 3),

previously calibrated, a suitable resistance, R, being placed in series. Owing to the fact that it was impossible either to have a point contact of the thermo-junction with the foil, or to ensure that the two points at which the junction wires left the foil should be on an equipotential line, it was always necessary to read the galvanometer deflection before and after reversing the direction of the heating current through the foil. The mean of these deflections (whose difference was never large) gave the true thermo-electric E.M.F. generated.

The thermo-electric junction itself was calibrated against a reliable mercury thermometer up to about 300° C. and at 800° C. by immersing in molten sodium chloride at its freezing point. A more accurate calibration was unnecessary for our purpose.

The platinum foil was heated by the current from two large secondary cells, *b*, a suitable rheostat, Rh, ammeter, A, and reversing switch, D, being placed in series. All these, as well as the instruments in the thermo-electric circuit, were carefully insulated from earth in order that the platinum foil might be raised to any desired potential by small storage cells (W. G. Pye's pattern).

The copper electrode, Cu, was connected directly to one pair of quadrants of a Dolezalek electrometer of high sensibility, and, except in the intervals required to take readings, to earth through the key K. A standard Muirhead condenser, the capacity of which could be varied from 0·001 to 0·2 microfarads, was also placed in parallel. The second pair of quadrants of the electrometer and the other plates of the condenser were permanently earthed, whilst all wires leading to the electrometer, the condenser, key, etc., were enclosed in earthed metallic casings.

The measurements of the photo-electric currents were made as follows:— The primary circuit of the induction coil was closed, starting the ultra-violet light; about three seconds later the pair of quadrants of the electrometer connected to the electrode Cu were insulated by breaking the earth connection at K electromagnetically, a stop watch being started simultaneously. The light was switched off exactly ten seconds later, and the electrometer deflection read at leisure as soon as it had become steady. Accuracy to 2 per cent. was always attained.

The platinum or sensitive electrode could be charged up to any potential up to 200 volts from small storage cells, *b*, and up to 435 volts by utilising the city mains.

The leak apparatus was connected to a Töpler pump, M'Leod gauge, and suitable drying tubes. Plugs of gold and silver leaf were inserted between the pump and leak apparatus in order to exclude any mercury

vapour. The apparatus was also connected through suitable stop-cocks with a Geryk vacuum pump, which enabled the exhaustion to be carried out with great rapidity and ease down to a pressure of 0·05 mms. mercury, below which, of course, the Töpler pump had to be employed. Arrangements were also made for filling the apparatus with other pure gases when required.

Experiments in Air.

(1) *Air at Atmospheric Pressure.*

Series of observations were made in air at atmospheric pressure in order to repeat Zeleny's determinations under the very different conditions under which we were working. The results obtained by us may perhaps be best seen from the following table, which gives the mean values of the photo-electric currents at different temperatures for one typical series of observations, in which the platinum was charged to -200 volts:—

Photo-electric current in arbitrary units.	Heating current in amperes.	Temperature in °C.
111	0	14°
113	10	78°
74	15	178°
24	20	319°
64	25	555°

The photo-electric currents were in all cases measured as the electrometer scale deflections obtained for ten seconds illumination.

In the above series the sensibility of the electrometer was such that 5720 scale divisions represented a potential difference of one volt between the quadrants. A capacity of 0·002 microfarad was placed in parallel with the electrometer, making a total capacity of 0·0024 microfarad, so that one scale division per 10 seconds represented a current of $4{\cdot}2 \times 10^{-14}$ amperes.

In the above experiments the final values of the photo-electric currents were not attained immediately after increasing the temperature. At 178° C. the value 74 was not reached until the heating current had been on for over ten minutes, although the platinum took up its final temperature within a few seconds. At 319° C., too, the current fell to 38 units one minute after applying the 20 amperes heating current, and only ten minutes later had reached the steady value 24. At 555° C. the current rose from 50 units, one minute after increasing the heating current, to 64 units twenty minutes

later. At this temperature a slight discharge took place from the negatively charged foil due to the heating alone, whether or no the foil was illuminated by ultra-violet light. The photo-electric discharge could not be investigated at higher temperatures owing to the rapid increase in this hot metal discharge with temperature.

The maximum reduction in the photo-electric current below the initial value is thus about 80 per cent., as against 44 per cent. found by Zeleny, whilst the temperature at which the discharge current became a minimum was about 200° C. higher than that observed by Zeleny.

After switching off the heating current the photo-electric current was found to have increased to 240 units, and was therefore more than double its previous value at the temperature of the room. In the course of a couple of days, however, it fell to its former value.

This increase was always obtained when the platinum had been heated to temperatures above 350° C., although when heated to say 150° C., at temperature at which the current is less than at ordinary temperatures, it does not even return immediately to its full original value on taking off the heating current, but requires several hours to do so.

(2) *Air at a Pressure of* 46 *mms. of Mercury.*

The potential difference applied, 200 volts, was high enough, as preliminary experiments showed, to give saturation currents—as nearly as possible independent of the voltage.

At this pressure we found that the discharge current of 58 units at 15° C. (the unit being about four times as great as in the experiments at 760 mms. pressure, since the sensibility of the electrometer was the same as before, whilst the capacity in parallel was now 0·01 mfd.) increased immediately to 65 units at 78° C. and to 69 units at 160° C., the heating current to give the last temperature being 10 amperes. An increase of current to 13 amperes gave a temperature of 250° C., and the photo-electric current decreased slowly to a final value of about 28 units in an hour. The actual readings obtained in one series at this pressure are given in the following table, which also gives the readings after breaking the heating circuit. It will be seen that the sensibility of the platinum slowly returns to practically its initial value.

Time, 7th March.	Photo-electric current in arbitrary units.	Heating current in amperes.	Temperature in ° C.
2.30 p.m. to 2.50 p.m.	56 57 58 57 58 58	0	15°
2.51	Heating current switched on.	7	78°
2.52	65		
2.55	66		
2.58	66		
3.1	65		
3.2	Heating current increased.	10	160°
3.3	70		
3.6	69		
3.8	70		
3.11	69		
3.13	69		
3.14	Heating current increased.	13	250°
3.15	64		
3.17½	56		
3.20	53		
3.22	50		
3.25	48		
3.27	46		
3.32	43		
3.35	40		
3.51	35		
3.53	31		
3.56	29		
4.16	28		
4.18	29		
4.19	Heating current switched off.	0	25° to 16°
4.20	35		
4.23	39		
4.26	39		
4.42	44		
4.45	45		
8th Mar., 4.10 p.m.	60		15°
4.13	60		

The gradual diminution in the photo-electric current with time at 250° is also shown by the curve in fig. 4.

This curve is a logarithmic curve, and can be represented by the equation

$$C - 27 = 69e^{-0\cdot054t}$$

where C is the photo-electric current in the above arbitrary units, and t is the time in minutes.

If log (C − 27) is plotted against t, a straight line is obtained—shown in fig. 4a.

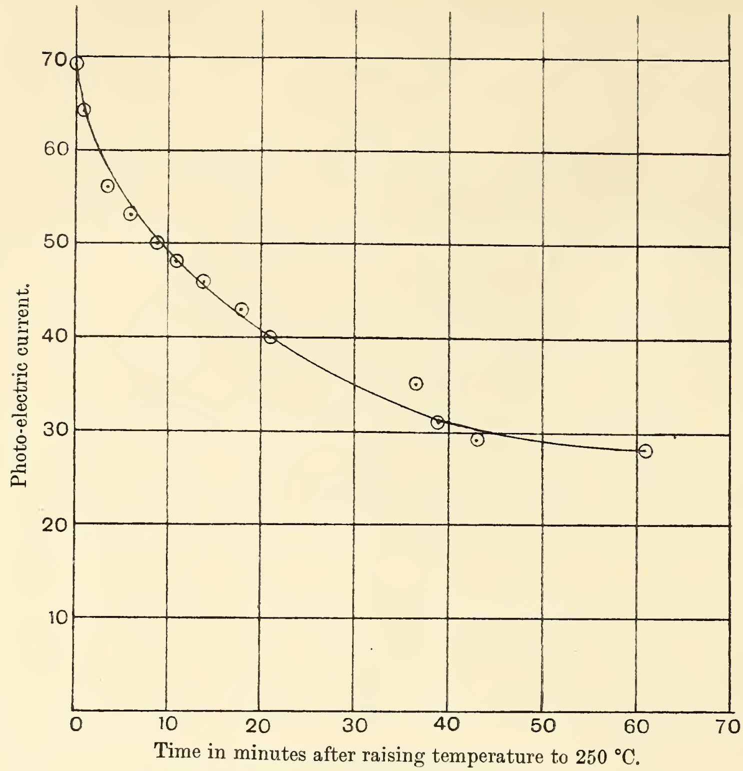
Photo-electric current.
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
Time in minutes after raising temperature to 250 °C.

FIG. 4.

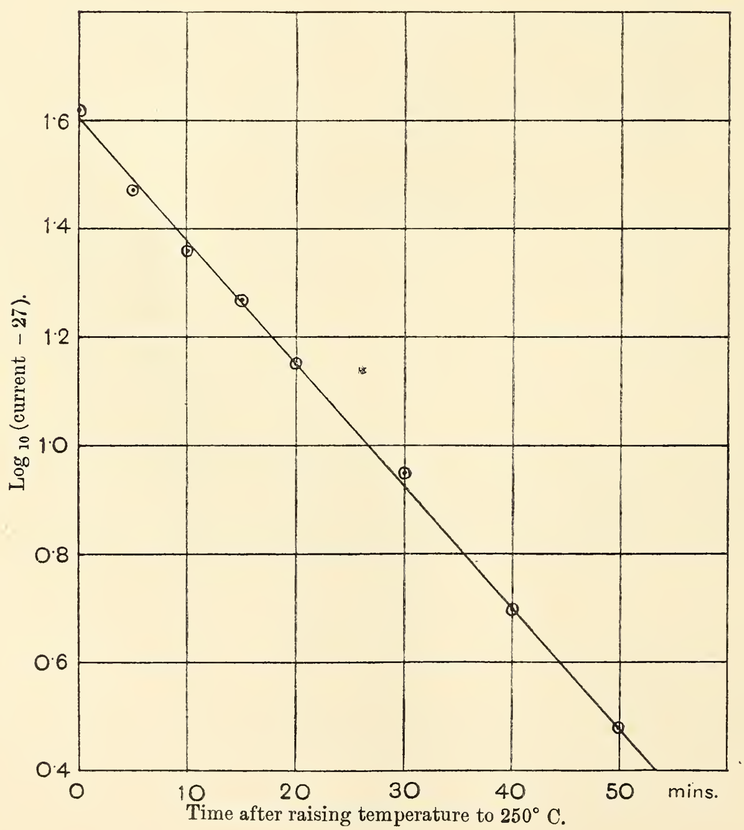
Log $_{10}$ (current − 27).
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0
10
20
30
40
50
mins.
Time after raising temperature to 250° C.

FIG. 4*a*.

(3) *Air at a Pressure of* 0·0035 *mm. Mercury.*

A large number of series of observations were made in air at a pressure of about 0·0035 mm. of mercury. At this pressure the mean free path of an ion in air is about 10 cms., and is therefore much greater than the distance between the electrodes (1·2 cms.). A potential of 80 volts was ample to give saturation, and the photo-electric current varied very slowly with the pressure, even doubling or halving the pressure only increasing or decreasing the current by about 7 to 8 per cent., whilst doubling or halving the potential gradient had a scarcely greater effect. Hence we may assume that under these conditions we are measuring the variations in the actual photo-electric discharge, quite independent of any charges due to secondary ionisation in the gas.

In all the experiments made under the above conditions, the sensibility of the electrometer was 5940 divisions per volt, and the capacity in parallel 0·05 mfd. A current, therefore, which gives to the electrometer a charge producing a deflection of 1 scale division per 10 seconds is equal to 8×10^{-13} amperes.

When the apparatus had been standing exhausted to $\frac{1}{300}$ mm. pressure for nineteen days, the photo-electric current was 36 units (1 unit $= 8 \times 10^{-13}$ amp.). On applying a heating current of 10 amperes, giving a temperature of 163° C., the photo-electric current rose at once to 129 units, and remained constant at that value for half an hour, readings being taken every few minutes.

On breaking the heating circuit the current fell to—

123	units in	4	minutes.
103	„	14	„
90	„	40	„
71	„	$5\frac{1}{2}$	hours.
65	„	24	„

On now applying a heating current of 5 amperes, giving a temperature of 50° C., the photo-electric current rose to 127 units in about fifteen minutes, and remained constant at that value for more than an hour. On increasing the heating current to 10 amperes, giving a temperature of 163° C., there was no appreciable change in the sensibility, the photo-electric current now having a value of 129 units. A further increase of temperature to 278° C. reduced it very slightly, to 122 units, whilst it returned to 129 units again on reducing the temperature to 163°.

On breaking the heating circuit the sensibility fell in twenty-four hours to 66 units again, and it was repeatedly found that after the platinum had

been heated to 200–300° C. the sensibility always fell to this value in twenty-four hours, falling to 50 units in three days, and to about 36 units in three weeks.

Between temperatures of 60° C. and 350° C. (the highest attainable in vacuo with our apparatus) the sensibility of the platinum remained always very nearly constant, perhaps falling at most 4 per cent. between these limits, though the accuracy of the method hardly justifies us stating definitely that such a small diminution actually occurs. Whatever the initial value of the sensibility of the platinum, a rise of temperature of 50° or 60° C. always restored it to the same constant value, and on switching off the heating current the sensibility always decreased regularly, falling to about half its maximum value in something like twenty-four hours.

It is possible that the heating current may affect the sensibility of the platinum directly, quite apart from the increase of temperature produced. In this case it would seem that, with the piece of platinum used, a current of 5 amperes is as effective as one of 15 or even 20 amperes, since an increase from one to the other produces no change in the photo-electric current. We made a number of experiments on this point, but the results obtained were not very conclusive.

Experiments in Carbon Dioxide.

(1) *Carbon Dioxide at Atmospheric Pressure.*

The results obtained in carbon dioxide at atmospheric pressure were very similar to those obtained in air at the same pressure, and are summarised as follows:—At 14° C. a photo-electric current of 91 units was obtained; at 140° C. the value fell rapidly to 71 units, at 263° C. to 34 units, and at 440° C. to 23 units. Above this temperature it rose again, and a current was also obtained due to the temperature alone, this latter current reaching 15 units at 670° C.

On stopping the heating current the sensibility was found to have increased, giving 190 units at the temperature of the room. The maximum diminution in sensibility observed is thus practically identical with that found in air under the same conditions, but the sensibility took up its final value rather more quickly than in the case of air.

(2) *Carbon Dioxide at a Pressure of* 46 *mms.*

The fall in the photo-electric current with temperature in carbon dioxide at this pressure was less marked than that in air at the same pressure and under precisely similar conditions.

The photo-electric current of 85 units at 13° C. fell to 83 units at 43° C., 67 units at 143° C., 61 units at 258° C., and 62 units at 368° C., rising almost immediately to its original value at 13° C. on switching off the heating current.

On a separate occasion a precisely similar result was obtained, a gradual fall from 85 units at 15° C. to 60 units at 370° C. being obtained.

The much shorter time required for the platinum to attain its final constant sensibility at any temperature in carbon dioxide than in air was strongly marked at this pressure, the steady values in the above experiments being attained within a minute or two at most after increasing or reducing the temperature of the platinum.

(3) *Carbon Dioxide at a Pressure of* 0·0035 *mm.*

Observations were made in carbon dioxide at a pressure of 0·0035 millimetres of mercury. The potential applied and other conditions were precisely the same as those for the experiments in air at the same pressure.

The results obtained were also identical with those obtained for air, a comparatively small heating current producing an increase in sensibility up to a certain value, beyond which no further increase in the heating current produced any effect. It is, therefore, unnecessary to go into further details of these series of observations.

Experiments in Hydrogen.

(1) *Hydrogen at Atmospheric Pressure.*

The results obtained in hydrogen at atmospheric pressure are summarised in the following table:—

Mean value of photo-electric current in arbitrary units.	Heating current in amperes.	Temperature in °C.
92	0	14°
110	15	91
120	20	137
135	25	225
181	30	340

and after the heating current had been switched off the photo-electric current fell to 105 in seven minutes and to 95 in twenty minutes.

We thus see that in hydrogen the photo-electric current increases steadily with the temperature from the ordinary temperature of the room—an absolutely different behaviour from that in either air or carbon dioxide.

(2) *Hydrogen at a Pressure of* 49 *mms.*

The platinum, as in all the experiments at high pressures, was charged to 200 volts negative, while the value of the arbitrary unit in which the photo-electric currents were measured was the same as in air and carbon dioxide at the same pressure. Owing to the high conductivity for heat of hydrogen, it was found impossible to raise the temperature of the foil above 200° C., a heating current of 20 amperes being required to give this temperature.

The results obtained are given in the following table:—

Mean value of photo-electric current in arbitrary units.	Heating current in amperes.	Temperature in °C.
85	0	14°
91	5	30
98	10	62
100	15	111
105	20	196

On switching off the heating current the photo-electric current fell to 96 in ten minutes, and to 88 some twenty hours later.

(3) *Hydrogen at very Low Pressures.*

The influence of temperature on the photo-electric currents in hydrogen at a pressure of 0·0035 mm. is similar to that in air and carbon dioxide—viz., a small increase of heating current, and therefore of temperature, produces an increase in the sensibility up to a maximum value, after which the heating current may be increased so as to raise the temperature up to at least 370° C. without producing any further increase in the sensibility.

This maximum sensibility appears to be at least 30 per cent. lower in hydrogen than in either of the other gases, whilst the change in sensibility produced is much less marked, the photo-electric current of 45 units at 14° C. (after the platinum had been left unheated for three days) rising only to 70 units at 370° C., this same ratio of increase being obtained on other occasions under the same conditions.

It is important for the success of these experiments that the vessel shall have been standing exhausted to the low pressure for some days before the observations are taken, in order that the quantity of hydrogen occluded in the platinum may have reached a constant value.

We noticed that when the apparatus, which had been standing filled with hydrogen at a high pressure for nearly a week, during which time the

platinum had been frequently raised to temperatures over 300° C., was exhausted to a very low pressure, 8 divisions on the M'Leod gauge (0·003 mm. of mercury), the sensibility steadily diminished with the time. The following table shows one set of observations taken :—

Time.	Photo-electric current.
12 noon.	Apparatus pumped out.
12.7 p.m.	98
12.12	90
12.15	88
12.18	85
12.21	82
12.24	78
12.30	73
12.34	74
12.50	69

The pressure during this series rose to 12 divisions on the gauge (which would tend to increase the current slightly), and continued to rise steadily, being 21 divisions seven hours later and 28 divisions twenty-seven hours later, and practically constant. The current was now 50 units, and the pressure upon reducing to 7 divisions (the original pressure after exhaustion), fell to 45 units. This rise in pressure could only be accounted for by the liberation of hydrogen occluded in the platinum foil. Calculations showed that at the higher pressures the platinum would easily absorb hydrogen in sufficient quantity to cause this increase in pressure (in all about 0·006 mm. mercury) when liberated. Application of heat to the foil produced a very slight liberation of hydrogen, 3 or 4 divisions on the gauge, after which the pressure remained perfectly constant even when the foil was raised to 370° C. It thus seems certain that the absorbed hydrogen increases the photo-electric sensibility of a platinum surface, and this may easily account for the difference between the effect of temperature on the photo-electric currents in this gas at high pressure and those in air and carbon dioxide.

Discussion of Results.

The variations with temperature of the photo-electric currents in air, carbon dioxide, and hydrogen at atmospheric pressure are shown graphically in fig. 5, the results obtained by Zeleny in air being also plotted for purposes of comparison. The actual photo-electric currents observed for carbon dioxide have been multiplied by 1·22 in order to compare the values better with those for air, the values at 14° C. now coinciding.

It will be noticed that the remaining points observed for carbon dioxide fall exactly on the curve for air, showing that the effect of temperature on the photo-electric currents in these gases is identical at this pressure. In

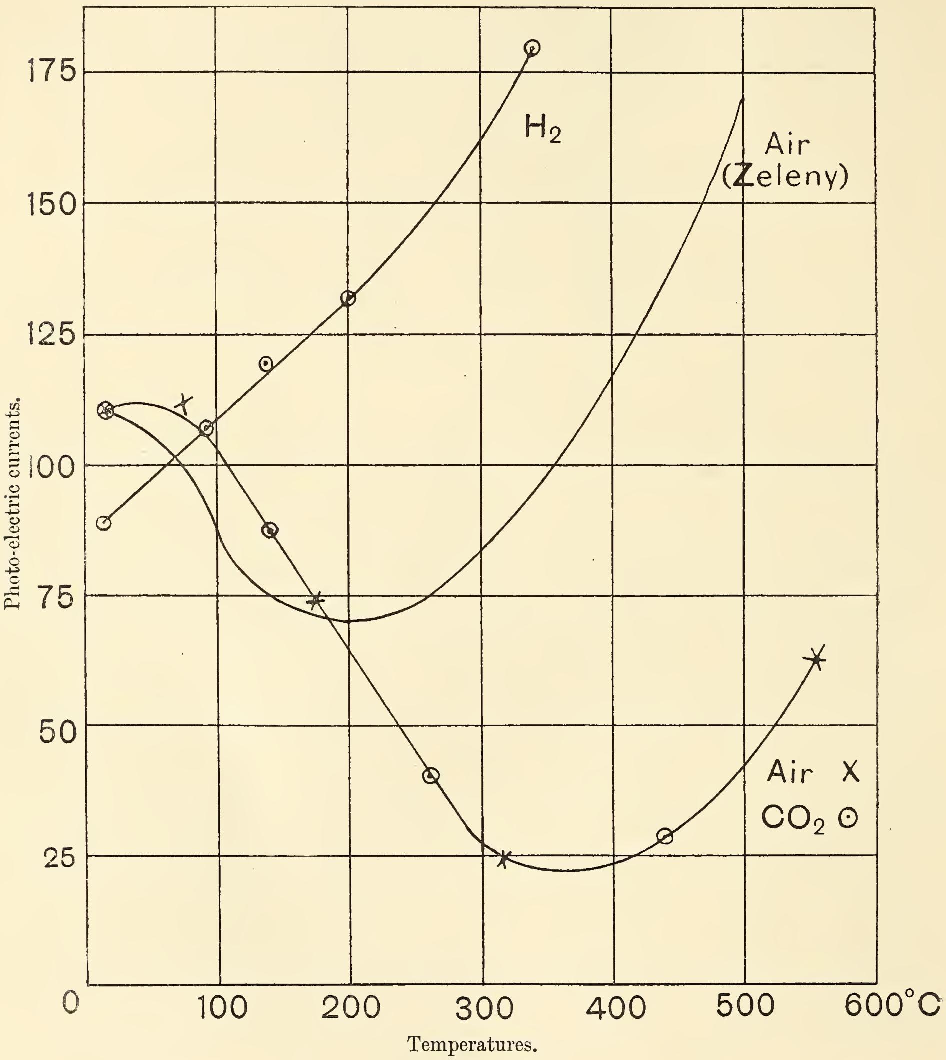

FIG. 5.

hydrogen, on the other hand, the photo-electric currents increase steadily with the temperature from the first.

The difference between our results and those of Zeleny is doubtless due to the fact that Zeleny was using a voltage far below that required to give the saturation current, only 72 volts for a distance of 1·6 cms.; also to the fact that he used as his sensitive electrode a bent piece of platinum

wire, measuring the leak to a piece of gauze, an arrangement which would hardly give a uniform field.

Furthermore, in Zeleny's experiments the readings were taken at intervals rapidly succeeding one another, only some twenty minutes being occupied in taking the complete series, one reading only being taken at each temperature, and no time being allowed for the current to take up its final value at any particular temperature—a factor of the highest importance.

The results in the different gases at 46 mms. pressure show no very marked difference from those at the higher pressure. In the case of neither carbon dioxide nor air was a point reached at which the current began to increase with the temperature, but this was doubtless due to the fact that the temperature could not be raised above 400° C. in gases at this pressure.

In no case could the currents be measured accurately at temperatures much above 500° C. owing to the constant emission of negative corpuscles due to the temperature alone. This effect commences about 500° C., and increases very rapidly as the temperature is further raised.

The experiments show that in air and carbon dioxide, and to a much less marked extent in hydrogen, time is required before the photo-electric current reaches its final value at any temperature, and on reducing the temperature to that of the atmosphere, many hours may elapse before the current returns to its original value. If in air or carbon dioxide the platinum was heated to above the temperature at which inversion took place (about 400° C.), a large increase in the sensibility occurred, the photo-electric current at 14° C. being twice as great after as before the heating, and requiring many hours to fall again to the original value. If heated to temperatures not exceeding 300° C. in these gases or in hydrogen, no such increase in sensibility was observed.

Some change in the surface of the metal must therefore take place at these high temperatures, for if the change with temperature was due to say a change in the secondary ionisation in the gas, near to the surface of the metal of course, we should naturally expect the readings at any particular temperature to be the same, irrespective of the previous history of the metallic surface itself. The increase cannot well be accounted for by an absorption of gas, since it occurs in carbon dioxide, which is not known to be absorbed by platinum.

The experiments carried out at very low pressures in the different gases show that under these conditions the behaviour of the platinum is independent, to a large extent at least, of the gas. In every case a comparatively small heating current was sufficient to increase the photo-electric sensibility

up to a certain maximum value, beyond which no further increase in the heating current produced any further appreciable result, the sensibility remaining constant for a range of temperature from 60° C. to at least 350° C. Between these limits we can then say that the specific photo-electric discharge from platinum is independent of the temperature.

On stopping the heating current the sensibility of the platinum slowly fell, decreasing to about half its value in twenty-four hours. The rate of decrease was proved to be in no way influenced by allowing ultra-violet light to illuminate the surface of the platinum.

Whether or no the passage of heating current through the platinum, apart from the rise in temperature produced, is a factor bringing about the increase in the sensibility of the platinum to its maximum value, cannot be definitely stated. We hope, however, at an early date to decide this point, an investigation to that end being in progress.

The apparatus required in the above investigation has, in part, been provided out of a grant made by the Carnegie Trustees, to whom our best thanks are due.

Our thanks must also be expressed to Professor Baily for granting us the very frequent use of certain machines in the alternating current laboratory.

(*Issued separately May* 31, 1907.)

XVI.—The Theory of Axisymmetric Determinants in the Historical Order of its Development up to 1860. By Thomas Muir, LL.D.

(MS. received December 24, 1906. Read January 21, 1907.)

Cayley (1841, May).

[On a theorem in the geometry of position. *Cambridge Math. Journ.*, ii. pp. 267–271: or *Collected Math. Papers*, i. pp. 1–4.]

A general account has already been given of this interesting paper—interesting as regards the subject, and interesting as being the author's first 'prentice effort. All that remains to be noticed here is what may be called Cayley's series of vanishing axisymmetric determinants. These we may write in the short form

$$\begin{vmatrix} \cdot & (x)_{12} & (x)_{13} & 1 \\ (x)_{21} & \cdot & (x)_{23} & 1 \\ (x)_{31} & (x)_{32} & \cdot & 1 \\ 1 & 1 & 1 & \cdot \end{vmatrix},$$

$$\begin{vmatrix} \cdot & (xy)_{12} & (xy)_{13} & (xy)_{14} & 1 \\ (xy)_{21} & \cdot & (xy)_{23} & (xy)_{24} & 1 \\ (xy)_{31} & (xy)_{32} & \cdot & (xy)_{34} & 1 \\ (xy)_{41} & (xy)_{42} & (xy)_{43} & \cdot & 1 \\ 1 & 1 & 1 & 1 & \cdot \end{vmatrix},$$

$$\begin{vmatrix} \cdot & (xyz)_{12} & (xyz)_{13} & (xyz)_{14} & (xyz)_{15} & 1 \\ (xyz)_{21} & \cdot & (xyz)_{23} & (xyz)_{24} & (xyz)_{25} & 1 \\ (xyz)_{31} & (xyz)_{32} & \cdot & (xyz)_{34} & (xyz)_{35} & 1 \\ (xyz)_{41} & (xyz)_{42} & (xyz)_{43} & \cdot & (xyz)_{45} & 1 \\ (xyz)_{51} & (xyz)_{52} & (xyz)_{53} & (xyz)_{54} & \cdot & 1 \\ 1 & 1 & 1 & 1 & 1 & \cdot \end{vmatrix},$$

. . . .

if we put

$$\begin{aligned} &(x)_{rs} \quad \text{for } (x_r - x_s)^2, \\ &(xy)_{rs} \quad \text{for } (x_r - x_s)^2 + (y_r - y_s)^2, \\ &(xyz)_{rs} \quad \text{for } (x_r - x_s)^2 + (y_r - y_s)^2 + (z_r - z_s)^2, \end{aligned}$$

. . . .

The fact that they are identically equal to zero is established by showing that each one is resolvable into two factors, of which one or both vanish; for example, that the first is equal to

$$\begin{vmatrix} x_1^2 & -2x_1 & . & 1 \\ x_2^2 & -2x_2 & . & 1 \\ x_3^2 & -2x_3 & . & 1 \\ 1 & . & . & . \end{vmatrix} \times_{rr} \begin{vmatrix} 1 & x_1 & . & x_1^2 \\ 1 & x_2 & . & x_2^2 \\ 1 & x_3 & . & x_3^2 \\ . & . & . & 1 \end{vmatrix},$$

where, for the moment, we use $\times_{rr}$ to indicate row by row multiplication.

It should be noted that not more than three of the series are contemplated by Cayley, as he viewed the identities mainly on their geometrical side, namely, as giving the relation between the distances of three points in a straight line, four points in a plane, and five points in three-dimensional space.*

Hesse (1844, Jan.).

[Ueber die Elimination der Variabeln aus drei algebraischen Gleichungen vom zweiten Grade mit zwei Variabeln. *Crelle's Journal*, xxviii. pp. 68–96.]

In this paper there appears the first reference to the special form of determinant which has for its elements the second differential-quotients of a function, and which consequently is axisymmetric. On account of its importance this form falls to be dealt with separately: we merely note here that Hesse himself viewed it as a special form of Jacobian, namely, the Jacobian whose originating functions are the first differential-quotients of a single function, and that he called it *the determinant of this single function*, a practice which, when the function is quadratic, is not at variance with that introduced by Gauss.

Cayley (1846).

[Problème de géométrie analytique. *Crelle's Journal*, xxxi. pp. 227–230: or *Collected Math. Papers*, i. pp. 329–331.]

The problem in question depends on an algebraic identity which, after a little examination, is seen to be a property of axisymmetric determinants. Cayley writes the identity in the form

$$F_{pp}(U+V^2)\cdot K(U) - F_{pp}(U)\cdot K(U+V^2) = \{F_{po}(U)\}^2$$

where

$$U = Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy + 2Lxw + 2Myw + 2Nzw + Pw^2,$$
$$V = \alpha x + \beta y + \gamma z + \delta w,$$

* To one taking this point of view Sylvester's paper "On Staudt's theorems . . . ." will be of interest. See *Philos. Magazine*, iv. (1852), pp. 335–345 : or *Collected Math. Papers*, i. pp. 382–391.

$$K(U) = \begin{vmatrix} A & H & G & L \\ H & B & F & M \\ G & F & C & N \\ L & M & N & P \end{vmatrix}, \qquad P_{po}(U) = \begin{vmatrix} \cdot & \xi & \eta & \zeta & \omega \\ \alpha & A & H & G & L \\ \beta & H & B & F & M \\ \gamma & G & F & C & N \\ \delta & L & M & N & P \end{vmatrix},$$

and P_{pp} (U) is what is obtained from P_{po} (U) on changing $\alpha, \beta, \gamma, \delta$ into ξ, η, ζ, ω respectively: but freed of all fresh notation it is nothing more nor less than

$$\begin{vmatrix} \cdot & \xi & \eta & \zeta & \omega \\ \xi & A+\alpha^2 & H+\alpha\beta & G+\alpha\gamma & L+\alpha\delta \\ \eta & H+\beta\alpha & B+\beta^2 & F+\beta\gamma & M+\beta\delta \\ \zeta & G+\gamma\alpha & F+\gamma\beta & C+\gamma^2 & N+\gamma\delta \\ \omega & L+\delta\alpha & M+\delta\beta & N+\delta\gamma & P+\delta^2 \end{vmatrix} \cdot \begin{vmatrix} A & H & G & L \\ H & B & F & M \\ G & F & C & N \\ L & M & N & P \end{vmatrix}$$

$$- \begin{vmatrix} \cdot & \xi & \eta & \zeta & \omega \\ \xi & A & H & G & L \\ \eta & H & B & F & M \\ \zeta & G & F & C & N \\ \omega & L & M & N & P \end{vmatrix} \cdot \begin{vmatrix} A+\alpha^2 & H+\alpha\beta & G+\alpha\gamma & L+\alpha\delta \\ H+\beta\alpha & B+\beta^2 & F+\beta\gamma & M+\beta\delta \\ G+\gamma\alpha & F+\gamma\beta & C+\gamma^2 & N+\gamma\delta \\ L+\delta\alpha & M+\delta\beta & N+\delta\gamma & P+\delta^2 \end{vmatrix}$$

$$= \begin{vmatrix} \cdot & \xi & \eta & \zeta & \omega \\ \alpha & A & H & G & L \\ \beta & H & B & F & M \\ \gamma & G & F & C & N \\ \delta & L & M & N & P \end{vmatrix}^2 .$$

Nothing is said about the mode of proving it.*

* If we note that the first determinant can be written in the form

$$- \begin{vmatrix} -1 & \cdot & \alpha & \beta & \gamma & \delta \\ \cdot & \cdot & \xi & \eta & \zeta & \omega \\ \alpha & \xi & A & H & G & L \\ \beta & \eta & H & B & F & M \\ \gamma & \zeta & G & F & C & N \\ \delta & \omega & L & M & N & P \end{vmatrix}$$

and the fourth in the form

$$- \begin{vmatrix} -1 & \alpha & \beta & \gamma & \delta \\ \alpha & A & H & G & L \\ \beta & H & B & F & M \\ \gamma & G & F & C & N \\ \delta & L & M & N & P \end{vmatrix}$$

light dawns at once, for the last three determinants of the identity are then seen to be principal minors of the first, and the identity itself to be a case of Jacobi's theorem regarding a minor of the adjugate.

"H (1)" (1846, Nov.).

[Mathematical notes, ii. *Camb. and Dublin Math. Journ.*, i. p. 286.]

A correspondent signing himself as above puts on record without proof two identities which, when the notation of determinants is used, may be written in the form

$$\begin{vmatrix} aa'-bb'-cc' & ab'+ba' & ca'+ac' \\ ab'+ba' & bb'-cc'-aa' & bc'+cb' \\ ca'+ac' & bc'+cb' & cc'-aa'-bb' \end{vmatrix} = (a^2+b^2+c^2)(aa'+bb'+cc')(a'^2+b'^2+c'^2)$$

and

$$\begin{vmatrix} 2aa' & ab'+ba' & ca'+ac' \\ ab'+ba' & 2bb' & bc'+cb' \\ ca'+ac' & bc'+cb' & 2cc' \end{vmatrix} = 0 .$$

It is seen that both determinants are axisymmetric, that the second is expressible as the product of two vanishing determinants, and that the first is formable from the second by subtracting $aa'+bb'+cc'$ from each diagonal element,—a fact which, taken along with the vanishing of the second, shows that $aa'+bb'+cc'$ is a factor of the first.

Cayley (1847).

[Note sur les hyperdéterminants. *Crelle's Journal*, xxxiv. pp. 148–152 : or *Collected Math. Papers*, i. pp. 352–355.]

The second paragraph of this note concerns the expression

$$6abcd+3b^2c^2-a^2d^2-4ac^3-4b^3d, \quad \text{or } \nabla \text{ say,}$$

soon afterwards (1851) to be called the "discriminant" of the binary cubic

$$ax^3+3bx^2y+3cxy^2+y^3 ;$$

and Cayley's proposition is that the determinant whose elements are the second differential-quotients of ∇ with respect to a, b, c, d, namely, the axisymmetric determinant

$$\begin{vmatrix} -2d^2 & 6cd & 6bd-12c^2 & 6bc-4ad \\ 6cd & 6c^2-24bd & 6ad+12bc & 6ac-12b^2 \\ 6bd-12c^2 & 6ad+12bc & 6b^2-24ac & 6ab \\ 6bc-4ad & 6ac-12b^2 & 6ab & -2a^2 \end{vmatrix},$$

is a numerical multiple of ∇^2. As a matter of fact he says the multiplier is 3 ; but this is because, instead of writing the determinant as here, he

removes from it the factors 2, 6, 6, 2. A verificatory proof, unsatisfactory to himself, is given, the determinant being as a preliminary again altered into a multiple (by 1296) of

$$\begin{vmatrix} a^2 & ab & 2b^2-ac & 3bc-2ad \\ ab & \frac{4}{3}ac-\frac{1}{3}b^2 & \frac{2}{3}bc+\frac{1}{3}ad & 2c^2-bd \\ 2b^2-ac & \frac{2}{3}bc+\frac{1}{3}ad & \frac{4}{3}bd-\frac{1}{3}c^2 & cd \\ 3bc-2ad & 2c^2-bd & cd & d^2 \end{vmatrix}$$

or

$$\begin{vmatrix} a^2 & ab & ac-3r & ad+9q \\ ba & b^2+2r & bc-q & bd-3p \\ ca-3r & cb-q & c^2+2p & cd \\ da+9q & db-3p & dc & d^2 \end{vmatrix},$$

where

$$p=\tfrac{2}{3}(bd-c^2)\,,\quad q=\tfrac{1}{3}(bc-ad)\,,\quad r=\tfrac{2}{3}(ac-b^2)\,,$$

the last change being probably due to the fact that it was known that

$$\nabla = 9(pr-q^2)$$

and that verification would thus be easier.

Sylvester (1850, Aug.).

[On the intersections, contacts and other correlations of two conics expressed by indeterminate co-ordinates. *Cambridge and Dublin Math. Journ.*, v. pp. 262–282: or *Collected Math. Papers*, i. pp. 119–137.]

In this paper an important property of axisymmetric determinants is incidentally brought to notice; but, unfortunately, the author's intended statement is almost hidden through want of care. He says (p. 270) that the determinant

$$\begin{vmatrix} A & C' & B' & l \\ C' & B & A' & m \\ B' & A' & C & n \\ l & m & n & 0 \end{vmatrix}$$

where

$$\begin{aligned} A &= \quad bc-a'^2, & B &= ca-b'^2, & C &= ab-c'^2, \\ A' &= -b'c'+aa', & B' &= -c'a'+bb', & C' &= -a'b'+cc', \end{aligned}$$

is "the product of the determinant

$$\begin{vmatrix} a & c' & b' \\ c' & b & a' \\ b' & a' & c \end{vmatrix}$$

by the quantity

$$al^2 + bm^2 + cn^2 - 2a'mn - 2b'ln - 2c'lm ."$$

Now the said four-line determinant is not resolvable unless A′, B′, C′ be changed in sign, and even then the second factor is not as printed, but is

$$-(al^2 + bm^2 + cn^2 + 2a'mn + 2b'nl + 2c'lm) .$$

The theorem, in fact, may be viewed as giving an expression for the product of a ternary quadric by its discriminant; and at a later date might have been written

$$\begin{array}{ccc|c} x & y & z & \\ \hline a & h & g & x \\ h & b & f & y \\ g & f & c & z \end{array} \cdot \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = - \begin{vmatrix} . & x & y & z \\ x & \mathrm{A} & \mathrm{H} & \mathrm{G} \\ y & \mathrm{H} & \mathrm{B} & \mathrm{F} \\ z & \mathrm{G} & \mathrm{F} & \mathrm{C} \end{vmatrix} .$$

No proof is given by Sylvester; but in a footnote we are told that it depends on "a theorem given by M. Cauchy, and which is included as a particular case in a theorem of my own, relating to compound determinants." What theorem of Cauchy's is thus referred to it is not easy to say. One would think that the most natural proceeding would be to show that the coefficients of x^2, y^2, . . . on the one side are identical with those on the other; and, in this case, the names to be mentioned would be Lagrange and Jacobi.

In a postscript Sylvester enunciates a theorem connected with the linear transformation of an n-ary quadric; and as this concerns the "determinant" of the quadric, or what a year later he named the "discriminant," it necessarily involves a property of axisymmetric determinants. His wording (p. 281) is:—"Let U be a quadratic function of any number of letters $x_1, x_2, \ldots, x_n$, and let any number r of linear equations of the general form

$$a_{1r}x_1 + a_{2r}x_2 + \ldots + a_{nr}x_n = 0$$

be instituted between them; and by means of these equations let U be expressed as a function of any $n-r$ of the given letters, say of $x_{r+1}, x_{r+2}, \ldots, x_n$, and let U so expressed be called M. Let

$$a_{1r}x_1 + a_{2r}x_2 + \ldots + a_{nr}x_n$$

be called L_r. Then the determinant of M in respect to the $n-r$ letters above given is equal to the determinant of

$$\mathrm{U} + \mathrm{L}_1x_{n+1} + \mathrm{L}_2x_{n+2} + \ldots + \mathrm{L}_rx_{n+r}$$

considered as a function of the $n+r$ letters

$$x_1, x_2, \ldots, x_{n+r}$$

divided by the square of the determinant

$$\left.\begin{matrix} a_{11} & a_{21} & \ldots & a_{r1} \\ a_{12} & a_{22} & \ldots & a_{r2} \\ \ldots & \ldots & \ldots & \ldots \\ a_{1r} & a_{2r} & \ldots & a_{rr} \end{matrix}\right\}\text{''}.$$

As regards this we have to remark (1) that again no proof is offered, and (2) that the discriminant of $U + L_1x_{n+1} + \ldots + L_rx_{n+r}$ is easily got by "bordering" the discriminant of U. Taking the case where U is

$$ax^2 + by^2 + cz^2 + dw^2 + 2fyz + 2gzn + 2hxy + 2pxw + 2qyw + 2rzw$$

with the discriminant

$$\begin{vmatrix} a & h & g & p \\ h & b & f & q \\ g & f & c & r \\ p & q & r & d \end{vmatrix},$$

and where the linear equations serving to eliminate x, y from U are

$$\mu_1x + \mu_2y + \mu_3z + \mu_4w = 0$$
$$\nu_1x + \nu_2y + \nu_3z + \nu_4w = 0,$$

we have, according to Sylvester, the discriminant of the altered U equal to

$$\begin{vmatrix} a & h & g & p & \mu_1 & \nu_1 \\ h & b & f & q & \mu_2 & \nu_2 \\ g & f & c & r & \mu_3 & \nu_3 \\ p & q & r & d & \mu_4 & \nu_4 \\ \mu_1 & \mu_2 & \mu_3 & \mu_4 & \cdot & \cdot \\ \nu_1 & \nu_2 & \nu_3 & \nu_4 & \cdot & \cdot \end{vmatrix} \div |\mu_1\nu_2|^2.$$

SYLVESTER (1852, July).

[A demonstration of the theorem that every homogeneous quadratic polynomial is reducible by real orthogonal substitutions to the form of a sum of positive and negative squares. *Philos. Magazine*, (4) iv. pp. 138–142: or *Collected Math. Papers*, i. pp. 378–381.]

What is really proved here is the important proposition in the theory of orthogonants regarding the reality of the roots of Lagrange's determinantal equation, or, as it was then called, the equation of the secular

inequalities. Our present interest in the demonstration, however, lies merely in the fact that it is based on two properties of axisymmetric determinants which it is desirable to isolate and to have more carefully formulated than it was Sylvester's wont to do. They are—

(1) If $|(11)\ (22)\ \ldots\ (nn)|$ be axisymmetric, and the result of multiplying it by itself be $|[11]\ [22]\ \ldots\ [nn]|$; and if $f(x)$ be the determinant got from the former by adding x to each element of the principal diagonal, and $F(x)$ the determinant got similarly from the latter; then

$$f(x)\cdot f(-x) = F(-x^2).$$

(2) If $F(x)$ be expanded and arranged according to descending powers of x, so that

$$F(x) = x^n + C_1x^{n-1} + C_2x^{n-2} + \ldots + C_n,$$

then C_r is the sum of the squares of all the r-line minors of the original determinant, it being understood that the one-line minors are the elements and the n-line minor the determinant itself.

Both are taken for granted,—a liberty which is not so defensible in the second case as in the first; for C_r is at the outset obtained merely as the sum of the r-line coaxial minors of $|[11]\ [22]\ \ldots\ [nn]|$, and use must thus be latently made of the not quite self-evident theorem that *if Δ be an axisymmetric determinant, the sum of the* r*-line coaxial minors of Δ^2 is the sum of the squares of all the* r*-line minors of Δ.* As an illustration of the whole, let us take the case where the given determinant and its square are

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} L & R & Q \\ R & M & P \\ Q & P & N \end{vmatrix}$$

and where therefore

$$\left.\begin{matrix} L & R, & Q \\ & M & P \\ & & N \end{matrix}\right\} \equiv \left\{\begin{matrix} a^2+h^2+g^2 & ah+hb+fg & ag+hf+gc \\ & h^2+b^2+f^2 & hg+bf+fc \\ & & g^2+f^2+c^2. \end{matrix}\right.$$

We have then

$$\begin{vmatrix} a+x & h & g \\ h & b+x & f \\ g & f & c+x \end{vmatrix} \cdot \begin{vmatrix} a-x & h & g \\ h & b-x & f \\ g & f & c-x \end{vmatrix} = \begin{vmatrix} L-x^2 & R & Q \\ R & M-x^2 & P \\ Q & P & N-x^2 \end{vmatrix},$$

$$= -x^6 + x^4(L+M+N) - x^2\left(\begin{vmatrix} L & R \\ R & M \end{vmatrix} + \begin{vmatrix} L & Q \\ Q & N \end{vmatrix} + \begin{vmatrix} M & P \\ P & N \end{vmatrix}\right) + \begin{vmatrix} L & R & Q \\ R & M & P \\ Q & P & N \end{vmatrix},$$

$$= -x^6 + x^4\left\{\begin{array}{l} a^2+h^2+g^2 \\ +h^2+b^2+f^2 \\ +g^2+f^2+c^2 \end{array}\right\} - x^2\left\{\begin{array}{l} \begin{vmatrix} a & h \\ h & b \end{vmatrix}^2 + \begin{vmatrix} a & g \\ h & f \end{vmatrix}^2 + \begin{vmatrix} h & g \\ b & f \end{vmatrix}^2 \\ + \begin{vmatrix} a & g \\ h & f \end{vmatrix}^2 + \begin{vmatrix} a & g \\ g & c \end{vmatrix}^2 + \begin{vmatrix} h & g \\ f & c \end{vmatrix}^2 \\ + \begin{vmatrix} h & g \\ b & f \end{vmatrix}^2 + \begin{vmatrix} h & g \\ f & c \end{vmatrix}^2 + \begin{vmatrix} b & f \\ f & c \end{vmatrix}^2 \end{array}\right\} + \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}^2 .$$

The fact that the coefficients here are negative and positive alternately is what Sylvester utilises for his main purpose, application being made of Descartes' rule of signs.

Sylvester (1852, Oct.).

[On Staudt's theorems concerning the contents of polygons and polyhedrons, with a note on a new and resembling class of theorems. *Philos. Magazine*, (4) iv. pp. 335–345: or *Collected Math. Papers*, i. pp. 382–391.]

As an illustration of his mode of expressing the product of two determinants of the n^{th} order as a determinant of the $(n+1)^{\text{th}}$ order, Sylvester gives the identity

$$\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix} \cdot \begin{vmatrix} \xi_1 & \eta_1 & \zeta_1 & 1 \\ \xi_2 & \eta_2 & \zeta_2 & 1 \\ \xi_3 & \eta_3 & \zeta_3 & 1 \\ \xi_4 & \eta_4 & \zeta_4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \Sigma x_1\xi_1 & \Sigma x_1\xi_2 & \Sigma x_1\xi_3 & \Sigma x_1\xi_4 & 1 \\ \Sigma x_2\xi_1 & \Sigma x_2\xi_2 & \Sigma x_2\xi_3 & \Sigma x_2\xi_4 & 1 \\ \Sigma x_3\xi_1 & \Sigma x_3\xi_2 & \Sigma x_3\xi_3 & \Sigma x_3\xi_4 & 1 \\ \Sigma x_4\xi_1 & \Sigma x_4\xi_2 & \Sigma x_4\xi_3 & \Sigma x_4\xi_4 & 1 \\ 1 & 1 & 1 & 1 & . \end{vmatrix}$$

where $\Sigma x_r\xi_s$ is put for $x_r\xi_s + y_r\eta_s + z_r\zeta_s$. Then performing on the last determinant the operations which we may denote by

$$\text{row}_1 - \tfrac{1}{2}\Sigma x_1^2 .\ \text{row}_5 , \quad \text{row}_2 - \tfrac{1}{2}\Sigma x_2^2 .\ \text{row}_5 , \ \ldots .$$
$$\text{col}_1 - \tfrac{1}{2}\Sigma \xi_1^2 .\ \text{col}_5 , \quad \text{col}_2 - \tfrac{1}{2}\Sigma \xi_2^2 .\ \text{col}_5 , \ \ldots .$$

he obtains

$$-\tfrac{1}{8}\begin{vmatrix} \Sigma(x_1-\xi_1)^2 & \Sigma(x_1-\xi_2)^2 & \Sigma(x_1-\xi_3)^2 & \Sigma(x_1-\xi_4)^2 & 1 \\ \Sigma(x_2-\xi_1)^2 & \Sigma(x_2-\xi_2)^2 & \Sigma(x_2-\xi_3)^2 & \Sigma(x_2-\xi_4)^2 & 1 \\ \Sigma(x_3-\xi_1)^2 & \Sigma(x_3-\xi_2)^2 & \Sigma(x_3-\xi_3)^2 & \Sigma(x_3-\xi_4)^2 & 1 \\ \Sigma(x_4-\xi_1)^2 & \Sigma(x_4-\xi_2)^2 & \Sigma(x_4-\xi_3)^2 & \Sigma(x_4-\xi_4)^2 & 1 \\ 1 & 1 & 1 & 1 & . \end{vmatrix} ;$$

so that, if x_r, y_r, z_r and ξ_r, η_r, ζ_r be rectangular co-ordinates of points in space, the result reached gives an expression for thirty-six times the product of the volumes of two tetrahedrons in terms of the distances of the angular points of the one from the angular points of the other. By proceeding to the case where the two tetrahedrons are coincident, and thence to the case where the four remaining points are situated in the same plane, we reach Cayley's relation connecting the mutual distances of four such points.

It is thus seen that whereas Cayley's vanishing axisymmetric determinant was originally got as a multiple of a peculiarly obtained square of the determinant

$$\begin{vmatrix} \Sigma x_1^2 & x_1 & y_1 & 0 & 1 \\ \Sigma x_2^2 & x_2 & y_2 & 0 & 1 \\ \Sigma x_3^2 & x_3 & y_3 & 0 & 1 \\ \Sigma x_4^2 & x_4 & y_4 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{vmatrix},$$

Sylvester arrives at it by squaring

$$\begin{vmatrix} x_1 & y_1 & 0 & 1 \\ x_2 & y_2 & 0 & 1 \\ x_3 & y_3 & 0 & 1 \\ x_4 & y_4 & 0 & 1 \end{vmatrix}$$

in a special fashion, and then performing certain transformations on the result.

Sylvester (1852, Nov.).

[Sur une propriété nouvelle de l'équation qui sert à déterminer les inégalités séculaires des planètes. *Nouv. Annales de Math.*, xi. pp. 434–440: or *Collected Math. Papers*, i. pp. 364–366.]

This paper of composite authorship probably originated in a letter from Sylvester giving his theorem and demonstration, with a remark or two additional. To these, which were made §§ 7, 7′, 8, the editor prefixed an introduction (§§ 1–6) on determinants and determinant-multiplication.*

The theorem is an extension of one which is the basis of his paper in the *Philosophical Magazine* of the same year, and may be shortly enunciated as follows: *If* $|(11)\ (22)\ \dots\ (nn)|$ *be axisymmetric and have* $|[11]\ [22]\ \dots\ [nn]|$ *for its* p^{th} *power, then the roots of the equation*

* In the *Coll. Math. Papers* §§ 1–6 are omitted, and §§ 7, 7′, 8 are numbered §§ 6, 7, 8. The theorem of the original § 6 is incorrect.

$$\begin{vmatrix} [11]-x & [12] & \dots & [1n] \\ [21] & [22]-x & \dots & [2n] \\ \dots & \dots & \dots & \dots \\ [n1] & [n2] & \dots & [nn]-x \end{vmatrix} = 0$$

are the p^{th} *powers of the roots of the equation*

$$\begin{vmatrix} (11)-x & (12) & \dots & (1n) \\ (21) & (22)-x & \dots & (2n) \\ \dots & \dots & \dots & \dots \\ (n1) & (n2) & \dots & (nn)-x \end{vmatrix} = 0 .$$

The "demonstration" leaves a good deal to be desired. In effect it amounts to saying that if $\zeta_1, \zeta_2, \dots, \zeta_p$ be the p^{th} roots of unity, and D_m the determinant got from $|(11), (22), \dots, (nn)|$ by subtracting $\zeta_m x$ from each of the diagonal elements, then

$$D_1 D_2 D_3 \dots D \equiv \begin{vmatrix} [11]-x^p & [12] & \dots & [1n] \\ [21] & [22]-x^p & \dots & [2n] \\ \dots & \dots & \dots & \dots \\ [n1] & [n2] & \dots & [nn]-x^p \end{vmatrix} .$$

Now it is well known that the multiplication of $D_1, D_2, \dots, D_p$ enables us to arrive at the equation whose roots are the p^{th} powers in question, but this and Sylvester's statement are not by any means identical. The separate points to be established are (1) that the element in the r^{th} row and s^{th} column of the determinant which is the product of $D_1, D_2, \dots, D_p$ consists of $[rs]$ and a tail of other terms, (2) that this tail vanishes in the case of every non-diagonal element, (3) that in the case of the diagonal elements it reduces to $-x$ and Sylvester's only justificatory statements are that the product of the p determinants is independent of the order in which they are taken, and that all the terms containing ζ in any other power than the p^{th} will vanish.

Another true proposition made on insufficient foundation is that *the* p^{th} *power of an axisymmetric determinant is itself axisymmetric.* The foundation here is the incorrect proposition of § 6.

Cayley (1852, Dec.).

[On the rationalisation of certain algebraical equations. *Cambridge and Dub. Math. Journ.*, viii. pp. 97–101: or *Collected Math. Papers*, ii. pp. 40–44.]

The equations first concerned with are of the type

$$a^{\frac{1}{2}} + b^{\frac{1}{2}} + c^{\frac{1}{2}} + \dots = 0 ,$$

and the fresh departure consists in viewing such an equation as the outcome of the set of equations

$$x+y+z+\ldots=0,\quad x^2=a,\quad y^2=b,\quad z^2=c,\quad \ldots .$$

Taking the case where the number of variables in the set is three, Cayley operates on the equation $x+y+z=0$ with the multipliers $1, yz, zx, xy$, thus obtaining with the help of the other equations a set from which the variables x, y, z, xyz may be eliminated with the result *

$$\begin{vmatrix} . & 1 & 1 & 1 \\ 1 & . & c & b \\ 1 & c & . & a \\ 1 & b & a & . \end{vmatrix} = 0 .$$

Also, and, so to say, conversely, he operates with the multipliers x, y, z, xyz, and eliminates $1, yz, zx, xy$, with the result

$$\begin{vmatrix} . & a & b & c \\ a & . & 1 & 1 \\ b & 1 & . & 1 \\ c & 1 & 1 & . \end{vmatrix} = 0 .$$

Similarly, when there are four variables, the multipliers are

$$1,\ yz,\ zx,\ xy,\ xw,\ yw,\ zw,\ xyzw,$$

and the eliminands

$$x,\ y,\ z,\ w,\ yzw,\ zwx,\ wxy,\ xyz,$$

or *vice versa*; but in this case the two resultants are not essentially distinct, the one being derivable from the other by mere transference of lines. Cayley then adds, "And in general for any *even* number of quadratic radicals the two forms are not essentially distinct,† but may be derived from each other by interchanging lines and columns, while for an *odd* number of quadratic radicals the two forms cannot be so derived from each other, but are essentially distinct."

The equations next dealt with are of the type

$$a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}}+\ \ldots . = 0,$$

Sylvester having suggested the extension of the process. Taking the case of three variables, that is to say, when the set of equations is

$$x+y+z=0,\quad x^3=a,\quad y^3=b,\quad z^3=c,$$

* Already thus formed by Cayley in his first paper of all (1841).

† Observe that, although Cayley considers the two determinants of the previous case to be essentially distinct, the second is derivable from the first by multiplying the columns by abc, a, b, c respectively, and then dividing the rows by $1, bc, ca, ab$ respectively.

he first uses the multipliers

$$1,\ xyz,\ x^2y^2z^2,\ x^2z,\ y^2x,\ z^2y,\ x^2y,\ y^2z,\ z^2x,$$

the eliminands then being

$$x,\ y,\ z,\ y^2z^2,\ x^2yz,\ y^2zx,\ z^2xy,\ z^2x^2,\ x^2y^2;$$

next he uses the said eliminands as multipliers, the new eliminands being

$$x^2,\ y^2,\ z^2,\ yz,\ zx,\ xy,\ xy^2z^2,\ yz^2x^2,\ zx^2y^2;$$

and finally using the new eliminands as multipliers, he eliminates

$$1,\ xyz,\ x^2y^2z^2,\ x^2z,\ y^2x,\ z^2y,\ x^2y,\ y^2z,\ z^2x,$$

that is to say, the first set of multipliers. Only in the case of the second elimination is the determinant axisymmetric, namely,

$$\begin{vmatrix} . & c & b & . & . & . & 1 & . & . \\ c & . & a & . & . & . & . & 1 & . \\ b & a & . & . & . & . & . & . & 1 \\ . & . & . & a & . & . & . & 1 & 1 \\ . & . & . & . & b & . & 1 & . & 1 \\ . & . & . & . & . & c & 1 & 1 & . \\ 1 & . & . & . & 1 & 1 & . & . & . \\ . & 1 & . & 1 & . & 1 & . & . & . \\ . & . & 1 & 1 & 1 & . & . & . & . \end{vmatrix}$$

which must thus be equal to $(a+b+c)^3-27abc$. In the two other cases the determinants are not essentially distinct,* the rows of the one being columns of the other; and this is said to be true of two of the three forms whatever be the number of variables.

Sylvester (1853, March).

[On the relation between the volume of a tetrahedron and the product of the sixteen algebraical values of its superficies. *Cambridge and Dub. Math. Journ.*, viii. pp. 171–178: or *Nouv. Annales de Math.*, xiii. pp. 203–209: or *Collected Math. Papers*, i. pp. 404–410.]

Denoting the vertices of a tetrahedron by a, b, c, d, its volume in terms of the edges by V, and the areas of its faces by $\frac{1}{4}\sqrt{\mathrm{F}}$, $\frac{1}{4}\sqrt{\mathrm{G}}$, $\frac{1}{4}\sqrt{\mathrm{H}}$, $\frac{1}{4}\sqrt{\mathrm{K}}$, we know that

* Cayley fails to notice, however, that each of those is readily transformable into the second. Thus, taking his first form, we have only to multiply the 4th, 8th, 9th columns of it by a, and divide the 2nd, 4th, 7th rows by a, when we obtain a determinant which by mere permutation of the rows $\begin{pmatrix}1&2&3&4&5&6&7&8&9\\7&4&1&9&3&6&8&5&2\end{pmatrix}$ and subsequently of the columns $\begin{pmatrix}1&2&3&4&5&6&7&8&9\\1&6&5&4&9&8&7&3&2\end{pmatrix}$ becomes identical with the axisymmetric form.

$$
\begin{aligned}
144V^2 = \quad & (bc)^2(da)^2\{(ca)^2+(db)^2+(ab)^2+(cd)^2-(bc)^2-(da)^2\} \\
& +(ca)^2(db)^2\{(ab)^2+(cd)^2+(bc)^2+(da)^2-(ca)^2-(db)^2\} \\
& +(ab)^2(cd)^2\{(bc)^2+(da)^2+(ca)^2+(db)^2-(ab)^2-(cd)^2\} \\
& -(bc)^2(ca)^2(ab)^2-(bc)^2(db)^2(cd)^2-(ca)^2(cd)^2(da)^2-(ab)^2(da)^2(db)^2 \\
= \quad & \text{W say,}
\end{aligned}
$$

and

$$
\begin{aligned}
F &= -(bc)^4-(cd)^4-(db)^4+2(cd)^2(db)^2+2(db)^2(bc)^2+2(bc)^2(cd)^2 \\
G &= -(ac)^4-(cd)^4-(da)^4+2(cd)^2(da)^2+2(da)^2(ac)^2+2(ac)^2(cd)^2 \\
H &= -(ab)^4-(bd)^4-(da)^4+2(ab)^2(bd)^2+2(bd)^2(da)^2+2(da)^2(ab)^2 \\
K &= -(ab)^4-(bc)^4-(ca)^4+2(bc)^2(ca)^2+2(ca)^2(ab)^2+2(ab)^2(bc)^2 .
\end{aligned}
$$

With this notation Sylvester points out that the condition for the vanishing of the surface of the tetrahedron is

$$\sqrt{F}+\sqrt{G}+\sqrt{H}+\sqrt{K} = 0,$$

and that this when freed of root-signs is

$$\sum F^4 - 4\sum F^2G + 6\sum F^2G^2 + 4\sum F^3GH - 40FGHK = 0,$$

or say

$$N=0,$$

where N consequently is of the eighth degree in the squared edges. His reasoning then is that as the vanishing of the surface and the vanishing of the volume are necessarily coincident, it follows that W, having no rational factors, must itself be a factor of N; and that, W being of the third degree in the squared edges, the quotient N/W must be of the fifth degree. Relying on this he proceeds to determine the quotient by considering the cases where (1) $ab=0=cd$, (2) $ab=0=ac$, (3) $ab=ac=ad=bc=bd=cd=1$, his result being

$$
\sum(ab)^2(bc)^2(ca)^2\left[\begin{array}{c}
(da)^4+(db)^4+(dc)^4 \\
-\{(da)^2+(db)^2+(dc)^2\}\{(ab)^2+(bc)^2+(ca)^2\} \\
+(ab)^2(bc)^2+(bc)^2(ca)^2+(aa)^2(ab)^2
\end{array}\right]
$$

$$+2\sum(ab)^2(bc)^2(cd)^2(da)^2(ac)^2,$$

where there are four expressions under the first Σ and six under the second; or

$$
\sum(ab)^2(bc)^2(ca)^2\left[\begin{array}{c}
(da)^4+(db)^4+(dc)^4 \\
-\{(da)^2+(db)^2+(dc)^2\}\{(ab)^2+(bc)^2+(ca)^2\} \\
+(da)^2(db)^2+(db)^2(dc)^2+(dc)^2(da)^2 \\
+(ab)^2(bc)^2+(bc)^2(ca)^2+(ca)^2(ab)^2
\end{array}\right]
$$

where there are four expressions under the Σ, one corresponding to each face.

Although there is no explicit mention here of determinants, it being

unnecessary, it has now to be noted that Sylvester had the determinant-form of W before him throughout: he even says that he had tried to express the quotient as a determinant, but had been unsuccessful. Without further restriction, then, as to form, his proposition is *If* F , G , H , K *be the complementary minors of the elements in the places* 11 , 22 , 33 , 44 *of the determinant*

$$\begin{vmatrix} \cdot & (ab)^2 & (ac)^2 & (ad)^2 & 1 \\ (ab)^2 & \cdot & (bc)^2 & (bd)^2 & 1 \\ (ac)^2 & (bc)^2 & \cdot & (cd)^2 & 1 \\ (ad)^2 & (bd)^2 & (cd)^2 & \cdot & 1 \\ 1 & 1 & 1 & 1 & \cdot \end{vmatrix} \quad \text{or } 2\mathrm{W} \text{ say},$$

then the result of rationalising

$$\sqrt{\mathrm{F}} + \sqrt{\mathrm{G}} + \sqrt{\mathrm{H}} + \sqrt{\mathrm{K}}$$

is divisible by W. This is not all, however; for Sylvester having noted the analogous case connected with the relation between the perimeter and area of a triangle, namely, the fact that

$$\begin{vmatrix} \cdot & (ab)^2 & (ac)^2 & 1 \\ (ab)^2 & \cdot & (bc)^2 & 1 \\ (ac)^2 & (bc)^2 & \cdot & 1 \\ 1 & 1 & 1 & \cdot \end{vmatrix} \quad i.e. \quad \sum(ab)^4 - 2\sum(ab)^2(bc)^2$$

is a divisor of the result of rationalising

$$\sqrt{2(bc)^2} + \sqrt{2(ca)^2} + \sqrt{2(ab)^2}$$

where the radicands are the complementary minors of the elements in the places 11 , 22 , 33 of the determinant, boldly extends the proposition (without proof) to any triangular number of arbitrary quantities, taking occasion also to point out that when we leave geometry (ab) , (ac) , ... may be written for $(ab)^2$, $(ac)^2$, ...

Hesse (1853, April).

[Ueber Determinanten und ihre Anwendung in der Geometrie, insbesondere auf Curven vierter Ordnung. *Crelle's Journal*, xlix. pp. 243–264.]

The main subject of the first half of Hesse's paper (pp. 243–253) is a property of axisymmetric determinants required for the establishment of the geometrical results contained in the second half. In the first three pages he considers the relations between the minors of two general

determinants A, B, and the minors of their product C; or, as he unfortunately feels himself compelled to put it, "wie die partiellen Differential-quotienten der Determinante C nach ihren Elementen c genommen durch die partiellen Differential-quotienten der Factoren A und B nach ihren Elementen genommen sich ausdrücken lassen." What follows thereafter may be described as the establishment of the simple identity

$$\begin{vmatrix} u_{11} & u_{12} & \alpha_1 \\ u_{21} & u_{22} & \alpha_2 \\ \alpha_1 & \alpha_2 & 0 \end{vmatrix} \begin{vmatrix} u_{11} & u_{12} & \gamma_1 \\ u_{21} & u_{22} & \gamma_2 \\ \gamma_1 & \gamma_2 & 0 \end{vmatrix} - \begin{vmatrix} u_{11} & u_{12} & \alpha_1 \\ u_{21} & u_{22} & \alpha_2 \\ \gamma_1 & \gamma_2 & 0 \end{vmatrix}^2 = \begin{vmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{vmatrix} \begin{vmatrix} \alpha_1 & \alpha_2 \\ \gamma_1 & \gamma_2 \end{vmatrix}^2,$$

where $u_{12}=u_{12}$, by multiplying together

$$\begin{vmatrix} \alpha_1 & -\alpha_2 \\ \gamma_1 & -\gamma_2 \end{vmatrix}, \quad \begin{vmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{vmatrix}, \quad \begin{vmatrix} -\alpha_2 & \alpha_1 \\ -\gamma_2 & \gamma_1 \end{vmatrix}$$

in row-by-row fashion; and then the generalisation of this identity in two different directions.

The first generalisation consists in the proposition that the two-line axisymmetric determinant

$$\begin{vmatrix} u_{11} & u_{12} & \cdots & u_{1n} & \alpha_1 \\ u_{21} & u_{22} & \cdots & u_{2n} & \alpha_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{n1} & u_{n2} & \cdots & u_{nn} & \alpha_n \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n & 0 \end{vmatrix} \begin{vmatrix} u_{11} & u_{12} & \cdots & u_{1n} & \gamma_1 \\ u_{21} & u_{22} & \cdots & u_{2n} & \gamma_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{n1} & u_{n2} & \cdots & u_{nn} & \gamma_n \\ \gamma_1 & \gamma_2 & \cdots & \gamma_n & 0 \end{vmatrix} - \begin{vmatrix} u_{11} & u_{12} & \cdots & u_{1n} & \alpha_1 \\ u_{21} & u_{22} & \cdots & u_{2n} & \alpha_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{n1} & u_{n2} & \cdots & u_{nn} & \alpha_n \\ \gamma_1 & \gamma_2 & \cdots & \gamma_n & 0 \end{vmatrix}^2$$

where $u_{\kappa\lambda}=u_{\lambda\kappa}$, contains

$$\begin{vmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ u_{21} & u_{22} & \cdots & u_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ u_{n1} & u_{n2} & \cdots & u_{nn} \end{vmatrix}$$

as a factor, and that the cofactor is an integral homogeneous function of the α's and likewise of the γ's. The case where n is equal to 3 is treated as follows. The determinant

$$\begin{vmatrix} u_{11} & u_{12} & u_{13} & \alpha_1 \\ u_{21} & u_{22} & u_{23} & \alpha_2 \\ u_{31} & u_{32} & u_{33} & \alpha_3 \\ \gamma_1 & \gamma_2 & \gamma_3 & \beta \end{vmatrix},$$

having $u_{\kappa\lambda}=u_{\lambda\kappa}$, is introduced and denoted by B, with the result that the two-line determinant in question is representable by

$$\begin{vmatrix} \frac{\partial B}{\partial \gamma_1}\alpha_1 + \frac{\partial B}{\partial \gamma_2}\alpha_2 + \frac{\partial B}{\partial \gamma_3}\alpha_3 & \frac{\partial B}{\partial \alpha_1}\alpha_1 + \frac{\partial B}{\partial \alpha_2}\alpha_2 + \frac{\partial B}{\partial \alpha_3}\alpha_3 \\ \frac{\partial B}{\partial \gamma_1}\gamma_1 + \frac{\partial B}{\partial \gamma_2}\gamma_2 + \frac{\partial B}{\partial \gamma_3}\gamma_3 & \frac{\partial B}{\partial \alpha_1}\gamma_1 + \frac{\partial B}{\partial \alpha_2}\gamma_2 + \frac{\partial B}{\partial \alpha_3}\gamma_3 \end{vmatrix}$$

and its predicated factor by $\frac{\partial B}{\partial \beta}$. It is then pointed out that the former is the differential-quotient of the product of the two determinants

$$\begin{vmatrix} \frac{\partial B}{\partial \gamma_1} & \frac{\partial B}{\partial \gamma_2} & \frac{\partial B}{\partial \gamma_3} \\ \frac{\partial B}{\partial \alpha_1} & \frac{\partial B}{\partial \alpha_2} & \frac{\partial B}{\partial \alpha_3} \\ m_1 & m_2 & m_3 \end{vmatrix}, \quad \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \\ n_1 & n_2 & n_3 \end{vmatrix},$$

called M and N, taken with respect to $m_1n_1 + m_2n_2 + m_3n_3$; and that consequently it is equal to

$$\frac{\partial M}{\partial m_1}\frac{\partial N}{\partial n_1} + \frac{\partial M}{\partial m_2}\frac{\partial N}{\partial n_2} + \frac{\partial M}{\partial m_3}\frac{\partial N}{\partial n_3}. \qquad (\varpi)$$

Since, however, we have

$$\frac{\partial B}{\partial \gamma_1}u_{11} + \frac{\partial B}{\partial \gamma_2}u_{12} + \frac{\partial B}{\partial \gamma_3}u_{13} + \frac{\partial B}{\partial \beta}\alpha_1 = 0,$$

and other similar identities, it follows that

$$M \cdot \frac{\partial B}{\partial \beta} \quad i.e. \quad \begin{vmatrix} \frac{\partial B}{\partial \gamma_1} & \frac{\partial B}{\partial \gamma_2} & \frac{\partial B}{\partial \gamma_3} \\ \frac{\partial B}{\partial \alpha_1} & \frac{\partial B}{\partial \alpha_2} & \frac{\partial B}{\partial \alpha_3} \\ m_1 & m_2 & m_3 \end{vmatrix} \cdot \begin{vmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{vmatrix}$$

$$= \begin{vmatrix} -\frac{\partial B}{\partial \beta}\alpha_1 & -\frac{\partial B}{\partial \beta}\gamma_1 & u_{11}m_1 + u_{12}m_2 + u_{13}m_3 \\ -\frac{\partial B}{\partial \beta}\alpha_2 & -\frac{\partial B}{\partial \beta}\gamma_2 & u_{21}m_1 + u_{22}m_2 + u_{23}m_3 \\ -\frac{\delta B}{\partial \beta}\alpha_3 & -\frac{\partial B}{\partial \beta}\gamma_3 & u_{31}m_1 + u_{32}m_2 + u_{33}m_3 \end{vmatrix},$$

and therefore

$$M = \frac{\partial B}{\partial \beta}\begin{vmatrix} \alpha_1 & \gamma_1 & u_{11}m_1 + u_{12}m_2 + u_{13}m_3 \\ \alpha_2 & \gamma_2 & u_{21}m_1 + u_{22}m_2 + u_{23}m_3 \\ \alpha_3 & \gamma_3 & u_{31}m_1 + u_{32}m_2 + u_{33}m_3 \end{vmatrix} = \frac{\partial B}{\partial \beta} \cdot P \text{ say.}$$

The expression (ϖ) then becomes

$$\frac{\partial B}{\partial \beta}\left\{\frac{\partial P}{\partial m_1}\frac{\partial N}{\partial n_1} + \frac{\partial P}{\partial m_2}\frac{\partial N}{\partial n_2} + \frac{\partial P}{\partial m_3}\frac{\partial N}{\partial n_3}\right\},$$

and all that remains is the evaluation of the bracketed factor after the equivalents of P and N have been substituted therein. The final result is

$$\begin{vmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{vmatrix} \cdot \left\{ \begin{aligned} & u_{11}(\alpha_2\gamma_3 - \alpha_3\gamma_2)^2 + u_{22}(\alpha_3\gamma_1 - \alpha_1\gamma_3)^2 + u_{33}(\alpha_1\gamma_2 - \alpha_2\gamma_1)^2 \\ & + (u_{23} + u_{32})(\alpha_3\gamma_1 - \alpha_1\gamma_3)(\alpha_1\gamma_2 - \alpha_2\gamma_1) \\ & + (u_{31} + u_{13})(\alpha_1\gamma_2 - \alpha_2\gamma_1)(\alpha_2\gamma_3 - \alpha_3\gamma_2) \\ & + (u_{12} + u_{21})(\alpha_2\gamma_3 - \alpha_3\gamma_2)(\alpha_3\gamma_1 - \alpha_1\gamma_3) \end{aligned} \right\}.$$

The case where $n=4$ is similarly dealt with; but as it is necessarily more complicated, it is not carried quite so far, the cofactor of $|u_{11}\ u_{22}\ u_{33}\ u_{44}|$ being merely stated to be of the desired form and easily calculable. "Sie hat aber zu viele Glieder, um sie berechnet hinzuschreiben." The general proposition, as above given, is then formally enunciated.

The other generalisation made of the case where $n = 2$ is to the effect that *the product of an axisymmetric determinant by the square of any other determinant is expressible as an axisymmetric determinant.* In connection with this the interesting point is the notation used for the elements of the product-determinants. Since the differential-quotient of

$$\tfrac{1}{2}\{u_{11}x_1^2 + u_{22}x_2^2 + \ . \ . \ . \ + u_{nn}x_n^2 + 2u_{12}x_1x_2 + \ . \ . \ . \ \},$$

$$\text{or} \quad F(x_1, x_2, \ . \ . \ . \ , x_n) \quad \text{say,}$$

with respect to x_p is

$$u_{1p}x_1 + u_{2p}x_2 + \ . \ . \ . \ + u_{pp}x_p + \ . \ . \ . \ + u_{np}x_n$$

the result of annexing q as a second suffix to the x's in this may be suitably denoted by

$$F'(x_{pq});$$

so that in accordance with this the product of

$$\begin{vmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ u_{21} & u_{22} & \dots & u_{2n} \\ \dots & \dots & \dots & \dots \\ u_{n1} & u_{n2} & \dots & u_{nn} \end{vmatrix}_{u_{\kappa\lambda} = u_{\lambda\kappa}} \quad \text{and} \quad \begin{vmatrix} x_{11} & x_{21} & \dots & x_{n1} \\ x_{12} & x_{22} & \dots & x_{n2} \\ \dots & \dots & \dots & \dots \\ x_{1n} & x_{2n} & \dots & x_{nn} \end{vmatrix}$$

will be

$$\begin{vmatrix} F'(x_{11}) & F'(x_{21}) & \dots & F'(x_{n1}) \\ F'(x_{12}) & F'(x_{22}) & \dots & F'(x_{n2}) \\ \dots & \dots & \dots & \dots \\ F'(x_{1n}) & F'(x_{2n}) & \dots & F'(x_{nn}) \end{vmatrix}$$

"Multiplicirt man diese Determinante nochmals mit der vorhergehenden und setzt:

$$F_{pq} = x_{1p}F'(x_{1q}) + x_{2p}F'(x_{2q}) + \ . \ . \ . \ + x_{np}F'(x_{nq}),$$

so erhält man

$$\begin{vmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ u_{21} & u_{22} & \cdots & u_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ u_{n1} & u_{n2} & \cdots & u_{nn} \end{vmatrix} \cdot \begin{vmatrix} x_{11} & x_{21} & \cdots & x_{n1} \\ x_{12} & x_{22} & \cdots & x_{n2} \\ \cdot & \cdot & \cdots & \cdot \\ x_{1n} & x_{2n} & \cdots & x_{nn} \end{vmatrix}^2 = \begin{vmatrix} F_{11} & F_{12} & \cdots & F_{1n} \\ F_{21} & F_{22} & \cdots & F_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ F_{n1} & N_{n2} & \cdots & F_{nn} \end{vmatrix}.$$

Da aber $F_{pq} = F_{qp}$ ist, so ist die letzte Determinante wieder *symmetrisch.*"

Brioschi (1853, July).

[Sur une propriété d'un produit de facteurs linéaires. *Cambridge and Dub. Math. Journ.*, ix. pp. 137–144: or *Opere Mat.*]

This paper of Brioschi's is avowedly an outcome of Cayley's "On the rationalisation of certain algebraical equations," published two months earlier. Starting with the equation $x+y+z=0$, and using on it the multipliers $1, yz, zx, xy$ in succession, he obtains

$$\left.\begin{array}{lllll} & x & + \; y & + \; z & = 0 \\ xyz \; + & & + \; z^2.y & + \; y^2.z & = 0 \\ xyz \; + & z^2.x & & + \; x^2.z & = 0 \\ xyz \; + & y^2.x & + \; x^2.y & & = 0 \end{array}\right\}$$

whence on elimination of xyz, x, y, z there results

$$0 = \begin{vmatrix} \cdot & 1 & 1 & 1 \\ 1 & \cdot & z^2 & y^2 \\ 1 & z^2 & \cdot & x^2 \\ 1 & y^2 & x^2 & \cdot \end{vmatrix} = \Delta \text{ say};$$

and as this equation has its origin in the equation $x+y+z=0$, he concludes that the determinant Δ must have $x+y+z$ for a factor. Then, since any one of the three other equations

$$x+y-z=0, \quad x-y+z=0, \quad -x+y+z=0$$

gives rise to the same result, it is easily seen how he reaches the identity

$$-\begin{vmatrix} \cdot & 1 & 1 & 1 \\ 1 & \cdot & z^2 & y^2 \\ 1 & z^2 & \cdot & x^2 \\ 1 & y^2 & x^2 & \cdot \end{vmatrix} = (x+y+z)(x+y-z)(x-y+z)(-x+y+z).$$

An alternative form of Δ is introduced by the words "Observons qu'on a évidemment *

* It would seem preferable to multiply the columns of Δ in order by xyz, x, y, z, the quantities just eliminated; and then divide the rows by the multipliers $1, yz, zx, xy$ used in obtaining the set of equations. The advantage of this method would be still greater in the next case.

$$\Delta = \frac{1}{x^2y^2z^2}\begin{vmatrix} . & x & y & z \\ x & . & z^2xy & y^2xz \\ y & z^2xy & . & x^2yz \\ z & y^2xz & x^2yz & . \end{vmatrix} = \begin{vmatrix} . & x & y & z \\ x & . & z & y \\ y & z & . & x \\ z & y & x & . \end{vmatrix} \text{''} .$$

Similarly from the equation $x+y+z+w=0$ or any one of its seven relatives by using the multipliers

$$1\,,\; zw\,,\; yw\,,\; xw\,,\; zy\,,\; zx\,,\; yx\,,\; xyzw$$

and eliminating

$$xyz\,,\; xyw\,,\; xzw\,,\; yzw\,,\; x\,,\; y\,,\; z\,,\; w$$

there is obtained the result

$$\begin{vmatrix} . & . & . & . & 1 & 1 & 1 & 1 \\ . & . & 1 & 1 & . & . & w^2 & z^2 \\ . & 1 & . & 1 & . & w^2 & . & y^2 \\ . & 1 & 1 & . & w^2 & . & . & x^2 \\ 1 & . & . & 1 & . & z^2 & y^2 & . \\ 1 & . & 1 & . & z^2 & . & x^2 & . \\ 1 & 1 & . & . & y^2 & x^2 & . & . \\ w^2 & z^2 & y^2 & x^2 & . & . & . & . \end{vmatrix}$$

$$= (x+y+z+w)\cdot(-x+y+z+w)(x-y+z+w)(x+y-z+w)(x+y+z-w) \cdot(-x-y+z+w)(-x+y-z+w)(-x+y+z-w),$$

an alternative form being stated to be the axisymmetric determinant

$$\begin{vmatrix} . & . & . & . & x & y & z & w \\ . & . & x & y & . & . & w & z \\ . & x & . & z & . & w & . & y \\ . & y & z & . & w & . & . & x \\ x & . & . & w & . & z & y & . \\ y & . & w & . & z & . & x & . \\ z & w & . & . & y & x & . & . \\ w & z & y & x & . & . & . & . \end{vmatrix} .$$

The general theorem is then formulated as follows: "Si en général on considère n éléments $x_1, x_2, \ldots, x_n$, en posant

$$X_n = x_1 + x_2 + \ldots + x_n\,,$$

et en désignant par

$$|X_n(1, 2, \ldots, m)|$$

le produit des facteurs linéaires qu'on deduit de X_n en changeant les signes à m des éléments $x_1, x_2, \ldots, x_n$; en aura pour n impair

$$X_n \cdot |X_n(1)| \cdot |X_n(1,2)| \;\ldots\; |X_n(1, 2, 3, \ldots, \tfrac{1}{2}(n-1)| = -\Delta,$$

et pour n pair

$$X_n \cdot |X_n(1)| \cdot |X_n(1\,,\,2)| \,.\,.\,.\,.\, |X_n(1\,,\,2\,,\,.\,.\,.\,,\,\tfrac{1}{2}(n-2)| \cdot |X_n(\bar{1},\,2,\,3,\,.\,.\,.\,,\,\tfrac{1}{2}n)| = \Delta$$

où le symbole

$$X_n(\bar{1}\,,\,2\,,\,3\,,\,.\,.\,.\,.\,,\,\tfrac{1}{2}n)$$

dénote que dans ce produit l'élément x_1 entre toujours parmi les éléments auxquels on a changé de signe. Le déterminant Δ résultera de la multiplication successive de l'équation $X_n = 0$ par l'unité, et par chacune des combinaisons deux à deux, quatre à quatre, . . . , $(n-1)$ à $(n-1)$ si n est impair: n à n si n est pair, des éléments $x_1\,,\,x_2\,,\,.\,.\,.\,,\,x_n$."

In addition, it is shown under this head that the number of equations in the set which originates the determinant is 2^{n-1}, and, a little unnecessarily, that the number of linear factors in the product is the same. It is also noted that if $x_1\,,\,x_2\,,\,.\,.\,.\,,\,x_n$ be quadratic radicals, the product of the linear factors is rational.

Following Cayley's paper still further, Brioschi similarly makes clear that one of the nine-line determinants there obtained, namely

$$\begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & 1 \\ \cdot & \cdot & 1 & \cdot & \cdot & 1 & \cdot & z^3 & \cdot \\ \cdot & \cdot & 1 & \cdot & 1 & \cdot & z^3 & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & y^3 \\ \cdot & 1 & \cdot & 1 & \cdot & \cdot & y^3 & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & x^3 \\ 1 & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & x^3 & \cdot \\ \cdot & \cdot & \cdot & z^3 & y^3 & x^3 & \cdot & \cdot & \cdot \\ 1 & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix},$$

may be viewed as originating in any one of the nine equations

$$\begin{array}{lll} x+y\;+z\;=0\,, & x+y\;+\alpha z=0\,, & x+y\;+\beta z=0\,, \\ x+\alpha y+\beta z=0\,, & x+\alpha y+z\;=0\,, & x+\beta y+z\;=0\,, \\ x+\beta y+\alpha z=0\,, & \alpha x+y\;+z\;=0\,, & \beta x+y\;+z\;=0\,, \end{array}$$

where $\alpha\,,\,\beta$ are the imaginary cube roots of unity, and could thus be shown to be equal to the product of the nine left-hand members of those equations.

Spottiswoode (1851, 1853).

[Elementary theorems relating to determinants. Second edition, rewritten and much enlarged by the author. *Crelle's Journal*, li. pp. 209–271, 328–381.]

The information given by Spottiswoode regarding axisymmetric determinants appears under a variety of headings. What little the first

edition contained (pp. 33–37) as a part of § vi. on "Inverse Systems" is placed in the second edition under "Compound Determinants" (pp. 368–372). Sylvester's mode of reaching Cayley's determinants connected with the mutual distances of points is given under "Multiplication" (pp. 250–253); and the chapter or "section" (§ iv.) on "Homogeneous Functions," which of course has to deal with quadrics, goes so far as to assign the name *determinant of a quadratic form* to any determinant possessed of axisymmetry (see pp. 328, 336).

The most interesting matter, however, is found in the last section of all, § 11, the contents of which are miscellaneous. There, on pages 376–380, the determinants

$$\begin{vmatrix} 1 & \cdot & a_1 & a_2 \\ \cdot & 1 & b_1 & b_2 \\ a_1 & b_1 & 1 & \cdot \\ a_2 & b_2 & \cdot & 1 \end{vmatrix}, \qquad \begin{vmatrix} 1 & \cdot & \cdot & a_1 & a_2 & a_3 \\ \cdot & 1 & \cdot & b_1 & b_2 & b_3 \\ \cdot & \cdot & 1 & c_1 & c_2 & c_3 \\ a_1 & b_1 & c_1 & 1 & \cdot & \cdot \\ a_2 & b_2 & c_2 & \cdot & 1 & \cdot \\ a_3 & b_3 & c_3 & \cdot & \cdot & 1 \end{vmatrix},$$

are considered, but, to one's regret, only with reference to the case where $|a_1b_2c_3|$ is an orthogonant. The first of the two determinants is given in the form

$$1 - a_1^2 - b_1^2 - a_2^2 - b_2^2 + (a_1b_2 - a_2b_1)^2;$$

and similar non-determinant forms are given for the whole of the thirty-six primary minors and for the first fifteen of the secondary minors of the second determinant. Thus, the primary minors which are the cofactors of the elements in the places (1 , 1), (1 , 3), (1 , 5) are

$$1 - b_1^2 - b_2^2 - b_3^2 - c_1^2 - c_2^2 - c_3^2 + (b_2c_3 - b_3c_2)^2 + (b_3c_1 - b_1c_3)^2 + (b_1c_2 - b_2c_1)^2,$$
$$(a_1c_1 + a_2c_2 + a_3c_3)(1 - b_1^2 - b_2^2 - b_3^2) + (b_1c_1 \div b_2c_2 + b_3c_3)(a_1b_1 + a_2b_2 + a_3b_3),$$
$$a_1b_2c_3 | \cdot | b_1c_3 | \;-\; a_1(1 - b_1^2 - b_2^2 - b_3^2 - c_1^2 - {}_2^2 - c_3^2) \;-\; b_1(a_1b_1 + a_2b_2 + a_3b_3)$$
$$-\; c_1(a_1c_1 + a_2c_2 + a_3c_3),$$

all the others being similar in form to one or other of these three; and in like manner the secondary minors are exemplified by

$$1 - c_1^2 - c_2^2 - c_3^2,$$
$$-(b_1c_1 + b_2c_2 + b_3c_3),$$
$$-b_1(1 - c_1^2 - c_2^2 - c_3^2) \;+\; c_1(b_1c_1 + b_2c_2 + b_3c_3),$$
$$a_1(b_1c_1 + b_2c_2 + b_3c_3) \;-\; b_1(c_1a_1 + c_2a_2 + c_3a_3),$$
$$-c_1 \,| a_1b_2c_3 | \;+\; | a_2b_3 | .$$

Brioschi (1854, March).

[La teorica dei determinanti, e le sue principali applicazioni; del Dr Francesco Brioschi; viii + 116 pp.; Pavia. Translation into French, by Combescure; ix + 216 pp.; Paris, 1856. Translation into German, by Schellbach; vii + 102 pp.; Berlin, 1856.]

Unlike Spottiswoode, Brioschi in methodical manner defines "un determinante simmetrico," and gives four known properties expressed in clear language, all within the space of one page (p. 70).

Brioschi (1854, Dec.).

[Sur quelques questions de la géométrie de position. *Crelle's Journal*, l. pp. 233–238: or *Opere Mat.*]

The title here recalls that of Cayley's maiden effort; and, as a matter of fact, the paper of 1854 had its origin in the paper of 1841. Cayley, it will be remembered, obtained the relation connecting the mutual distances of five points in space by multiplying the two determinants

$$\begin{vmatrix} \Sigma x_1^2 & -2x_1 & -2y_1 & -2z_1 & -2w_1 & 1 \\ \Sigma x_2^2 & -2x_2 & -2y_2 & -2z_2 & -2w_2 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \Sigma x_5^2 & -2x_5 & -2y_5 & -2z_5 & -2w_5 & 1 \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}, \quad \begin{vmatrix} 1 & x_1 & y_1 & z_1 & w_1 & \Sigma x_1^2 \\ 1 & x_2 & y_2 & z_2 & w_2 & \Sigma x_2^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & x_5 & y_5 & z_5 & w_5 & \Sigma x_5^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{vmatrix},$$

the first of which is -16 times the second, and then putting the w's equal to zero. Brioschi now follows on the same lines, but with an interesting difference. Having shown that the determinant

$$\begin{vmatrix} x_1^2+y_1^2+z_1^2 & (x_6-x_1)^2+(y_6-y_1)^2+(z_6-z_1)^2 & 1 & -2x_1 & -2y_1 & -2z_1 \\ x_2^2+y_2^2+z_2^2 & (x_6-x_2)^2+(y_6-y_2)^2+(z_6-z_2)^2 & 1 & -2x_2 & -2y_2 & -2z_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_5^2+y_5^2+z_5^2 & (x_6-x_5)^2+(y_6-y_5)^2+(z_6-z_5)^2 & 1 & -2x_5 & -2y_5 & -2z_5 \\ 1 & 1 & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

vanishes identically, the simple fact being that the second column is a sum of multiples of the other columns, he multiplies it by $\frac{1}{8}$ of itself, namely by

$$\begin{vmatrix} 1 & \Sigma(x_6-x_1)^2 & \Sigma x_1^2 & x_1 & y_1 & z_1 \\ 1 & \Sigma(x_6-x_2)^2 & \Sigma x_2^2 & x_2 & y_2 & z_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \Sigma(x_6-x_5)^2 & \Sigma x_5^2 & x_5 & y_5 & z_5 \\ \cdot & 1 & 1 & \cdot & \cdot & \cdot \end{vmatrix}$$

and, putting $d_{61}, \ldots$ for $\Sigma(x_6 - x_1)^2, \ldots$, obtains the relation

$$\begin{vmatrix} d_{61}{}^2 & d_{61}d_{62} + d_{12} & \ldots & d_{61}d_{65} + d_{15} & d_{61} + 1 \\ d_{61}d_{62} + d_{12} & d_{62}{}^2 & \ldots & d_{62}d_{65} + d_{25} & d_{62} + 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ d_{61}d_{65} + d_{15} & d_{62}d_{65} + d_{25} & \ldots & d_{65}{}^2 & d_{65} + 1 \\ d_{61} + 1 & d_{62} + 1 & \ldots & d_{65} + 1 & 1 \end{vmatrix},$$

which degenerates into Cayley's result when we put $x_6, y_6, z_6 = x_1, y_1, z_1$, and make certain easy transformations.

In a similar manner the relation between the distances of five points on an ellipsoid is found, and the relation "entre les plus courtes distances respectives et les inclinaisons mutuelle de sept lignes quelconques."

Sylvester (1855, April).

[On the change of systems of independent variables. *Quarterly Journ. of Math.*, i. pp. 42–56: or *Collected Math. Papers,* ii. pp. 65–85.]

Having reached in the course of his investigation (p. 55) a determinant of the form

$$\begin{vmatrix} a_1 + a_2 + a_3 & -b_3 & -c_2 \\ -a_2 & b_1 + b_2 + b_3 & -c_3 \\ -a_3 & -b_2 & c_1 + c_2 + c_3 \end{vmatrix},$$

the final expansion of which, he says, contains only positive terms with the coefficient unity, Sylvester naturally notes that the number of such terms must be

$$\begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}.$$

He is thus led to the consideration of the n-line axisymmetric determinant

$$\begin{vmatrix} a & -1 & -1 & \ldots & -1 \\ -1 & a & -1 & \ldots & -1 \\ -1 & -1 & a & \ldots & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -1 & -1 & -1 & \ldots & a \end{vmatrix}$$

to which he assigns the value

$$(a - n + 1)(a + 1)^{n-1}.$$

FERRERS (1855, Dec.).

[Two elementary theorems in determinants. *Quarterly Journ. of Math.*, i. p. 364: or *Nouv. Annales de Math.*, xvi. pp. 402–403, xvii. pp. 190–191.]

The first theorem referred to is

$$\begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 1+a_1 & 1 & \dots & 1 \\ 1 & 1 & 1+a_2 & \dots & 1 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 1 & 1 & 1 & \dots & 1+a_n \end{vmatrix} = a_1a_2\dots a_n\,,$$

the proof being dependent on the fact that, if any one of the a's be put equal to 0, the determinant vanishes. The second is

$$\begin{vmatrix} 1+a_1 & 1 & 1 & \dots & 1 \\ 1 & 1+a_2 & 1 & \dots & 1 \\ 1 & 1 & 1+a_3 & \dots & 1 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 1 & 1 & 1 & \dots & 1+a_n \end{vmatrix}$$

$$= a_1a_2\dots a_n\left(1+\frac{1}{a_1}+\frac{1}{a_2}+\dots+\frac{1}{a_n}\right),$$

which is made to rest mainly on the fact that if any one of the a's be put equal to 0 the determinant takes the form of the preceding determinant. Note is taken that Sylvester's theorem on p. 55 of the same volume is a special case of this second result.

FAÀ DI BRUNO (1855, Dec.).

[Addizione alla nota inserita nel fascicolo di ottobre ultimo. *Annali di sci. mat. e fis.*, vi. pp. 476–479: or § vi. of his THÉORIE GÉNÉRALE DE L'ÉLIMINATION, x + 224 pp., Paris, 1859.]

The note referred to in the title professed to be "Sulle funzioni simmetriche delle radici di un' equazione," and contained, besides other things, the final expansions of the resultants of two quadrics, two cubics, and two quartics. The "addizione," on the other hand, draws attention to the axisymmetric determinants which represent those resultants, the author being apparently unaware that Cauchy had already done this in

1840. His rule of formation, which is different from Cauchy's, may be paraphrased in the case of the quartics

$$\left.\begin{array}{l} a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0 \\ b_0x^4 + b_1x^3 + b_2x^2 + b_3x + b_4 = 0 \end{array}\right\}$$

by saying that the resultant is got by superposing on the central two-line minor of the symmetric (*persymmetric*, rather) determinant

$$\begin{Vmatrix} |a_0b_1| & |a_0b_2| & |a_0b_3| & |a_0b_4| \\ |a_0b_2| & |a_0b_3| & |a_0b_4| & |a_1b_4| \\ |a_0b_3| & |a_0b_4| & |a_1b_4| & |a_2b_4| \\ |a_0b_4| & |a_1b_4| & |a_2b_4| & |a_3b_4| \end{Vmatrix}$$

the determinant

$$\begin{Vmatrix} |a_1b_2| & |a_1b_3| \\ |a_1b_3| & |a_2b_3| \end{Vmatrix},$$

the latter being obtained from the array

$$\begin{array}{ccc} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{array}$$

in the same manner as the former is obtained from the array

$$\begin{array}{ccccc} a_0 & a_1 & a_2 & a_3 & a_4 \\ b_0 & b_1 & b_2 & b_3 & b_4 . \end{array}$$

In his "Théorie Générale de l'Élimination" the matter is gone into in greater detail, the rule of formation occupying a full page (pp. 55–56).

Cayley (1856, March).

[Note upon a result of elimination. *Philos. Magazine,* (4) xi. pp. 378–379: or *Collected Math. Papers,* iii. pp. 214–215.]

If the quadric

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy$$

have a linear factor, $\xi x + \eta y + \zeta z$ say, it must vanish identically when we make the substitution

$$x, y, z = \beta\zeta - \gamma\eta, \gamma\xi - \alpha\zeta, \alpha\eta - \beta\xi,$$

where α, β, γ are any quantities whatever; and consequently the co-factors of $\alpha^2, \beta^2, \ldots$ in the result must vanish,—that is to say, we must have

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$$\left.\begin{array}{llllllll}
 & c\eta^2 & + b\zeta^2 & -2f\eta\zeta & & & & = 0 \\
c\xi^2 & & + a\zeta^2 & & -2g\zeta\xi & & & = 0 \\
b\xi^2 & + a\eta^2 & & & & -2h\xi\eta & & = 0 \\
-2f\xi^2 & & & -2a\eta\zeta & +2h\zeta\xi & +2g\xi\eta & & = 0 \\
 & -2g\eta^2 & & +2h\eta\zeta & -2b\zeta\xi & +2f\xi\eta & & = 0 \\
 & & -2h\zeta^2 & +2g\eta\zeta & +2f\zeta\xi & -2c\xi\eta & & = 0
\end{array}\right\}$$

and therefore

$$\begin{vmatrix}
\cdot & c & b & -2f & \cdot & \cdot \\
c & \cdot & a & \cdot & -2g & \cdot \\
b & a & \cdot & \cdot & \cdot & -2h \\
-2f & \cdot & \cdot & -2a & 2h & 2g \\
\cdot & -2g & \cdot & 2h & -2b & 2f \\
\cdot & \cdot & -2h & 2g & 2f & -2c
\end{vmatrix} = 0,$$

or, say, $8\Delta = 0$.

But on account of the breaking up of the quadric into linear factors the discriminant must likewise vanish. It is thus suggested that the discriminant is a factor of Δ; and by actual trial it is found that

$$\Delta = -2\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}^2.$$

It has only in addition to be noted that Δ originally made its appearance in Sylvester's second paper on dialytic elimination.

Faà di Bruno (1857, April).

[Sopra il volume della piramide triangolare. *Annali di sci. mat. e fis.*, viii. pp. 77–78.]

With an eye on Sylvester's paper of October 1852, Faà di Bruno first expresses the volume (V) of a tetrahedron in the form

$$\frac{1}{6}\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ x_1-x_3 & y_1-y_3 & z_1-z_3 \\ x_1-x_4 & y_1-y_4 & z_1-z_4 \end{vmatrix},$$

and then by squaring obtains the result

$$288\,V^2 = \begin{vmatrix} 2d_{12}^2 & d_{12}^2+d_{13}^2-d_{23}^2 & d_{12}^2+d_{14}^2-d_{24}^2 \\ d_{12}^2+d_{13}^2-d_{23}^2 & 2d_{13}^2 & d_{13}^2+d_{14}^2-d_{34}^2 \\ d_{12}^2+d_{14}^2-d_{24}^2 & d_{13}^2+d_{14}^2-d_{34}^2 & 2d_{14}^2 \end{vmatrix}$$

as a consequence of the relation

$$2\sum(x_1-x_2)(x_1-x_3) = \sum(x_1-x_2)^2+\sum(x_1-x_3)^2-\sum(x_2-x_3)^2.$$

Another form of the relation between the mutual distances of four points in a plane is thus brought to light.

Rubini (1857).

[Applicazioni della teorica dei determinanti. *Annali di sci. mat. e fis.*, viii. pp. 179–200.]

Rubini starts with the theorem which expresses a determinant with binomial elements as a sum of determinants with monomial elements, and then considers a long series of special cases. Among these Ferrers' theorems of the year 1855 occupy the first place (§§ 2, 3, pp. 181–184).

Bellavitis (1857, June).

[Sposizione elementare della teorica dei determinanti. *Mem. . . . istituto veneto* . . . . vii. pp. 67–144.]

Besides the paragraphs (§§ 42, 43, 44) specifically devoted in the second half of the exposition to axisymmetric determinants, there are two others (§§ 9, 35) connected with the same subject in the first half. One of the latter (§ 35) draws attention to the fact that any coaxial minor of the axisymmetric determinant which is the square of a determinant is expressible as a sum of squares. What is new in the former is the definite reference to determinants which are doubly axisymmetric ("doppiamente simmetrici"), the examples given being *

$$\begin{vmatrix} a & b \\ b & a \end{vmatrix}, \quad \begin{vmatrix} a & b & c \\ b & d & b \\ c & b & a \end{vmatrix}, \quad \begin{vmatrix} a & b & c & d \\ b & a & d & c \\ c & d & a & b \\ d & c & b & a \end{vmatrix},$$

the last of which is noted as being equal to

$$-(a+b+c-d)(a+b-c+d)(a-b+c+d)(-a+b+c+d).$$

* The third, by reason of its central two-line minor which might have been $\begin{vmatrix} \alpha & \delta \\ \delta & \alpha \end{vmatrix}$, is more specialised than a doubly axisymmetric determinant.

Baltzer (1857).

[Theorie und Anwendung der Determinanten, mit Beziehung auf die Originalquellen, dargestellt von Dr Richard Baltzer: vi+129 pp., Leipzig, 1857. French translation by J. Houel, xii+235 pp., Paris, 1861.]

In five different sections (§ **3**, 8, 9; § **5**, 2; § **6**, 2, 5; § **7**, 5; § **18**, 12) Baltzer gives attention to determinants whose elements a_{ik}, a_{ki} are identical. In § 3 he notes that conjugate minors, as they came to be called at a later date, are equal, and proves Jacobi's theorem regarding the differential-quotient of a determinant with respect to a non-diagonal element by using the fact that if u be a function of x and y, and y be a function of x, then

$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dx},$$

the whole matter being

$$\frac{\partial \Delta}{\partial a_{1k}} = \mathrm{A}_{ik} + \mathrm{A}_{ki}\frac{\partial a_{ki}}{\partial a_{ik}} = \mathrm{A}_{ik} + \mathrm{A}_{ki} = 2\mathrm{A}_{ik},$$

where it will be observed ∂'s only are employed. In § **7**, 5 is given the result which we have already seen in Lebesgue's paper of 1837, namely, that for a vanishing axisymmetric determinant

$$\mathrm{A}_{ik} = \sqrt{\mathrm{A}_{ii}\mathrm{A}_{kk}};$$

and there is thence deduced

$$\mathrm{A}_{i1} : \mathrm{A}_{i2} : \mathrm{A}_{i3} : \ldots\ldots = \sqrt{\mathrm{A}_{11}} : \sqrt{\mathrm{A}_{22}} : \sqrt{\mathrm{A}_{33}} : \ldots\ldots$$

Lastly, in § **18**, 12 he applies this to Cayley's vanishing determinants of the year 1841; for example, to the determinant

$$\begin{vmatrix} . & 1 & 1 & 1 & 1 \\ 1 & . & d_{12} & d_{13} & d_{14} \\ 1 & d_{12} & . & d_{23} & d_{24} \\ 1 & d_{13} & d_{23} & . & d_{34} \\ 1 & d_{14} & d_{24} & d_{34} & . \end{vmatrix}.$$

This being equal to zero, if we write $[r, s]$ for the cofactor of the element in the place (r, s), we have of course

$$[12] + [13] + [14] + [15] = 0,$$

and consequently also

$$\sqrt{\overline{22}} + \sqrt{\overline{33}} + \sqrt{\overline{44}} + \sqrt{\overline{55}} = 0,$$

—a result hitherto only obtained independently from geometry.*

Cayley (1859, June).

[Note on the value of certain determinants, the terms of which are the squared distances of points in a plane or in space. *Quart. Journ. of Math.*, iii. pp. 275–277: or *Collected Math. Papers*, iv. pp. 460–462.]

The determinants referred to are those occurring in his first paper of the year 1841; but the expansions of them which are given do not assume that $\overline{12} = \overline{21}$, etc. Sylvester's related paper of March 1853 is also referred to, the 2W of which is put in the formed

$$\begin{vmatrix} . & c & b & f & 1 \\ c & . & a & g & 1 \\ b & a & . & h & 1 \\ f & g & h & . & 1 \\ 1 & 1 & 1 & 1 & . \end{vmatrix},$$

with the result that Q takes the form

$$\begin{aligned} & a^2b^2c^2\{f^4 + g^4 + h^4 + g^2h^2 + h^2f^2 + f^2g^2 + b^2c^2 + c^2a^2 + a^2b^2 - (f^2 + g^2 + h^2)(a^2 + b^2 + c^2)\} \\ & + a^2g^2h^2\{f^4 + b^4 + c^4 + b^2c^2 + c^2f^2 + f^2b^2 + g^2h^2 + h^2a^2 + a^2g^2 - (f^2 + b^2 + c^2)(a^2 + g^2 + h^2)\} \\ & + b^2h^2f^2\{g^4 + c^4 + a^4 + c^2a^2 + a^2g^2 + g^2c^2 + h^2f^2 + f^2b^2 + b^2h^2 - (g^2 + c^2 + a^2)(b^2 + h^2 + f^2)\} \\ & + c^2f^2g^2\{h^4 + a^4 + b^4 + a^2b^2 + b^2h^2 + h^2a^2 + f^2g^2 + g^2c^2 + c^2f^2 - (h^2 + a^2 + b^2)(c^2 + f^2 + g^2)\}. \end{aligned}$$

* Baltzer gives (p. 20) Cayley's determinant form for

$$-(\sqrt{a}+\sqrt{b}+\sqrt{c})(-\sqrt{a}+\sqrt{b}+\sqrt{c})(\sqrt{a}-\sqrt{b}+\sqrt{c})(\sqrt{a}+\sqrt{b}-\sqrt{c}),$$

placing in front of it what looks like a generalisation, namely

$$\begin{vmatrix} . & a_1 & b_1 & c_1 \\ a_1 & . & c_2 & b_2 \\ b_1 & c_2 & . & a_2 \\ c_1 & b_2 & a_2 & . \end{vmatrix},$$

but is not really such. We can easily show that if a_1, b_1, c_1 be multiplied and a_2, b_2, c_2 be divided by x, y, z respectively, the determinant is unaltered; consequently it

$$= \begin{vmatrix} . & a_1a_2 & b_1b_2 & c_1c_2 \\ a_1a_2 & . & 1 & 1 \\ b_1b_2 & 1 & . & 1 \\ c_1c_2 & 1 & 1 & . \end{vmatrix} = \begin{vmatrix} . & 1 & 1 & 1 \\ 1 & . & c_1c_2 & b_1b_2 \\ 1 & c_1c_2 & . & a_1a_2 \\ 1 & b_1b_2 & a_1a_2 & . \end{vmatrix} = \begin{vmatrix} . & \sqrt{a_1a_2} & \sqrt{b_1b_2} & \sqrt{c_1c_2} \\ \sqrt{a_1a_2} & . & \sqrt{c_1c_2} & \sqrt{b_1b_2} \\ \sqrt{b_1b_2} & \sqrt{c_1c_2} & . & \sqrt{a_1a_2} \\ \sqrt{c_1c_2} & \sqrt{b_1b_2} & \sqrt{a_1a_2} & . \end{vmatrix} = \ldots$$

SALMON (1859, July).

[On the relation which connects the mutual distances of five points in space. *Quart. Journ. of Math.*, iii. pp. 282–288.]

Salmon starts from the elimination of a, b, c from the set of trigonometrical equations

$$\left.\begin{array}{rcl} a &=& b\cos C + c\cos B \\ b &=& c\cos A + a\cos C \\ c &=& a\cos B + b\cos A \end{array}\right\}$$

and proceeding to similar eliminants of higher order reaches Cayley's results of 1841 and others related to them. The interest of the paper is mainly geometrical. Use is made of the proposition that *if the quadric*

$$\begin{array}{l} ax^2 + by^2 + cz^2 + dw^2 \\ \qquad + 2lyz + 2mzx + 2nxy + 2pxw + 2qyw + 2rzw \end{array}$$

be increased by $(ax + \beta y + \gamma z + \delta w)^2$, *the discriminant*

$$\begin{vmatrix} a & n & m & p \\ n & b & l & q \\ m & l & c & r \\ p & q & r & d \end{vmatrix}$$

is increased by *

$$-\begin{vmatrix} \cdot & \alpha & \beta & \gamma & \delta \\ \alpha & a & n & m & p \\ \beta & n & b & l & q \\ \gamma & m & l & c & r \\ \delta & p & q & r & d \end{vmatrix}$$

* The new discriminant being

$$\begin{vmatrix} a+\alpha^2 & n+\alpha\beta & m+\alpha\gamma & p+\alpha\delta \\ n+\alpha\beta & b+\beta^2 & l+\beta\gamma & q+\beta\delta \\ m+\alpha\gamma & l+\beta\gamma & c+\gamma^2 & r+\gamma\delta \\ p+\alpha\delta & q+\beta\delta & r+\gamma\delta & d+\delta^2 \end{vmatrix}$$

is easily seen to be equal to

$$-\begin{vmatrix} -1 & \alpha & \beta & \gamma & \delta \\ \alpha & a & n & m & p \\ \beta & n & b & l & q \\ \gamma & m & l & c & r \\ \delta & p & q & r & d \end{vmatrix},$$

and therefore to be separable into the two expressions referred to.

LIST OF AUTHORS

whose writings are herein dealt with.

(*Issued separately July* 19, 1907.)

XVII.—The Composition of the Red Clay. By F. W. Clarke, D.Sc., LL.D., *Chief Chemist, U.S. Geological Survey. Communicated by* Sir John Murray, K.C.B., F.R.S., etc.

(MS. received April 3, 1907. Read May 20, 1907.)

In the volume upon *Deep-Sea Deposits*, issued as one of the reports of the *Challenger* Expedition, there are published twenty-five analyses of the "red clay." * This sediment is now recognised as the most extensive and important of all the oceanic deposits, for it covers 51,500,000 square miles of the sea-floor, and is characteristic of the greatest depths. It is therefore obviously desirable to know its average composition as exactly as possible, and for that reason the following investigation was undertaken.

Of the analyses above mentioned, twenty-one were by Brazier, two by Hornung, one by Klement, and one by Renard. They are, however, not strictly comparable, as a glance at the recorded data will show; nor are they, from the point of view of the modern analyst, so complete as they should be. For example, that ubiquitous element, titanium, was not determined in any of the analyses, for at the time they were made its importance and relative abundance were not appreciated. Other substances which are common in clays were neglected for similar reasons; but their significance is now better understood, and improvements in analytical methods have made it easier to search for them. In Brazier's analyses the alkalies were not estimated, but were reported by the other analysts—an omission in the first group that was not due to oversight, but to the limitations of the purposes for which the work was done. The general nature of the red clay was well established, its great variability in composition was clearly shown, and its relations to other clays were made sufficiently plain to satisfy all ordinary requirements.

Of late years, however, it has become a matter of interest to determine the relative abundance and distribution of the chemical elements; and in an inquiry of that sort so notable a substance as the red clay could not well be neglected. A new and more elaborate analysis of it, therefore, seemed to be required, and to that end two methods of investigation were available. First, it was possible to make a number of individual analyses

* *Deep-Sea Deposits*, pp. 198, 201, and 425–435.

of separate samples, from which an average might be computed. Secondly, one composite sample could be analysed, giving the desired information once for all. The first method, evidently, would have involved much labour —too much, indeed, to be justifiable. The second method would solve the problem equally well, but with greater ease and vastly less expenditure of time. The second, therefore, was chosen. Through the kindness of Sir John Murray, and his assistants, Dr G. W. Lee and Mr James Chumley, fifty-one samples of the red clay, from as many localities, and approximately equal in weight, were combined into a single sample, and that was analysed by my associates in the laboratory of the United States Geological Survey.* The results of the composite analysis will be given presently.

The composite sample, as made up by Sir John Murray, contained thirty-five samples from the dredgings of the *Challenger* Expedition, twelve collected by the *Egeria*, two by the *Waterwitch*, and two by the *Penguin*. Of these, eight samples were collected in the Atlantic, two in the Indian Ocean, and forty-one in the Pacific. The *Challenger* localities were stations Nos. 5, 9, 26, 27, 29, 160, 165, 181, 215, 221, 226, 228, 229, 230, 238, 240, 241, 244, 247, 251, 253, 254, 255, 256, 258, 259, 275, 277, 285, 286, 288, 294, 329, 330, and 353. These stations can be identified by reference to the published reports of the expedition.† The geographic range of the collection is evidently large enough to give a significant average, and the number of individual samples was also adequate. Twelve of the localities enumerated above are represented among the analyses already published in the volume on *Deep-Sea Deposits*, and are there indicated by their station numbers. The other localities furnished material hitherto unstudied chemically.

The new analysis of the clay was made upon the air-dried and unwashed sample. It therefore included adherent sea-salts and hygroscopic moisture, varying in these respects from the earlier analyses. The magnitude of these variations, however, was determined, and appears in the figures given below. The final data are as follows:

A. General analysis, by Mr George Steiger.
B. Portion soluble in cold water, Steiger.
C. Special determinations, by Dr W. F. Hillebrand.
D. Special determinations, on material concentrated from 150 grammes of clay, by Dr E. C. Sullivan.

* The analytical methods employed were those prescribed by Hillebrand in *U.S.G.S. Bulletin* 305.

† A chart showing the position at which each sample was taken was also furnished with the material sent for analysis.

These last determinations were checked by blank experiments upon equal quantities of the reagents employed in the research.

	A.	B.		C.		D.
SiO_2	45·32	none	BaO	·16	CuO	·02
Al_2O_3	13·26	none	Cr_2O_3	·011	PbO	·007
Fe_2O_3	7·20	none	V_2O_3	·035	ZnO	·004
FeO	·70	none	MoO_3	trace	As_2O_3	·0007
MgO	3·05	·21				
CaO	6·82	·19				
Na_2O	3·63	2·01				
K_2O	2·43					
H_2O at 106°	3·28	...				
H_2O above 106°	5·93	...				
TiO_2	·82	...				
CO_2	3·91	...				
P_2O_5	·25	...				
SO_3	·48	·39				
Cl	2·77	2·73				
F	none	...				
Cr_2O_3	·01	...				
NiO, CoO	·032	...				
MnO_2	1·01	...				
BaO	·17	...				
SrO	·046	...				
Li_2O	none	...				
V_2O_3	·023	...				
	101·141	5·53				
Less O = Cl	·62	·62				
	100·521	4·91				

Zirconia and the rare earths were sought for by Dr Hillebrand, but not found. Titanium, chromium, vanadium, barium, strontium, nickel, copper, lead, zinc, arsenic, and molybdenum were not reported in the *Challenger* analyses. Their widespread distribution in the igneous rocks is, however, well recognised. The absence of lithium and fluorine is noteworthy. The unusually high proportion of manganese suggests the presence of disseminated or incipient manganese nodules, like that so admirably analysed by Gibson.* In his work several of the rarer elements were determined; so that their existence in the oceanic sediments is no new discovery. The fact of their general distribution, however, remained to be proved.

In order to establish the true composition of the clay substance, some deductions must be made from the analysis as it is now stated. Hygroscopic water and soluble matter must be eliminated, and also the calcium carbonate which is represented by the CO_2. The composition of the

* *Deep-Sea Deposits*, pp. 422, 423.

aqueous extract, as given by Mr Steiger's figures, is very near that of the sea-salts. I have therefore taken Dittmar's analysis of sea-salts as a standard, and subtracted their equivalent, as measured by the chlorine in the unleached clay, from the analysis of the latter. A slight excess of SO_3 remained, which I have also thrown out as representing gypsum. In short, from the general analysis I have withdrawn the water lost below 106°, the sea-salts, the calcium carbonate, and a little gypsum, and recalculated the remainder to 100 per cent.

For comparison, I have combined the twenty-five analyses of the *Challenger* report into an average, from which similar subtractions, so far as they were needed, have been made. In this case only gypsum and calcium carbonate were rejectable. As for the combination, its value is not very great, because of the inequality shown by the individual analyses. Only in four of the latter were alkalies determined, and the mean of those four I have assumed to be representative of all. The ferrous oxide is even more doubtful, for it was only determined in one of Hornung's analyses, and neglected in the others. Still, as will be seen in the subjoined table, the comparison is not without significance, and even if it is not perfect, it is better than none at all.

REDUCED ANALYSES.

	Composite Analysis.	*Challenger* Average.
SiO_2	54·48	54·28
TiO_2	·98	...
Al_2O_3	15·94	16·41
Cr_2O_3	·012	...
Fe_2O_3	8·66	13·58
FeO	·84	1·26
NiO, CoO	·039	...
MnO_2	1·21	1·62
MgO	3·31	1·76
CaO	1·96	·74
SrO	·056	...
BaO	·20	...
K_2O	2·85	1·61
Na_2O	2·05	1·37
V_2O_3	·035	...
As_2O_3	·001	...
MoO_3	trace	...
P_2O_5	·30	·35
CuO	·024	...
PbO	·008	...
ZnO	·005	...
H_2O	7·04	7·02
	100·000	100·00

The minor elements, not shown in the *Challenger* analyses, only sum up altogether to 1·36 per cent. Apart from these, the comparison is satisfactory in some respects, not so in others. In silica, alumina, and water the agreement is fairly good, but in iron the old analyses range much higher than the new. Possibly for the individual analyses the reddest, and therefore the most ferruginous, samples were chosen for examination, as being presumably the most typical.

On the proximate or mineralogical composition of the red clay I have no important suggestions to offer. That problem was discussed at some length in the *Challenger* report, on the basis of Brazier's analyses. In those analyses there was discrimination between the portion of the clay soluble in strong hydrochloric acid, and the portion insoluble. In nearly every case the soluble ferric oxide was reported in excess of the insoluble; from which I am inclined to suspect that the iron of the clay is present, at least partially, in limonitic or glauconitic combination. The new composite analysis shows a quantity of potash which suggests the presence of glauconite. The latter compound may well exist in a diffused form, quite unlike its common granular variety, and therefore not so readily recognised. This is a mere suspicion, not entitled to much weight at present, but worth considering in future investigations. My specific problem has been to study the distribution of the chemical elements in nature, and to that end the composite analysis of the red clay is a step forward.

(*Issued separately July* 19, 1907.)

XVIII.—The Physical Properties of Mixed Solutions of Independent Optically-Active Substances. By **Clerk Ranken**, B.Sc., Carnegie Research Scholar, and **W. W. Taylor**, M.A., D.Sc. *Communicated by* Professor Crum Brown, F.R.S.

(MS. received May 6, 1907. Read same date.)

The two systems containing one of a pair of optically-active stereoisomers and an independent optically-active substance present many points of interest and importance, but have not hitherto been investigated with any degree of completeness. What is known about them may fairly be said to consist of a series of isolated facts. The present communication also contributes a few more isolated observations, and is, in reality, a preliminary to a more systematic examination of the whole subject.

So far as has been ascertained up to the present time, there seems to be some justification for the notion that there is an essential distinction between those systems in which the independent optically-active substance is capable of entering into definite chemical combinations with each of the enantiomorphs, and those systems in which chemical combination is either absent or doubtful.

The method of resolution of racemic acid into its optically-active constituents by means of an alkaloid, one of the methods first made use of by Pasteur, depends upon the difference in solubility, in water or some other solvent, of the salt formed by the alkaloid with the *dextro*-acid, and of the salt formed by the *laevo*-acid with that alkaloid. There are now known innumerable instances of a similar kind.

Further, Marckwald and M'Kenzie * found that the rate of esterification of *l*-mandelic acid and *l*-menthol is slower than that of *d*-mandelic acid and *l*-menthol. Similarly, the rate of saponification of *l*-menthyl *d*-mandelate by solutions of alkalies in ethyl alcohol is greater than that of the *l*-menthyl *l*-mandelate.† M'Kenzie and Thompson ‡ have just shown that the same holds with respect to the corresponding *l*-bornyl mandelates. In these examples there is no doubt about the chemical combination between the two optically-active substances.

On the other hand, van't Hoff § suggested the possibility of optical

* *Berl. Ber.*, 34, p. 469, 1901.
† M'Kenzie, *Chem. Soc. Jour.*, 85, p. 1249, 1904.
‡ *Proc. Chem. Soc.*, 23, p. 113, April 1907.
§ *Lagerung der Atome im Raume*, p. 30, 1894.

antipodes possessing different solubilities in optically-active media; no such difference has yet been observed. Tolloczko * shook up aqueous solution of racemic acid with active amyl alcohol, which is almost insoluble in water; he observed no selective extraction of the one or other stereoisomer. Goldschmidt and Cooper † determined the solubility of *d*- and of *l*-carvoxim in active limonen at various temperatures, and found identity of solubility in the two cases. Cooper ‡ further determined the solubility of the *l*-sodium hydrogen *d*- and *l*-tartrates in *d*-glucose solutions and found no differences in solubility. Quite recently H. O. Jones § found that the solubilities of *d*- and of *l*-camphor, and of *d*- and *l*-camphoroximes in purified turpentine (*d*-pinene) and in *l*-amyl bromide, were identical.

Then, again, Emil Fischer ‖ was unable to detect any difference in the rate of inversion of cane sugar by *d*- and *l*-camphoric acids. Caldwell ¶ very carefully measured the rates of inversion of cane sugar by the β-sulphonic acids of *d*- and *l*-camphor, and found them to be the same. Unpublished measurements, made by one of us in conjunction with Mr T. Rettie, on the inversion of cane sugar by the tartaric acids lead to the same conclusion.

In these instances it will be generally conceded that there is no definite chemical combination between the two optically-active substances; if it does exist, it is almost certainly not of the same character as that obtaining in the systems of the other class.

There still remain two cases of special interest, as they do not belong with certainty to either of the two classes discussed above. Pasteur ** found that ammonium hydrogen *d*-tartrate formed a well-defined crystalline substance with ammonium hydrogen *l*-malate, whereas ammonium hydrogen *l*-tartrate forms no such crystalline substance with ammonium hydrogen *l*-malate. Other similar differences are also known in the case of derivatives of tartaric acid and malic acid. Whether this crystalline substance can be regarded as a chemical compound is doubtful, and it has been recently stated by Bruni †† not to be a double salt but an isomorphous mixture.

* *Zeit. Phys. Chem.*, 20, p. 412, 1896.

† *Zeit. Phys. Chem.*, 26, p. 711, 1898.

‡ *Amer. Chem. Jour.*, 23, p. 253, 1900.

§ *Proc. Cam. Phil. Soc.*, 14, p. 27, 1907.

‖ *Zeit. Physiol. Chem.*, 26, p. 83, 1898.

¶ *Proc. Roy. Soc.*, 74, p. 184, 1904.

** *Ann. Chem. Phys.* (3), 38, p. 461 1853.

†† See Meyerhoffer, *Gleichgewichte der Stereomeren*, 1906, p. 62.

Finally, Kipping and Pope * observed that the addition of *d*-glucose to a solution of sodium chlorate always caused a preponderance of dextro-rotatory crystals of sodium chlorate to separate out on spontaneous crystallisation; and later, they extended their experiments to solutions of sodium ammonium racemate.†

In this case also they found that a solution of sodium ammonium racemate to which *d*-glucose had been added, if allowed to crystallise spontaneously below 27° C. (*i.e.* at a temperature at which the racemate is the unstable form and the tartrates the stable form), always deposited crystals of *d*-tartrate in greater quantity than of *l*-tartrate; in some of the experiments almost pure *d*-tartrate was obtained.

It is difficult to see in what way the sugar brings about this result. In view of the results of the researches mentioned above on the solubility of optically-active stereoisomers in optically-active liquids, investigations which were subsequent to and, in all probability, consequent upon these crystallisation experiments of Kipping and Pope, the glucose cannot be considered to diminish the solubility of the dextro-salt, nor yet to form a more soluble complex with the laevo-salt. If the aqueous solution of sugar be looked upon as an optically-active solvent for the salt, it is not easy to see why an optically-active liquid, which is at the same time a pure substance, does not cause an inactive $d+l$ mixture to deposit on crystallisation an excess of the one or the other isomer; nevertheless H. O. Jones ‡ was unable to obtain any trace of separations of inactive camphoroxim, or of inactive mandelic acid by crystallisation of its solutions of them in *d*-pinene.

Whatever the explanation may be, the interesting fact remains that the glucose influenced the two optically-active isomers in a different manner or to a different extent.

The experiments described in this paper were directed to the mixed solutions of the two independent optically-active substances rather than to the heterogeneous systems-solid and solution, certain physical properties of the solutions being measured in order to find out if there were any recognisable differences in the two sets of solutions.

* *Chem. Soc. Jour.*, 73, p. 606, 1898.
† *Proc. Chem. Soc.*, 1898, p. 113.
‡ *Loc. cit.*

Experimental.

The properties of the solutions which were chosen for measurement were electrical conductivity, viscosity, and, as incidental to the viscosity determinations, density.

The viscosity measurements were made with the apparatus described in detail in a recently published paper * on the Viscosity of Solutions. It may be mentioned here that what is experimentally determined is the ratio, η/η_0, of the viscosity of the solution to the viscosity of the solvent at the temperature of experiment. This ratio may be taken to be accurate to less than 1 in 1000.

The density determinations were made in the usual manner by means of an Ostwald-Sprengel pycnometer; a high degree of accuracy was not aimed at in the density determinations.

The electric conductivity of the solutions was also measured in the usual alternating-current method with telephone and Wheatstone bridge.

Every measurement given in the tables is the mean of several concordant determinations, made, as a rule, on duplicate solutions. In some cases the duplicate solutions were made up from different samples of the substances.

Two pairs of optically-active stereoisomers were used, the normal potassium tartrates and the tartar emetics; for the sake of comparison, potassium racemate and racemic emetic were also included.

The independent optically-active substance was cane sugar, but in one additional set of experiments maltose was employed; the reason for selecting maltose will be given later on.

Water was the solvent in all cases.

The *l*-tartaric acid was made from racemic acid by resolution with cinchonin. For the potassium tartrate solutions the three acids were converted into the potassium hydrogen salts, which were then repeatedly recrystallised. Weighed quantities of the dry salt were exactly neutralised with pure potassium hydroxide, and the solutions then were made up to the requisite volume.

The *d*-tartar emetic was a specimen of commercial pure emetic which was recrystallised several times.

The *l*-emetic and *r*-emetic were made from the corresponding potassium hydrogen salts by boiling up with pure antimonious oxide and much water for two to three hours; the liquid was then filtered and allowed to crystallise. The product was then recrystallised several times. Owing to

* *Trans. Roy. Soc. Edin.*, 45, p. 397, 1906.

their sparing solubility and well-defined crystalline character, the emetics were easily obtained in a very pure state.

The solutions were always tested polarimetrically, and in every case the rotations of the equivalent *dextro-* and *laevo*-solutions were found to be the same in amount; the racemic solutions were tested and were inactive. As all that was wanted was a comparison of the effect produced by cane sugar on equivalent solutions of the *d-* and *l*-stereoisomers, the method adopted was as follows:—Equivalent solutions of the three, *d-*, *l-*, and *r-*, substances were made; their density, viscosity, and electric conductivity were determined at 15° C. and 25° C. A definite weight (1 gram) of cane sugar was then added to a definite volume (35 c.c.) of each solution, and the density, viscosity, and electric conductivity of the resulting solutions again determined.

The results are given in full in the following tables. As already explained, it is the relative viscosity (η/η_0) which is actually determined; the values of η are obtained from this ratio by multiplying it by η_0, the viscosity of water at that temperature expressed in absolute units. The values of η_0 are taken from Thorpe and Rodger's table,* and are as follows, η_0 at 15° C. = 0·011335, and at 25° C. 0·00891.

The electric conductivities are expressed in Kohlrausch units.

Owing to their small solubility, the tartar emetics could not be investigated at a higher concentration than that given in the table, and nothing, apparently, was to be gained by working with more dilute solutions.

Viscosity.

Table I.—Potassium Tartrates and Cane Sugar.

Temp. 15° C.

Mols. per litre.	Optical Activity.	Solution alone.		Solution + Sugar.		Difference in η.
		η/η_0	η	η/η_0	η	
·5	*d*	1·1890	·013477	1·2935	·014661	·001184
	l	1·1886	·013473	1·2916	·014640	·001167
	r	1·1870	·013455	1·2902	·014624	·001169
·25	*d*	1·0870	·012321	1·1740	·013307	·000986
	l	1·0865	·012316	1·1732	·013298	·000982
	r	1·0865	·012304	1·1730	·013296	·000992
·125	*d*	1·0410	·011800	1·1255	·012758	·000958
	l	1·0405	·011794	1·1245	·012746	·000952
	r	1·0402	·011792	1·1245	·012746	·000954

* *Phil. Trans.*, 185, p. 449, 1894.

Table II.—Potassium Tartrates and Cane Sugar.

Temp. 25° C.

Mols. per litre.	Optical Activity.	Solution alone. η/η_0	Solution alone. η	Solution + Sugar. η/η_0	Solution + Sugar. η	Difference in η.
·5	*d*	1·2045	·010732	1·3055	·011632	·000900
	l	1·2042	·010729	1·3052	·011630	·000901
	r	1·2025	·010715	1·3028	·011608	·000893
·25	*d*	1·0945	·009752	1·1800	·010514	·000762
	l	1·0955	·009761	1·1815	·010527	·000766
	r	1·0940	·009748	1·1790	·010505	·000757
·125	*d*	1·0470	·009329	1·1245	·010019	·000690
	l	1·0462	·009322	1·1240	·010015	·000693
	r	1·0460	·009320	1·1240	·010015	·000695

Table III.—Potassium Tartrates and Maltose.

Temp. 15° C.

·5	*d*	1·1890	·013477	1·2910	·014634	·001157
	l	1·1886	·013472	1·2915	·014639	·001167
	r	1·1870	·013455	1·2900	·014622	·001167

Table IV.—Tartar Emetics and Cane Sugar.

Temp. 15° C.

·08	*d*	1·0245	·011613	1·1055	·012531	·000918
	l	1·0242	·011609	1·1050	·012525	·000916
	r	1·0232	·011598	1·1042	·012516	·000918

Table V.—Tartar Emetics and Cane Sugar.

Temp. 25° C.

·08	*d*	1·0260	·009142	1·1042	·009838	·000696
	l	1·0255	·009137	1·1045	·009841	·000704
	r	1·0250	·009133	1·1037	·009835	·000702

Density and Specific Conductivity.

Table VI.—Potassium Tartrates and Cane Sugar.

Temp. 15° C.

Mols. per litre.	Optical Activity.	Solution alone.		Solution + Sugar.	
		d	κ	d	κ
·5	d	1·0719	·0549	1·0812	·05085
	l	1·0718	·0550	1·0812	·05095
	r	1·0718	·0551	1·0811	·05105
·25	d	1·0362	·03165	1·0463	·02935
	l	1·0361	·03165	1·0462	·02925
	r	1 0361	·03165	1·0463	·02935
·125	d	1·0177	·01745	1·0283	01615
	l	1·0178	·01745	1·0282	·01610
	r	1·0178	·01745	1·0283	·01615

Table VII.—Potassium Tartrates and Cane Sugar.

Temp. 25° C.

·5	d	1·0684	·06825	1·0778	·06330
	l	1·0685	·06835	1·0779	·06330
	r	1·0683	·06835	1·0778	·06315
·25	d	1·0333	·03925	1·0434	·03660
	l	1·0334	·03925	1·0434	·03665
	r	1·0333	·03925	1·0433	·03665
·125	d	1·0154	·02185	1·0257	·02005
	l	1·0154	·02185	1·0257	·02005
	r	1·0153	·02185	1·0258	·02005

Table VIII.—Potassium Tartrates and Maltose.

Temp. 15° C.

·5	d	1·0719	·0549	1·0809	·05105
	l	1·0718	·0550	1·0809	·05105
	r	1·0718	·0551	1 0809	·05105

Table IX.—Tartar Emetics and Cane Sugar.

Temp. 15° C.

Mols. per litre.	Optical Activity.	Solution alone.		Solution + Sugar.	
		d	κ	d	κ
·08	d	1·0157	·00487	1·0261	·004665
	l	1·0157	·00487	1·0261	·004665
	r	1·0156	·00487	1·0261	·004665

Table X.—Tartar Emetics and Cane Sugar.

Temp. 25° C.

·08	d	1·0135	·00626	1·0239	·005900
	l	1·0135	·00623	1·0239	·005900
	r	1·0134	·00628	1·0238	·005900

In the first place, it will be noticed that addition of cane sugar has precisely the same effect on the d-, l-, and r-solutions, so far as viscosity, density, and specific electric conductivity are concerned. In only one case does the difference of viscosity caused by the sugar exceed 5 units in the fifth place; and in that one instance it may be concluded that there is a slight error, for at 25° C. the same solutions agree exactly.

In one series of experiments maltose was used in place of cane sugar. The reason for this was that maltose on hydrolysis gives rise to glucose only, whereas cane sugar on hydrolysis gives glucose and fructose; and it seemed not impossible that, owing to this difference, the two sugars might have different effects upon the d-salts and the l-salts.

The maltose results are not strictly comparable with the others, because the water of crystallisation of the maltose was not taken into account, 1 gram of crystallised maltose being dissolved in 35 c.c. of the solutions as usual.

The maltose did not appear to have any different effect on the d-tartrate and l-tartrate solutions.

There is, however, an unmistakable difference throughout between the optically-active solutions and the racemic solutions. This difference is greater in the more concentrated solutions, and is, no doubt, an indication of the existence of racemic ions or molecules in the solutions. This conclusion is supported by the fact that in every case the viscosity of the racemic solution is smaller than that of the corresponding dextro- or laevo-solutions.

It is curious that the viscosity of the solutions show this difference so much more clearly than the density or the electric conductivity. Hitherto the differences observed between corresponding solutions of racemic and dextro- or laevo-compounds have been very small, and necessitated very accurate work.

For example, Perkin * found that an 8·333 per cent. solution of tartaric acid had the density at 15°/15° C. of 1·03703, whilst the density of racemic acid of the same concentration was 1·03712. Again, Marchlewski † found that 14·018 per cent. solutions of tartaric acid and of racemic acid had the densities at 15°/4° of 1·06600 and 1·06623 respectively. In dilute solutions these small differences entirely disappear. Ostwald found that the electric conductivities of tartaric acid and of racemic acid were the same, but the solutions were all dilute. Other examples are given in Meyerhoffer's *Gleichgewichte der Stereomeren*, p. 20.

In none of these instances are the differences so pronounced as we have found to be the case with the viscosities. It is somewhat surprising that there does not appear, in this instance, to be any close connection between viscosity and electric conductivity. During the course of the investigation it was noticed that the viscosity was affected by small traces of impurities in the substances, to a much greater extent than were the density or electric conductivity.

In conclusion, as the addition of an independent optically-active substance to solutions containing optically-active stereoisomers has not caused any recognisable differences, it would be of special interest to know if, in the other class, in which chemical combination undoubtedly occurs between the two optically-active substances, there are recognisable differences in the properties of the two compounds. Nothing appears to be known with certainty regarding these systems beyond the fact that, in general, the two compounds have different solubilities, and in one or two cases that the two compounds can form a partially-racemic compound.

In continuing this investigation we intend to include an examination of solutions of typical substances comprised in these systems.

We desire to express our thanks to the Carnegie Trust for a grant towards the expenses of this research.

* *Chem. Soc. Jour.*, 51, p. 362, 1887.
† *Berl. Ber.*, 25, p. 1561, 1892.

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University of Edinburgh.

(*Issued separately July* 19, 1907.)

XIX.—On Vibrating Systems which are not subject to the Boltzmann-Maxwell Law. (Second Paper.) By Dr W. Peddie.

(Read January 7, 1907. MS. received March 7, 1907.)

1. In the first part of this paper (*Proc. Roy. Soc. Edin.*, vol. xxvi., 1905–6, p. 130), a method was suggested by means of which it might be found possible to discriminate amongst certain oscillatory dynamical systems according as they did, or did not, follow the law of equipartition of energy in their respective vibratory freedoms. The fundamental assumption, implicitly made in the proposed process, was that *the mean value* of the kinetic energy of the system could be expressed as a sum of mean squares of the time-rates of circular functions of the time; which functions were *not* "normal" co-ordinates, for their products in the expression for the kinetic energy were supposed only to vanish *on the average* over sufficient time. The results obtained are therefore subject to such limitations as that postulate may be found to impose. It is of importance to settle the matter decisively; for the method, if valid, gives a test which is independent of any question as to difference of the time-average for a single system and the number-average for many similar systems, and it is also applicable to dissipative systems, provided that the dissipation function is quadratic.

The investigation is continued in the present paper, and the result confirms the previous general conclusion as to the existence of systems which do not exhibit equipartition of energy amongst their freedoms.

2. A distinction was drawn in the previous paper between cases in which the third law of motion held and those in which it did not hold. It was pointed out in § 4 that equipartition did not hold in a biperiodic system exempt from the third law, and this conclusion is quite general. It applies equally to a system possessing any number of freedoms, for the masses can always be chosen so that their ratio is either greater than the greatest, or less than the least, of the ratios b'/a', etc., as in § 6.

It is important to remark that this absence of equality between action and reaction merely necessitates the existence of a suitably constituted medium through which that action is propagated with finite speed.

3. Dr Paul Ehrenfest has kindly communicated to me an elegant proof showing that, in the case of *normal* co-ordinates, the expression (see § 6 of the former paper)

$$\sum_1^n{}^r (m_p a'^2_{pr} - m_q a'^2_{qr}) n_r^2 \mathrm{A}_r^2$$

can never have all its terms of like sign. Therefore, although the case of normal co-ordinates was excluded from consideration, it is necessary to inquire whether, in any of the vibratory systems under consideration, the fundamental assumption referred to in § 1 above may be impossible of fulfilment. Should this be so, it will not follow that equipartition of energy holds in such systems, but it will follow that the test applied becomes indiscriminative.

4. The equations of motion are of the type

$$\ddot{\xi}_p = a_{p1}\xi_1 + a_{p2}\xi_2 + \cdots + a_{pp}\xi_p + \cdots + a_{pn}\xi_n ,$$

and the solution is of the type

$$\xi_p = \sum_1^m {}^r a'_{pr} \mathrm{A}_r \sin(n_r t + \alpha_r) ,$$

where $-n_r^2 = a_{11} + \lambda_{2r}a_{21} + \lambda_{3r}a_{31} + \ldots + \lambda_{mr}a_{m1}$, and the λs are subject to the $m-1$ relations

$$\lambda_{q'q}(a_{11} + \lambda_{2q}a_{21} + \lambda_{3q}a_{31} + \cdots + \lambda_{mq}a_{m1}) = a_{1q'} + \lambda_{2q}a_{2q'} + \cdots + \lambda_{mq}a_{mq'} \quad . \quad (1)$$

obtained by giving q' values from 2 to m inclusive. The quantities $\lambda_2 \ldots \lambda_m$ are the usual undetermined multipliers used in the integration, and the second suffix, r or q, which has values from 1 to m inclusive, merely indicates one of the m values which each λ may have.

Using the notation $a_{g1} = k_{1g}a_{1g}$, and postulating observance of the third law of motion, so that $k_{1g}M_g = M_1$, where M_g is the mass associated with the co-ordinate ξ_g, we have

$$a_{pg} = a_{gp}\frac{k_{1p}}{k_{1g}},$$

and the equation connecting the λs becomes

$$\lambda_{q'q}(a_{11} - a_{q'q'}) = a_{1q'} + \sum_2^{q'-1} {}^r \lambda_{rq}a_{rq'} + \sum_{q'+1}^m {}^r \frac{k_{1r}}{k_{1q'}} a_{q'r}\lambda_{rq} .$$

Forming the similar equation in $q+1$, multiplying it by $\lambda_{q'q}$, and subtracting from it the above equation multiplied by $\lambda_{q',q+1}$, we get

$$\lambda_{q'q}\lambda_{q',q+1}\sum_2^m {}^r k_{1r}(\lambda_{rq} - \lambda_{r,q+1})a_{1r} - (\lambda_{q',q+1} - \lambda_{q'q})a_{1q'} + \sum_2^{q'-1} {}^r (\lambda_{q',q+1}\lambda_{rq} - \lambda_{q'q}\lambda_{r,q+1})a_{rq'}$$
$$+ \sum_{q'+1}^m {}^r \frac{k_{1r}}{k_{1q'}}(\lambda_{q',q+1}\lambda_{rq} - \lambda_{q'q}\lambda_{r,q+1})a_{q'r} = 0 .$$

Multiplying this equation by $k_{1q'}$, and summing for all values of q' from 2 to m, there results

$$\sum_2^m {}^{q'} \sum_2^m {}^r k_{1r}(\lambda_{rq} - \lambda_{r,q+1})(1 + k_{1q'}\lambda_{q'q}\lambda_{q',q+1})a_{1r} - \sum_2^m {}^{q'} \sum_2^{q'-1} {}^r k_{1q'}(\lambda_{q',q+1}\lambda_{rq} - \lambda_{q'q}\lambda_{r,q+1})a^{rq'}$$
$$- \sum_2^m {}^{q'} \sum_{q'+1}^m {}^r k_{1r}(\lambda_{q',q+1}\lambda_{rq} - \lambda_{q'q}\lambda_{r,q+1})a_{q'r} = 0 .$$

Remembering now the condition $k_{1q'}a_{rq} = k_{1r}a_{q'r}$, and noting that the quantity in the bracket in the last two double sums reverses its sign on interchange of q' and r, we see that the terms in these sums cancel in pairs, so that we finally obtain

$$\sum_2^m{}^{q'}\sum_2^m{}^{\lambda} k_{1r}(\lambda_{rq} - \lambda_{r,\,q+1})(1 + k_{1q'}\lambda_{q'q}\lambda_{q',\,q+1})a_{1r} = 0\,.$$

Hence we have one or other (or both) of the conditions

$$\sum_2^m{}^{q'}(1 + k_{1q'}\lambda_{q'q}\lambda_{q',\,q+1}) = 0\ ; \quad \sum_2^m{}^{r} k_{1r}(\lambda_{rq} - \lambda_{r,\,q+1})a_{1r} = 0\ ;$$

that is, one or other (or both) of the conditions

$$\sum_2^m{}^{q'}(1 + k_{1q'}\lambda_{q'q}\lambda_{q',\,q+1}) = 0\ ; \quad n_q{}^2 - n_{q+1}{}^2 = 0 \quad . \quad . \quad . \quad (2)$$

Each of the $m-1$ possible values of q contributes a similar pair of alternative or simultaneous conditions. If, in any case, two frequencies specified by n_q and n_{q+1} are unequal, the left-hand alternative must be satisfied.*

It is necessary now to investigate the bearing of these conditions on the question of "normality" above alluded to.

5. We have

$$2\left\{\dot{\xi}_p{}^2\right\} = \sum_1^m{}^{r} a'^2{}_{pr}n_r{}^2\mathrm{A}_r{}^2 + \sum_1^m{}^{r}\sum_1^m{}^{s}\left\{a'_{pr}a'_{ps}n_rn_s\mathrm{A}_r\mathrm{A}_s\cos(n_rt+\alpha_r)\cos n_st+\alpha_s)\right\}$$

where $\{\ \}$ indicates an average taken over a time which is long in comparison with the longest of the fundamental periods, which we presume to be distinct. It is also implied that the average time between successive collisions is similarly large. In accordance with our postulate, every term in the double sum vanishes. If E be the total kinetic energy in the system in the interval between two successive collisions, its value is given by

$$2\sum_1^m{}^{p}\left\{\mathrm{M}_p\dot{\xi}_p{}^2\right\} = \sum_1^m{}^{p}\sum_1^m{}^{r}\mathrm{M}_pa'^2{}_{pr}n_r{}^2\mathrm{A}_r{}^2$$
$$+\sum_1^m{}^{r}\sum_1^m{}^{p}\mathrm{M}_p\sum_1^m{}^{s}a'_{pr}a'_{ps}n_rn_s\mathrm{A}_r\mathrm{A}_s\left\{\cos(n_rt+\alpha_r)\cos(n_st+\alpha_s)\right\}.$$

The triple sum would vanish *at every instant* if we had

$$\sum_1^m{}^{p}\mathrm{M}_pa'_{pr}a'_{ps} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

* Similar formulæ hold in the $m-2$ cases in which we take $q+2$ instead of q, and so on. Altogether there are $m(m-1)$ equations of type (2).

This, therefore, is the condition for "normality"; and if it be satisfied, Dr Ehrenfest's result applies, and our test of non-equipartition, *i.e.*

$$M_p a'^2_{pr} - M_q a'^2_{qr} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

of the same sign, whatever be the value of r, when p and q are given, cannot be satisfied. If, on the other hand, equation (3) does not hold, the test is valid.

6. Still presuming distinctness of periods, so that the first of equations (2) applies, we shall now use that equation to discriminate cases in which (3) will hold.

Taking first the case of two freedoms, (2) gives

$$1 + k_{12}\lambda_{21}\lambda_{22} = 0 ,$$

or, in the notation of § 3 of the first paper,

$$1 - k_{12}rr' = 0 = b_1 - b_2 rr' .$$

But (3) becomes $M_1 rr' - M_2$, so, since $k_{12} = M_1/M_2$ the conditions (2) and (3) are identical, and normality is necessary provided that $M_1 b_1 = M_2 b_2$ in accordance with the third law of motion. This verifies the conclusion in the former paper.

7. In the case of three freedoms with distinct periods (2) gives

$$1 + k_{12}\lambda_{21}\lambda_{22} + k_{13}\lambda_{31}\lambda_{32} = 0 ,$$
$$1 + k_{12}\lambda_{22}\lambda_{23} + k_{13}\lambda_{32}\lambda_{33} = 0 ,$$
$$1 + k_{12}\lambda_{23}\lambda_{21} + k_{13}\lambda_{33}\lambda_{31} = 0 ,$$

while (3) gives

$$M_1 a'_{11} a'_{12} + M_2 a'_{21} a'_{22} + M_3 a'_{31} a'_{32} = 0 , \text{ etc.}$$

Now, referring to the formulæ of § 6 of the former paper, we have

$$\Delta a'_{11} = \begin{vmatrix} \lambda_{22} & \lambda_{32} \\ \lambda_{23} & \lambda_{33} \end{vmatrix} , \quad \Delta a'_{12} = \begin{vmatrix} \lambda_{23} & \lambda_{33} \\ \lambda_{21} & \lambda_{31} \end{vmatrix} , \quad \Delta a'_{21} = \begin{vmatrix} \lambda_{32} & 1 \\ \lambda_{23} & 1 \end{vmatrix} ,$$

$$\Delta a'_{22} = \begin{vmatrix} \lambda_{33} & 1 \\ \lambda_{31} & 1 \end{vmatrix} , \quad \Delta a'_{31} = \begin{vmatrix} 1 & \lambda_{22} \\ 1 & \lambda_{23} \end{vmatrix} , \quad \Delta a'_{32} = \begin{vmatrix} 1 & \lambda_{23} \\ 1 & \lambda_{21} \end{vmatrix} .$$

Hence, using the first two of the three equations just deduced from (2), we obtain the results

$$k_{12} = \frac{a'_{22}}{\lambda_{22} a'_{12}} , \quad k_{13} = \frac{a'_{32}}{\lambda_{32} a'_{12}} .$$

Similarly,

$$k_{12} = \frac{a'_{21}}{\lambda_{21} a'_{11}} , \quad k_{13} = \frac{a'_{31}}{\lambda_{31} a'_{11}} .$$

Therefore, substituting in the first of the three equations, we establish the first of equations (3), and the others can be established similarly. Thus normality obtains necessarily, and Dr Ehrenfest's conclusion shows that, contrary to the expectation expressed in § 8 of the former paper, while

equipartition may not hold in the case of three freedoms with distinct periods, we cannot prove by the present method that it does not hold.

8. We shall next take the case of three freedoms with two periods identical and the third distinct. In this case the three left-hand alternatives of (2) may still hold. If they do hold, no conclusion can be drawn; but if one does not hold, we cannot establish the condition for normality, and so Dr Ehrenfest's result does not apply.

We have now to make the triple sum, expressed in § 5, vanish by extending the average so as to include many collisions, and making the assumption

$$\{A_rA_s \cos(\alpha_r - \alpha_s)\} = 0 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

with regard to the term in which $n_r = n_s$.

Because of the random nature of the collisions relatively to the actual co-ordinates, this seems to be the proper assumption to make. Yet it must be recognised as an assumption to be verified by tests. If we can construct a three-freedom system, having two periods coincident, and satisfying the conditions which secure unequal partition of kinetic energy on the average amongst the freedoms, the assumption will be justified.

We shall take $n_1 = n_3 \neq n_2$; $\lambda_{21} = 1$, $\lambda_{22} = 1$, $\lambda_{23} = 2$; $\lambda_{31} = 2$, $\lambda_{32} = -\frac{1}{2}$, $\lambda_{33} = 3$. With these values, the conditions

$$1 + k_{12}\lambda_{21}\lambda_{22} + k_{13}\lambda_{31}\lambda_{32} = 0$$
$$1 + k_{12}\lambda_{22}\lambda_{23} + k_{13}\lambda_{32}\lambda_{33} = 0$$

give $k_{12} = 1$, $k_{13} = 2$, and the remaining quantity $1 + k_{12}\lambda_{23}\lambda_{21} + k_{13}\lambda_{33}\lambda_{31}$ is not zero, so that the condition for normality is not satisfied. Going back now to equation (1), we find

$$\lambda_{2q}(a_{11} + \lambda_{2q}a_{21} + \lambda_{3q}a_{31}) = a_{12} + \lambda_{2q}a_{22} + \lambda_{3q}a_{32}$$
$$\lambda_{3q}(a_{11} + \lambda_{2q}a_{21} + \lambda_{3q}a_{31}) = a_{13} + \lambda_{2q}a_{23} + \lambda_{3q}a_{33}$$

in which q takes the values 1, 2, 3. Taking the values $a_{11} = -3$, $a_{12} = -2$, $a_{13} = 1$, $a_{22} = -3$, $a_{23} = 1$, $a_{33} = -2$, and remembering the relations $a_{21}k_{11} = a_{12}k_{12}$, etc., it is easily found that the above equations are identically satisfied. Therefore the cubic in n^2 (see § 3 of the first paper) is

$$\begin{vmatrix} -3+n^2 & -2 & 1 \\ -2 & -3+n^2 & 1 \\ 2 & 2 & -2+n^2 \end{vmatrix} = 0,$$

the roots of which are $n_1^2 = n_3^2 = 1$, $n_2^2 = 6$. This may be verified from the expression $-n_q^2 = a_{11} + \lambda_{2q}a_{21} + \lambda_{3q}a_{31}$. If we put $M_1 = 1$, we get $M_2 = 1$, $M_3 = \frac{1}{2}$, and the dynamical system is completely specified.*

* The numerical example given in the first paper was fallacious.

The signs of the various quantities (4), § 5, have now to be determined. The chosen values of the λs give

$$\Delta a'_{11}=4,\ \Delta a'_{12}=1,\ \Delta a'_{13}=-5/2;\ \Delta a'_{21}=-7/2,\ \Delta a'_{22}=1,\ \Delta a'_{23}=5/2;$$
$$\Delta a'_{31}=1,\ \Delta a'_{32}=-1,\ \Delta a'_{33}=0.$$

So we find

$$\Delta^2(M_1a'^2_{11}-M_2a'^2_{21})=\ 3\tfrac{3}{4};\ \ \Delta^2(M_1a'^2_{12}-M_2a'^2_{22})=0;\ \ \Delta^2M_1a'^2_{13}-M_2a'^2_{23})=0;$$
$$\Delta^2(M_2a'^2_{21}-M_3a'^2_{31})=11\tfrac{3}{4};\ \ \Delta^2(M_2a'^2_{22}-M_3a'^2_{32})=\tfrac{1}{2};\ \ \Delta^2M_2a'^2_{23}-M_3a'^2_{33})=6\tfrac{1}{4};$$
$$\Delta^2(M_3a'^2_{31}-M_1a'^2_{11})=-15\tfrac{1}{2};\ \Delta^2(M_3a'^2_{32}-M_1a'^2_{12})=-\tfrac{1}{2};\ \Delta^2M_3a'^2_{33}-M_3a'^2_{13})=-6\tfrac{1}{4}.$$

The sign of every one of the three quantities in any one of these three rows is similar. Therefore there is no equipartition of energy between any pair of masses. The average energy of M_1 exceeds that of M_2, which exceeds that of M_3.

If the postulate (5) were not granted, it is not easy to see how the quantities A and a, which are the arbitrary constants of integration, could be regarded as truly arbitrary within their allowable ranges. The arbitrariness is supplied by the randomness of collisions.

9. We shall now take an example in four freedoms with the conditions $n_2=n_3=n_4\neq n_1$. With the values

$$\begin{aligned}
\lambda_{21}&=\ \ 1,\ \lambda_{31}=-1,\ \lambda_{41}=1,\\
\lambda_{22}&=-1,\ \lambda_{32}=\ \ 1,\ \lambda_{42}=2,\\
\lambda_{23}&=\ \ 1,\ \lambda_{33}=\ \ 2,\ \lambda_{43}=1,\\
\lambda_{24}&=-3,\ \lambda_{34}=\ \ 0,\ \lambda_{44}=3,
\end{aligned}$$

the conditions (2) for inequality of periods give

$$\begin{aligned}
1-k_{12}-k_{13}+2k_{14}&=0,\\
1-3k_{12}\qquad+3k_{14}&=0,\\
2k_{12}-k_{13}-\ k_{14}&=0.
\end{aligned}$$

Hence we have $k_{12}=2/3$, $k_{13}=1$, $k_{14}=1/3$; and also, by the expression for n_r in § 4, if we take $a_{12}=-a_{13}=a_{14}=1$, we have $-n_1^2=a_{11}+2$, $-n_2^2=-n_3^2=-n_4^2=a_{11}-1$. Taking $a_{11}=-3$, these give $n_1=1$, $n_2=n_3=n_4=2$; and the conditions (2) for equality of periods are identically satisfied. Taking $a_{22}=-10/3$, $a_{23}=-2/3$, $a_{24}=2/3$, $a_{33}=-3$, $a_{34}=-1$, $a_{44}=-11/3$, and remembering $a_{pq}k_{1q}=a_{qp}k_{1p}$, we find that equations (1) connecting the λs, and the as are identically satisfied. The quartic in n^2 is therefore

$$\begin{vmatrix}
-3+n^2 & 1 & -1 & 1\\
\tfrac{2}{3} & -\tfrac{10}{3}+n^2 & -\tfrac{2}{3} & \tfrac{2}{3}\\
-1 & -1 & -3+n^2 & -1\\
\tfrac{1}{3} & \tfrac{1}{3} & -\tfrac{1}{3} & -\tfrac{11}{3}+n^2
\end{vmatrix}=0,$$

the roots being 1 and 4 (triple) as above. The values of the quantities a'_{rs} are as follows:—

$$\Delta^2 a'^2_{11}=0\,,\ \Delta^2 a'^2_{12}=18^2\,,\ \Delta^2 a'^2_{13}=9^2\,,\ \Delta^2 a'^2_{14}=9^2\,;$$
$$\Delta^2 a'^2_{21}=0\,,\ \Delta^2 a'^2_{22}=\ 6^2\,,\ \Delta^2 a'^2_{23}=3^2\,,\ \Delta^2 a'^2_{24}=3^2\,;$$
$$\Delta^2 a'^2_{31}=0\,,\ \Delta^2 a'^2_{32}=\ 0\ \,,\ \Delta^2 a'^2_{33}=0\ \,,\ \Delta^2 a'^2_{34}=0\ \,;$$
$$\Delta^2 a'^2_{41}=0\,,\ \Delta^2 a'^2_{42}=12^2\,,\ \Delta^2 a'^2_{43}=6^2\,,\ \Delta^2 a'^2_{44}=6^2\,.$$

Hence, remembering the values of the ks in terms of the masses which give $M_2=3M_1/2$, $M_3=M_1$, $M_4=3M_1$, we get

$$\begin{aligned}
&\Delta^2(M_1 a'^2_{11}-M_2 a'^2_{21})=0 &&;\ \Delta^2(M_2 a'^2_{21}-M_3 a'^2_{31})=0\\
&\Delta^2(M_1 a'^2_{12}-M_2 a'^2_{22})=\Delta^2 M_1(18^2-\tfrac{3}{2}6^2)&&;\ \Delta^2(M_2 a'^2_{22}-M_3 a'^2_{32})=\Delta^2 M_1(\tfrac{3}{2}6^2)\\
&\Delta^2(M_1 a'^2_{13}-M_2 a'^2_{23})=\Delta^2 M_1(\ 9^2-\tfrac{3}{2}3^2)&&;\ \Delta^2(M_2 a'^2_{23}-M_3 a'^2_{33})=\Delta^2 M_1(\tfrac{3}{2}3^2)\\
&\Delta^2(M_1 a'^2_{14}-M_2 a'^2_{24})=\Delta^2 M_1(\ 9^2-\tfrac{3}{2}3^2)&&;\ \Delta^2(M_2 a'^2_{24}-M_3 a'^2_{34})=\Delta^2 M_1(\tfrac{3}{2}3^2)
\end{aligned}$$

$$\begin{aligned}
&\Delta^2(M_1 a'^2_{11}-M_4 a'^2_{41})=0\,,\\
&\Delta^2(M_1 a'^2_{12}-M_4 a'^2_{42})=\Delta^2 M_1(18^2-3(12)^2)\,,\\
&\Delta^2(M_1 a'^2_{13}-M_4 a'^2_{43})=\Delta^2 M_1(\ 9^2-3(\ 6)^2)\,,\\
&\Delta^2(M_1 a'^2_{14}-M_4 a'^2_{44})=\Delta^2 M_1(\ 9^2-3(\ 6)^2)\,,
\end{aligned}$$

which show that the average energies of the masses are never equal, the energies of M_4, M_1, M_2, M_3 being in descending order of magnitude.

By change of the value of a_{11} a single infinity of such examples is readily specified.

10. The general values of a'_{11}, a'_{21}, a'_{31}, are given by

$$\Delta a'_{11}=\begin{vmatrix}\lambda_{22} & \lambda_{32} & \cdots & \lambda_{m2}\\ \vdots & & & \\ \vdots & & & \\ \lambda_{2m} & & & \lambda_{mm}\end{vmatrix}\ ;\ \Delta a'_{21}=\begin{vmatrix}\lambda_{32} & \cdots & \lambda_{m2} & 1\\ \vdots & & & \\ \vdots & & & \\ \lambda_{3m} & & & 1\end{vmatrix}\ ;$$

$$\Delta a'_{31}=\begin{vmatrix}\lambda_{42} & \cdots & \lambda_{m2} & 1 & \lambda_{22}\\ \vdots & & & & \\ \vdots & & & & \\ \lambda_{4m} & & \lambda_{mm} & 1 & \lambda_{2m}\end{vmatrix}\ ;$$

and the expressions for other as can be written down by symmetry. Hence, selecting the $m-1$ equations involving $\lambda_{21}, \ldots \lambda_{m1}$ out of the $m(m-1)$ equations (2) when all periods are distinct, we find

$$k_{12}=\frac{1}{\lambda_{12}}\ \frac{a'_{21}}{a'_{11}}\,.$$

Similarly, selecting the $m-1$ equations which involved $\lambda_{22}, \ldots \lambda_{m2}$, we get

$$k_{12} = \frac{1}{\lambda_{22}} \frac{a'_{22}}{a'_{12}},$$

and so on. Finally,

$$k_{12} = \frac{1}{\lambda_{21}} \frac{a'_{21}}{a'_{11}} = \frac{1}{\lambda_{22}} \frac{a'_{22}}{a'_{12}} = \ldots\ldots = \frac{1}{\lambda_{2m}} \frac{a'_{2m}}{a'_{1m}}.$$

Generally,

$$k_{1p} = \frac{1}{\lambda_{p1}} \frac{a'_{p1}}{a'_{11}} = \frac{1}{\lambda} \frac{a'_{p2}}{a'_{12}} = \ldots\ldots = \frac{1}{\lambda_{pm}} \frac{a'_{pm}}{a'_{1m}}.$$

Therefore the equation

$$1 + k_{12}\lambda_{2r}\lambda_{2s} + k_{13}\lambda_{3r}\lambda_{3s} + \ldots\ldots + k_{1m}\lambda_{mr}\lambda_{ms} = 0$$

becomes

$$1 + \frac{1}{k_{12}} \frac{a'_{2r}}{a'_{1r}} \frac{a'_{2s}}{a'_{1s}} + \frac{1}{k_{13}} \frac{a'_{3r}}{a'_{1r}} \frac{a'_{3s}}{a'_{1s}} + \ldots\ldots + \frac{1}{k_{1m}} \frac{a'_{mr}}{a'_{1r}} \frac{a'_{ms}}{a'_{1s}} = 0,$$

or

$$M_1 a'_{1r} a'_{1s} + M_2 a'_{2r} a'_{2s} + M_3 a'_{3r} a'_{3s} + \ldots\ldots + M_m a'_{mr} a'_{ms} = 0,$$

which shows that, in a system having m degrees of freedom with m distinct periods, the property of normality always exists, and therefore Dr Ehrenfest's conclusion applies, and we can neither prove nor disprove the doctrine of equipartition of energy. If, on the other hand, even only two periods coincide, $n_r = n_s$, the above condition is not binding, and we may deduce infinities of cases where equipartition is not observed. At least this is so if we can postulate free response of the As and as to random impacts so as to satisfy (5). That condition being granted, the above numerical examples are conclusive.

No doubt we can introduce, in an infinite number of ways, a linear transformation of co-ordinates, which would impose "normality." It would be impossible then to prove, by the present method, anything definite regarding the partition of energy amongst these new variables. But these variables are not those corresponding to the actual freedoms of the individual masses. Even if it could be otherwise proved that the energy is equipartitioned amongst them, it is not clear how any conclusion could be drawn from that fact regarding the partitioning amongst the actual freedoms. Unequal salaries, due to A, B, and C, can be equipartitioned, in an infinity of ways, amongst A′, B′, and C′; but law prevents the process.

11. In this connection it is important to examine analogous questions of partition in non-vibratory systems.

Lord Kelvin has pointed out, and it seems to be fully admitted, that

the meaning of the Boltzmann-Maxwell doctrine, as applied to a single particle moving, under none but collisional forces, on a smooth surface whose boundary is such as to ensure fulfilment of Maxwell's assumption regarding phases, is that a geodetic drawn long enough will pass infinitely near every point within the boundary, as often in nearly one direction as in another, as often within a given small range in the neighbourhood of one point as in that of another. The possibility of passage infinitely near any point in infinitely nearly any direction being assumed, the doctrine asserts absolute equality in space distribution and direction distribution. To settle this point, Lord Kelvin has introduced experimental tests (*Baltimore Lectures*, App. B) which indicate marked deviation from the Boltzmann-Maxwell condition. Lotteries were used to bring in the random element which the kinetic theory requires.

In a recent address to the American Association for the Advancement of Science, Professor W. F. Magie says, "I do not pretend to be able to show that these results of Lord Kelvin are of no value as evidence against the truth of the theorem, but I would remark that we can at least justify a doubt about them by noticing how small a deviation in the experiments from perfect impartiality of conditions will suffice to produce a large deviation from the expectation of the theory of probabilities" (*Science*, Feb. 2, 1906). He then shows that great bias may arise through slight inequalities in the sizes of cards in a pack, and adds, "I think we may fairly suppose that the discrepancies of 15 per cent. or so, which appeared in his experiments, may have been due, not to a failure of the theorem of equipartition, but to trifling departures from impartiality in his methods of experimentation."

We shall consider specially the case, dealt with by Lord Kelvin in § 38 of the Appendix, of a plane surface with a circular boundary of infinitesimal semicircular corrugations hollow inwards. He says, "The Boltzmann-Maxwell doctrine asserts that the time-integrals of the kinetic energies of the two components, radial and transversal, according to polar co-ordinates, would be equal." In a set of 143 flights, in which the total time-integral of kinetic energy was 121·3, he found an excess of the radial over the transverse component amounting to 10·7, and remarks, "Out of fourteen sets of ten flights I find that the time-integral of the transverse component is less than half the whole in twelve sets, and greater in only two. This seems to prove beyond doubt that the deviation from the Boltzmann-Maxwell doctrine is genuine; and that the ultimate time-integral of the transverse component is certainly smaller than the time-integral of the radial component."

The fact of the constancy of the result in twelve out of fourteen equal portions of 140 flights bears against Professor Magie's supposition; but, in view of the entire legitimacy of that supposition, it seems to be desirable to test the question farther mathematically. The following investigation indicates how Lord Kelvin's result may be entirely correct. An extension on similar lines could fix the exact proportion of radial to transverse components of the kinetic energy. Without carrying out that extension, we obtain a disproof of the conclusion that all phases are equally distributed in space and time provided that all phases are possible.

12. An incident path, making a fixed angle θ with the line drawn from the centre of the circular area to the centre C of a semicircular corrugation, may cross the diameter A B of that corrugation at any point between A and B. All points are supposed to be equally probable. The diagram represents the case $n=5$. The reflected path coincides with the incident path when it passes through C. As the crossing point passes downwards from C, the reflected path makes a smaller angle, θ', with C O until the point A_1 is reached, when the paths are contra-parallel. (Points such as A_1, etc., are indicated in the figure by their suffixes alone.) As the crossing point passes farther down, θ' increases until reflection occurs at B. The magnitude of θ' then diminishes until, when A_2 is reached, parallelism again occurs. Similarly A_3 and A_4 correspond respectively to contra-parallelism and parallelism; and so on. When the crossing point lies in the regions C A_1, A_2 A_3, etc., the path which leaves the corrugation makes a smaller angle with C O than the entering path makes. When the point lies in the regions A_1 A_2, etc., the emergent path makes a larger angle with C O than the entering path makes.

In the same way we find that, when the crossing point is in the regions C B_1, B_2 B_3, etc., the angle of emergence exceeds the angle of entrance; while, when the crossing point lies in the regions B_1 B_2, B_3 B_4, etc., the angle of emergence is less than the angle of entrance. If the radius of the corrugation be unity, twice the probability that the emergent path shall pass nearer the centre of the circular area having the corrugated boundary than the entering path passes is given by the sum of the lengths of the regions C A_1, A_2 A_3, ... , B_1 B_2, B_3 B_4, ... The number of these regions depends on the value of the entrant angle, θ. When θ has the value $\pi/2n$, the points A_{2n-3}, $A_{2(n-1)}$, $B_{2(n-1)}$, B_{2n-1}, coincide in pairs respectively with the extremities A, B, of the diameter.

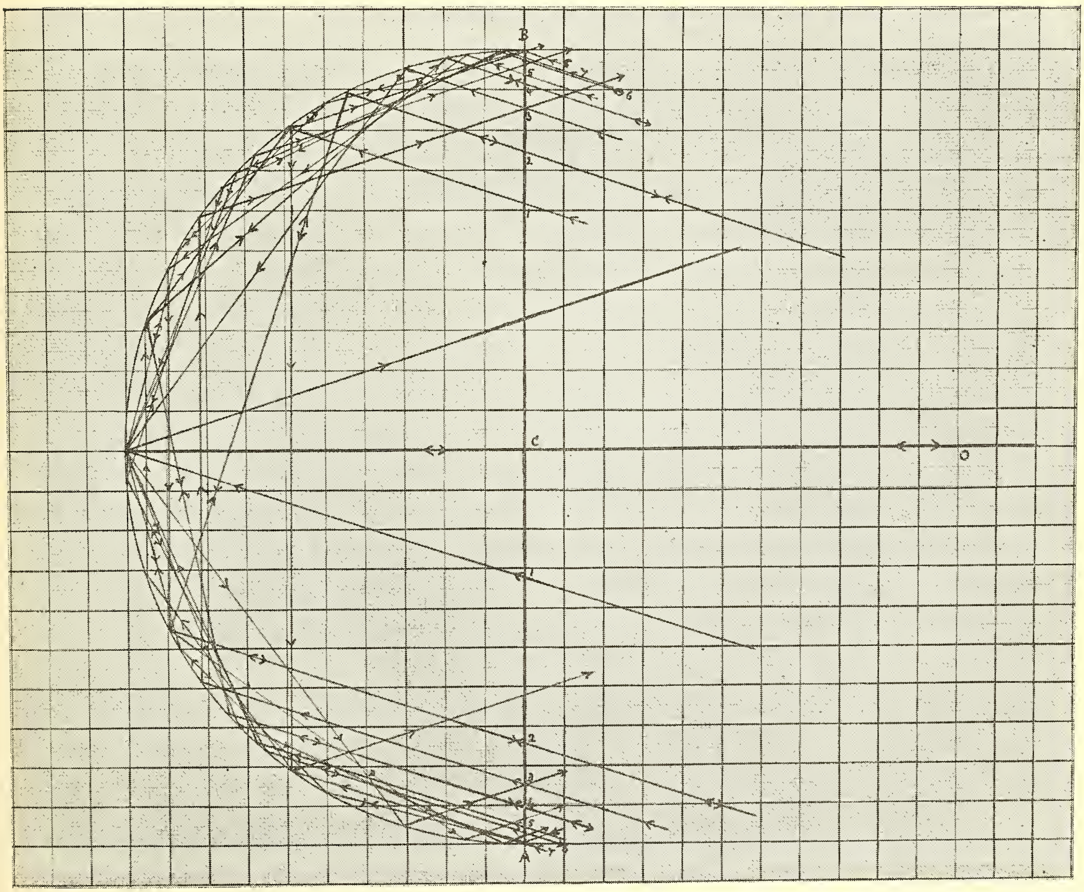
If $p+1$ be the number of reflections of a path between its entrance

into, and its emergence from, the semicircle, we have the following expressions:

$$CA_{2p+1}\cos\theta = \sin\frac{p\pi+2\theta}{2(p+1)},$$

$$CA_{2p}\cos\theta = CB_{2p}\cos\theta = \sin\frac{p}{p+1}\frac{\pi}{2},$$

$$CB_{2p-1}\cos\theta = \sin\frac{p\pi-2\theta}{2(p+1)}.$$



Twice the probability that the emergent path shall pass nearer to the centre of the circular area than the entrant path passes is represented by

$$2 - \left\{ \sum_{p=1}^{n-2}\left[\sin\frac{(p+1)\pi+2\theta}{2(p+2)} - \sin\frac{(p-1)\pi+2\theta}{2p}\right] + \sin\frac{\pi-2\theta}{4} \right\} \Big/ \cos\theta\,.$$

When n has a given value, θ lies between $\pi/2n$ and $\pi/2\,(n-1)$.

13. The full curve in the accompanying diagram represents the probability of the emergent path being nearer to the centre than the entrant path; the dotted curve represents the probability of the emergent path being further from the centre than the entrant path. Ordinates represent the probability, abscissæ represent the angle θ. The probability is almost exactly 0·5 at $\theta = 26°$.

If a and a' are the complements of θ and θ', where θ' is the smallest emergent angle corresponding to θ as the entrant angle, we have $a' = 3a$, provided that a is less than $\pi/4$. Conversely, no path a can be reached from a path outside the angle $3a$. Thus, with all possible directions of entrant path, the probability that the emergent path shall have a small a is small. On the contrary, emergent paths for which a is large have large probability.

Now the path for which the average radial and transverse components of kinetic energy are equal is given by

$$\tan \theta = 2\theta$$

and corresponds to θ slightly larger than 23°. It therefore lies slightly inside the path ($\theta = 26°$) which is equally readily reached from both smaller and larger values of θ. Therefore this latter path has a slight preponderance of the transverse component of kinetic energy over the radial component. The excess is scarcely one-fifth of the total kinetic energy. Consequently, all directions of incident path at the corrugated boundary being made initially equally probable, the large probability of emergent path with large a, that is, small θ; and the small probability of emergent paths with small a, that is, large θ; make it almost certain that the initially emergent paths will show an average preponderance of the radial over the transverse component of kinetic energy. This distribution of emergent paths constitutes a distribution of entrant paths still more favourable to that preponderance than was the initially postulated distribution.

Without making a more detailed investigation, it therefore appears likely that the preponderance of the radial component, which was exhibited in Lord Kelvin's tests, is a result to be expected.

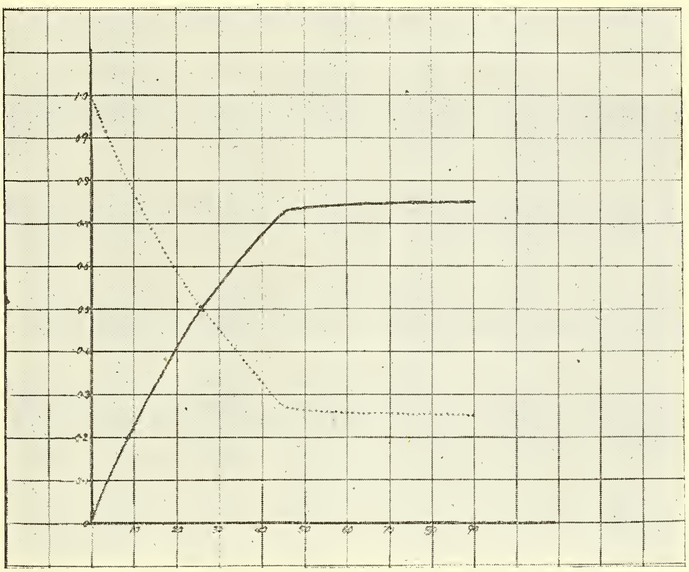
14. The corresponding result can be obtained more readily with a different law of reflection at the boundary. A molecule moving in the interior of a liquid or solid spherical cavity has probably no direction of reflection preponderant over another. We shall assume that all directions are equally probable for the reflected path whatever be the direction of the incident paths. If we assume, for simplicity, that the cavity has unit radius, and that the mass of the moving particle is 2 while its speed is

always unity, the excess of the radial over the transverse component of kinetic energy is

$$\frac{\tan\theta - 2\theta}{\tan\theta}$$

where θ is the angle which the path makes with the tangent plane at its extremity. The element of solid angle is $2\pi\cos\theta d\theta$, so that the average excess of the radial over the transverse component of kinetic energy is

$$\int_0^{\frac{\pi}{2}} \frac{\tan\theta - 2\theta}{\tan\theta}\cos\theta d\theta\,,$$



which can be put into the form

$$3 + 4\int_{-\infty}^{0} \frac{x\epsilon^x dx}{1+\epsilon^{2x}}\,;$$

or, again, into the form

$$3 - 4\int_0^1 \frac{dx}{1+x^2}\sum_{n=8}\left[1 + \frac{2}{3}\frac{x^2}{1+x^2} + \frac{2.4}{3.5}\left(\frac{x^2}{1+x^2}\right)^2 + \cdot\cdot\cdot\cdot + \frac{2.4\cdot\cdot\cdot\cdot(2n)}{3.5\cdot\cdot\cdot\cdot(2n+1)}\left(\frac{x^2}{1+x^2}\right)^n\right].$$

Evaluation of the first two terms of the integral shows that the whole quantity is negative; so that, in the motion of a single particle throughout a sufficient time, there is excess of the transverse component of kinetic energy over the radial component; or, in the motion of many non-colliding particles, there is, on the number-average, a like excess.

In this case all phases occur, and that with equal frequency, yet there is not equipartition of the radial and transverse components. In the previous example all phases occurred, but not apparently with equal frequency.

When collisions occur amongst the particles, equipartition becomes more and more nearly satisfied as the mean free path becomes smaller and smaller relatively to the diameter of the sphere, unless some cause other than a boundary condition intervenes to prevent equipartition, or to prevent that condition of "molecular chaos," the postulation of which seems to be equivalent to the postulation of equipartition. Such a cause is that co-ordination of velocities which Burbury contemplates as due to the tendency to release local variations of pressure, on a molecular scale, along paths of least resistance. It does not appear that, either in rare gases whose mean free path is comparable to the linear dimensions of the enclosing vessel, or in dense gases where it is not comparable to these dimensions, there is necessary equipartition of energy amongst all the freedoms.

(*Issued separately August* 28, 1907.)

XX.—On the Partition of Heat Energy in the Molecules of Gases. By Dr Paul Ehrenfest. (*Communicated by* Dr W. Peddie.)

(Read January 7, 1907. MS. received March 7, 1907.)

In a paper published some time since,* Mr Peddie takes up the following question: Given a gas whose molecules contain atoms which are held together by purely elastic forces, do there exist, even in this simplest type of multiply-atomic molecules, cases in which, when there is equilibrium of heat, the average equipartition of kinetic energy demanded by the Boltzmann-Maxwell Law does not exist?

Mr Peddie's formulation of the question possesses, above all, the merit of having directed the discussion of the Boltzmann-Maxwell Law of energy partition to a case which adapts itself readily to calculation. This discussion of elastically oscillating systems assumes a special interest in regard to the difficult problem of the partition of energy between ether and matter.

Having in view the great importance which, in this way, this formulation of the question possesses, I propose to show—

1. There actually exist, in the sense of the general question, a class of cases in which elastically oscillating systems certainly exhibit non-equipartition. These cases are obtained by means of special *simultaneous* assumptions: (*a*) regarding the mechanical structure of the oscillating molecule; (*b*) regarding the character of the collisions amongst the molecules. But this class of non-equipartition cases does not contradict Boltzmann's line of thought. It verifies, rather, certain restricting remarks which, without developing them farther, Boltzmann made in his first work on the H-theorem.

2. Mr Peddie seeks, in the special development of his work, to construct an essentially different class of non-equipartition cases. Their existence would in fact be irreconcilable with Boltzmann's results. Yet, as I shall show farther on, this special presentment, at least in its present form, does not suffice to prove the existence of the second class of non-equipartition cases.

It would thus appear that *the construction, postulated by Mr Peddie for elastically oscillating molecules which do not exhibit equipartition,*

* W. Peddie, "On Vibrating Systems which are not subject to the Boltzmann-Maxwell Law," *Proc. Roy. Soc. Edin.*, vol. xxvi. 1906 (pp. 130–141).

can in fact be carried out in a somewhat modified manner. But this construction then forms no objection to Boltzmann's general line of thought.

§ 1. Boltzmann has given, in §§ 74–86 of the second volume of his *Gastheorie,* on the basis of the H-theorem, the theory of the equilibrium of heat in multiply-atomic gases. We shall frequently refer in what follows to this presentation. Boltzmann makes the following remarks in preface to the deduction of the law of distribution (page 241):—"The complete development of the proof that the distribution, $\epsilon^{-h.\text{Energy}}$, is the only possible one which can maintain itself in a stationary gas, whose molecules are multiply-atomic, has not hitherto been successful in complete generality. Yet the development of this proof is possible, in the simplest and practically most important cases, to the same extent as for mon-atomic gases."

The deduction of the distribution from the H-theorem involves two distinctly separate steps:—

A. The deduction of the known equation

$$f_1 f_2 = f_1^1 f_2^1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

(see §§ 74–82).

B. The solution of this (functional) equation. Whilst this second step is very simple in the case of monatomic molecules, it offers special difficulty in the case of multiply-atomic molecules. (For the discussion of particular cases, see Boltzmann in §§ 83–86.) But, at the same time, this second step exhibits the possibility, *e.g.* through special assumptions regarding the nature of the collisions between the molecules, of finding cases of non-equipartition.

§ 2. Each molecule is made up of atoms A, B, C . . . . With reference to the nature of the collisions between the molecules, Boltzmann follows out two essentially different assumptions.

Assumption I.—Every atom of every molecule can collide with every other (§§ 83–84).

Assumption II.—In each molecule the atom A alone is susceptible of collision; the remaining atoms are influenced only through the changes of motion of the atoms A.

In the case of assumption I., the H-theorem furnishes a complete estimation of the speed distribution for atoms of all kinds. As well the atoms A, as B, as C, ... , obey Maxwell's speed distribution, and certainly are such that the mean kinetic energy (time, and number, mean), for the atoms of all kinds, has the same value (p. 243).

In the case of Assumption II., *on the contrary, the H-theorem furnishes too few equations, apart from supplementary assumptions, for the deter-*

mination of the speed distribution of all atoms (p. 246). In connection with this point, the calculation is carried out for diatomic molecules in the case in which A and B act upon each other in the direction of the central line, and only the atoms A strike each other. In this special case we can so complete the result of the H-theorem, by apparently plausible assumptions, that there results a fixed speed distribution. It appears that here also, both for A and B, Maxwell's distribution law holds, and certainly that the mean kinetic energy (time, and number, mean) has the same value.

§ 3. The calculation by which, in this special case, Boltzmann reached the result, does not settle whether the same result would hold in somewhat more general cases, *e.g.* for a molecule with more than two atoms or with another law of force. But the calculation which Boltzmann carried out in § 86 for the two-planet case can be put in a somewhat more general form. This shows at once that, in the case of assumption II., for observance or non-observance of equipartition, it is very essential, on account of the structure of the molecule, that the *free* motion (between two collisions) admit an integral of the form

$$f(\dot{\xi}_4, \ldots . \dot{\xi}_{3n}, \xi_1, \xi_2, \xi_3 \ldots . \xi_{3n}) = \text{const.}, \quad . \quad . \quad . \quad (2)$$

that is to say, an integral which does not depend explicitly either on the time, or the velocity components $\dot{\xi}_1$, $\dot{\xi}_2$, $\dot{\xi}_3$, of the atom A.* The farther pursuit of this question leads, then, to a corresponding systematic determination of non-equipartition cases. For the present purpose, it suffices to give one case. We choose one such which furnishes the immediate answer to Mr Peddie's question.

§ 4. We consider a mixture of two gases of the following nature. The molecules of the first gas are monatomic, and do not act upon each other with any force (except on collision). The molecules of the second gas are constituted of n atoms A, B, . . . which are bound together, and to their equilibrium position, by elastic forces. Here also the atoms A alone are to suffer blows, and these can come from the freely moving molecules of the first kind.

The cartesian co-ordinates of the n-atoms, measured from the equilibrium position of each atom, may be denoted throughout by

$$\xi_1, \xi_2, \ldots . , \xi_{3n},$$

where ξ_1, ξ_2, ξ_3, are the co-ordinates of the atom A. As elastically oscillating

[1] Moreover, it can then be shown that the integral does not contain the co-ordinates ξ_1, ξ_2, ξ_3. For the proof, one has to use the supposition that the forces depend only on the co-ordinates.

systems, the molecules of the second kind possess $3n$ normal-vibrations. The normal co-ordinates may (in arbitrary order) be denoted by

$$X_1, X_2, \ldots, X_{3n}.$$

For the kinetic and potential energies we have the following representations:

$$2T = \sum_1^{3n} {}^h m_h \dot{\xi}^2{}_h = \sum_1^{3n} {}^i \dot{X}^2{}_i \quad \ldots \quad (3)$$

$$2V = \sum_1^{3n} {}^h \sum_1^{3n} {}^k \beta_{hk} \xi_h \xi_k = \sum_1^{3n} {}^i B_i X^2{}_i \quad \ldots \quad (4)$$

There are, farther, the following substitutions:—

$$\left.\begin{array}{l} X_1 = l_{11}\xi_1 + l_{12}\xi_2 + l_{13}\xi_3 + \ldots + l_{1,3n}\xi_{3n} \\ X_2 = l_{21}\xi_1 + \ldots \\ \ldots \\ X_{3n} = l_{3n,1}\xi_1 + \ldots + l_{3n,3n}\xi_{3n} \end{array}\right\} \quad \ldots \quad (5)$$

The substitution coefficients l_{ih} depend on the mechanical structure of the molecule. We make now the following special assumption regarding this structure. Postulate that

$$l_{11} = l_{12} = l_{13} = 0 \quad \ldots \quad (6)$$

All other coefficients l_{ih} shall have any values, different from zero, so far as they are compatible with positive values of m and real values of β_{hk}. The physical meaning of this assumption is this:—

In the expression for the energy connected with the first normal vibration

$$E_1 = \tfrac{1}{2}(\dot{X}^2{}_1 + B_1 X^2{}_1), \quad \ldots \quad (7)$$

the speed components $\dot{\xi}_1$, $\dot{\xi}_2$, $\dot{\xi}_3$, *of the atom A do not appear* (nor also its co-ordinates).

5. One sees easily now that this assumption regarding the structure, together with the restricting assumption regarding the collisions, brings in non-equipartition.

To this end we show next that the quantity E_1 retains for each molecule its original value (E_1 is an integral of the type defined in equ. 2).

(*a*) During the free motion the quantities $E_1, \ldots, E_{3n}$ remain constant.

(*b*) E_1 also does not change by the blows. For the quantities

$$\dot{\xi}_4, \ldots, \dot{\xi}_{3n}, \xi_1, \ldots \xi_{3n}$$

possess immediately after the blow the same values as they had immediately before the blow. Only the quantities

$$\dot{\xi}_1, \dot{\xi}_2, \dot{\xi}_3$$

suffer certain sudden changes. But, since the quantities $\dot{\xi}_1$, $\dot{\xi}_2$, $\dot{\xi}_3$, do not

enter into E_1, that quantity possesses, immediately before and after the blow (therefore in general always), the same value.

We consider at present the simplest case: E_1 originally zero for all molecules. It remains then lastingly zero, and with it also the value of $\dot{X}_1^2$. So the mean value of $\dot{X}_1^2$ taken over all molecules is lastingly zero.

$$\overline{\dot{X}_1^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

however one may change the temperature of the first gas.*

On the other hand, the law of equipartition would demand that

$$\overline{\dot{X}_1^2} = \overline{\dot{X}_2^2} = \ldots\ldots = \overline{\dot{X}_{3n}^2} = \alpha \,.\, \mathrm{T} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

for, according to (3), the kinetic energy is made up of sums of squares of $\dot{X}_i$, which are the so-called "momentoids" (Boltz., b. ii. § 33).

For certain questions it would be interesting to settle: Does this manifest non-equipartition for the momentoid $\dot{X}_1$ necessitate that non-equipartition holds also for $\sqrt{m_h}\dot{\xi}$? To be able to deduce this consequence, one must exclude all the special structures, which, besides the integral $E_1 = \text{const.}$, admit other integrals of the form (2). Yet that would lead as far. It is sufficient for the preceding object to settle that at any rate *the momentoid* $\dot{X}_1$ *no longer complies with the law of equipartition.* Throughout, it is essential for the attainment of the result that we make the assumption, previously taken into consideration by Boltzmann, regarding the limitation of the collisions.

If one, on the contrary, made regarding the blows the assumption I., *the H-theorem would give equipartition for both series of momentoids* ($\sqrt{m_h}\dot{\xi}_h$ *and* $\dot{X}_i$).

§ 6. We pass now to the consideration of the test of that class of non-equipartition cases which, in the introduction, we denoted as the second class of non-equipartition cases. Apart from making an assumption regarding the limitation of the blows to the atom A, equipartition can only be excluded by an assumption regarding the structure of the oscillating molecule.

We make now the assumption that the frequencies of the 3_n normal vibrations possess distinct values $p_1, \ldots\ldots, p_{3n}$.† We have

$$X_i = A_i \sin p_i(t + t_i) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\dot{X}_i = p_i A_i \cos p_i(t + t_i) \quad . \quad . \quad . \quad . \quad . \quad (11)$$

* One can also proceed from any other original value of $\bar{E}_1$. Only one must then specially prove first that $\overline{\dot{X}_1^2}$ remains constant with E_1. There exists here an analogy to the case of rigid molecules with rotational symmetry.

† In § 8 this restriction will be removed.

There subsist farther the substitutions *

$$\left.\begin{array}{l}\xi_1 = a_{11}X_1 + a_{12}X_2 + \ldots \ldots + a_{1,3n}X_{3n} \\ \xi_2 = a_{21}X_1 + \ldots \ldots \ldots \ldots \ldots \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ \xi_{3n} = a_{3n,1}X_1 + \ldots \ldots + a_{3n,3n}X_{3n}\,. \end{array}\right\} \quad . \quad . \quad . \quad (12)$$

with analogous substitutions for $\dot{\xi}_h$. We introduce now the following time average symbol

$$\left\{ m_h\dot{\xi}^2_h - m_k\dot{\xi}^2_k \right\} = \frac{1}{\theta}\int_0^\theta (m_h\dot{\xi}^2_h - m_k\dot{\xi}^2_k)dt \quad . \quad . \quad . \quad (13)$$

Here θ denotes a time which is large relatively to the longest of the periods of the normal vibrations. The integration is extended over a free motion of the molecule of the second kind. We shall correspondingly make for the following application, the assumption that a molecule of the second kind makes numerous oscillations between two collisions.† From the assumption that all frequencies, p_i, are different from each other, it follows that

$$\{\dot{X}_i\dot{X}_j\} = 0 \qquad \text{for } i \neq j \quad . \quad . \quad . \quad . \quad . \quad (14)$$

On the other hand we have

$$\{\dot{X}_i^2\} = \frac{p^2{}_i A^2{}_i}{2}\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

One has also, for the difference of the time averages of the kinetic energies $\frac{1}{2}m_h\dot{\xi}_h^2$ and $\frac{1}{2}m_k\dot{\xi}_k^2$ the expression

$$\{m_h\dot{\xi}^2_h\} - \{m_k\dot{\xi}^2_k\} = \tfrac{1}{2}\sum_1^{3n}{}^i p^2{}_i A^2{}_i(m_h a^2{}_{hi} - m_k a^2{}_{ki}) \quad . \quad . \quad . \quad (16)$$

The quantities

$$P^i{}_{hk} = m_h a^2{}_{hi} - m_k a^2{}_{ki} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

are determined by the structure of the molecule, the quantities $A^2{}_i$ by the occasional excitation.

One can now put the question ‡—Is it possible so to choose the structure of the molecule, that is, the quantities $m_1 \ldots m_{3n}$ and β_k, that, at least for *one* pair of indices h, k, all the $3n$ quantities P shall satisfy

$$P^i_{h,k} > 0 \qquad i = 1, \ldots, 3n \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Let us assume that this might actually be possible. Then, from equation (16), for each arbitrary excitation (for each value of the quantities $A_1 \ldots A_{3n}$) we would have

$$\{m_h\dot{\xi}^2_h\} > \{m_k\dot{\xi}^2_k\}\,, \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

* Here, naturally, the earlier assumptions regarding l_{11}, l_{12}, l_{13} are no longer made.

† *Cf.* Peddie, *l.c.*, § 13.

‡ *Cf.* Peddie, *l.c.*, §§ 2, 3, 13, 14.

and only in the case of rest ($A_1 = A_2 \ldots\ldots = A_{3n} = 0$) would both expressions be simultaneously zero. Let us use farther

$$\{\overline{m_h \dot{\xi}^2_h}\} \quad \ldots\ldots \quad (20)$$

to denote the number average of the time average, taken over all molecules of the second kind.

For molecules of such structure, if they could exist, equipartition for the momentoids $\sqrt{m_h}\dot{\xi}_h$ would never occur, even with arbitrary collisions throughout. Equipartition would demand

$$\{\overline{m_h \dot{\xi}^2_h}\} = \{\overline{m_k \dot{\xi}^2_k}\} \quad \ldots\ldots \quad (21)$$

which, according to 19, cannot occur.

To attain, free from contradiction, to a specification of molecules of such structure, would, moreover, be to contradict a much more fundamental consequence of Boltzmann's theory, which says: Given a multitude of multiply-atomic molecules, of arbitrary structure, which obeys the law of distribution $\epsilon^{-h.\text{Energy}}$, equipartition holds *for every system of momentoids* (Boltzmann, *Gasth.*, b. ii. § 42). For the molecules here discussed there is no distribution which can permit of equipartition for the momentoids $\sqrt{m_h}\dot{\xi}_h$.

§ 7. Having regard specially to this contradiction of one of the weightiest fundamental laws of Boltzmann's theory, I would show in what follows that the $3n$ inequalities (18) can never be fulfilled simultaneously. If, by means of the substitutions (12), the kinetic energy

$$T = \tfrac{1}{2}\sum_1^{3n}{}^h m_h \dot{\xi}^2_h \quad \ldots\ldots \quad (22)$$

be transformed to the form

$$T = \tfrac{1}{2}\sum_1^{3n}{}^i \dot{X}^2_i \quad \ldots\ldots \quad (23)$$

the quantities a_{hi} must satisfy the following orthogonal conditions,

$$\sum_1^{3n}{}^h m_h a^2_{hi} = 1 \qquad i = 1, \ldots\ldots, 3n \quad \ldots\ldots \quad (24)$$

and

$$\sum_1^{3n}{}^h m_h a_{hi} a_{hj} = 0 \qquad i \neq j;\ \left.\begin{matrix} i \\ j \end{matrix}\right\} = 1, \ldots\ldots, 3n \quad \ldots\ldots \quad (25)$$

But, from (24) and (25) it follows that *

$$m_h \sum_1^{3n}{}^i a^2_{hi} = 1 \qquad h = 1, \ldots\ldots, 3n \quad \ldots\ldots \quad (26)$$

and

$$\sqrt{m_h m_k}\sum_1^{3n}{}^i a_{hi} a_{ki} = 0 \qquad h \neq k;\ \left.\begin{matrix} h \\ k \end{matrix}\right\} = 1, \ldots\ldots, 3n \quad \ldots \quad (27)$$

* To see this readily, one introduces new quantities $j_{hi} = \sqrt{m_h} a_{hi}$.

Equations (26) give (see eq. 16) the relation

$$\sum_{1}^{3n} {}_i P^i_{hk} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

for each pair of indices h, k. *Therefore the* $3n$ *inequalities* (18) *can never be satisfied simultaneously, and the system postulated in* § 6 *cannot exist.*

We remark further, that the conclusion of this paragraph, and also equation (28), are independent of the frequencies p_i being different from each other, or some of them being equal.

§ 8. Equations (28) and (16) show farther: If equipartition exists for the momentoids $\dot{X}_1, \ldots \dot{X}_{3n}$, and all the frequencies p_i are different from each other, equipartition also holds for the momentoids $\sqrt{m_h}\dot{\xi}_h$.

For if

$$\overline{\{\dot{X}^2\}} = \overline{\{\dot{X}^2_2\}} = \ldots = \overline{\{\dot{X}^2_{3n}\}}, \quad . \quad . \quad . \quad (29)$$

i.e. if

$$p^2_1 A^2_1 = p^2_2 A^2_2 = \ldots = p^2_{3n} A^2_{3n}, \quad . \quad . \quad . \quad (30)$$

equations (16) and (28) give

$$\overline{\{m_1\dot{\xi}^2_1\}} = \overline{\{m_2\dot{\xi}^2_2\}} = \ldots = \overline{\{m_{3n}\dot{\xi}^2_{3n}\}} \quad . \quad . \quad . \quad (31)$$

The case in which not all the p_i are different from each other requires some complementary presumptions. Take, *e.g.*, $p_1 = p_2$. Then the relation (14) no longer suits for $i = 1, j = 2$. We have instead

$$\{\dot{X}_1\dot{X}_2\} = A_1A_2\cos\phi_{12} \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

where ϕ is the phase-difference between the two fundamental oscillations X_1, X_2, which remains constant during the free motion. In order to arrive at equation (16) again, we make the farther assumption regarding the distribution, that

$$\overline{A_1A_2\cos\phi_{12}} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

It is always fulfilled if the distribution referred to allows contrary phase differences to occur, with equal frequency, in the various molecules. But assumption (33), together with (29), then leads to equation (31) in the case of partly equal frequencies also. *Consequently it is possible to arrange equipartition for every structure of the vibrating molecules, between every set of momentoids.*

§ 9. The above discussion is intentionally confined to the two definite questions which were formulated in the introduction; for my intention in this connection was merely to refer to the restricting remarks which Boltzmann himself made on the H-theorem. On this account the remaining questions and objections which have recently been formulated in reference to the equipartition law can be left aside in the meantime.

(*Issued separately August* 28, 1907.)

XXI.—Preliminary Note on the Internal Structure of Sigillaria mamillaris, Brongniart, and Sigillaria scutellata, Brongniart.

By **R. Kidston**, F.R.S. L. & E., F.G.S., *Foreign Mem. Kaiserl. Mineral. Gesell. zu St Petersburg.*

(MS. received July 2, 1907. Read July 15, 1907.)

A FEW months ago I received from Mr G. H. Knott three specimens of *Sigillaria* from the Halifax Hard Bed of Yorkshire having the structure of their steles and outer cortex well preserved.

In two of these specimens the coal-ball containing the fossils so split that the outer surface of the cortex became exposed, exhibiting the leaf scars in a fine state of preservation, from which a specific identification of the two plants was easily made.

One of these was *Sigillaria mamillaris*, Brongt., and the other *Sigillaria scutellata*, Brongt. The third specimen from the structure of the stele is seen also to belong to *Sigillaria mamillaris*, Brongt.

Both these species belong to the *Rhytidolepis*, or ribbed section of the genus *Sigillaria*, of which group the internal structure was hitherto only known of *Sigillaria elongata*, Brongt.,* and *Sigillaria elegans*, Brongt.†

SIGILLARIA MAMILLARIS, Brongniart.

The stele of the specimen showing the outer cortex from which the specific identification was made has suffered from lateral pressure. The continuous xylem ring is fractured and the separated portions somewhat displaced. In certain parts of the stele an original tare in the specimen has separated the centripetal from the centrifugal xylem, but in other portions they are still in organic union. The xylem ring is 4 mm. wide, of which the primary xylem occupies about 1·50 mm. (fig. 1).

The primary xylem consists of a continuous band whose outer margin is distinctly crenulated, the crenulations having slightly flattened apices. These apices consist of the small protoxylem elements which are arranged at their widest part in three or four rows, the smallest tracheids being outer-

* Bertrand, *Annals of Botany*, vol. xiii., 1899, p. 607.

† Kidston, *Trans. Roy. Soc. Edin.*, vol. xli., p. 533, 1905, Pls. i.–iii.

most. These are succeeded inwards by large tracheids, which have a tendency to arrange themselves in irregular rows (fig. 1).

The secondary xylem consists of regular rows of tracheids of smaller size than those of the primary xylem, but they increase in size from within outwards, though they never attain to the dimensions of those of the primary wood. Passing out through the secondary xylem are the medullary rays.

If the structure of the primary wood of *Sigillaria mamillaris* be compared with that of *Sigillaria elegans*, their great similarity is very striking. The only point in the primary xylem by which the two species could be separated is the protoxylem groups, which in *Sigillaria mamillaris* contain

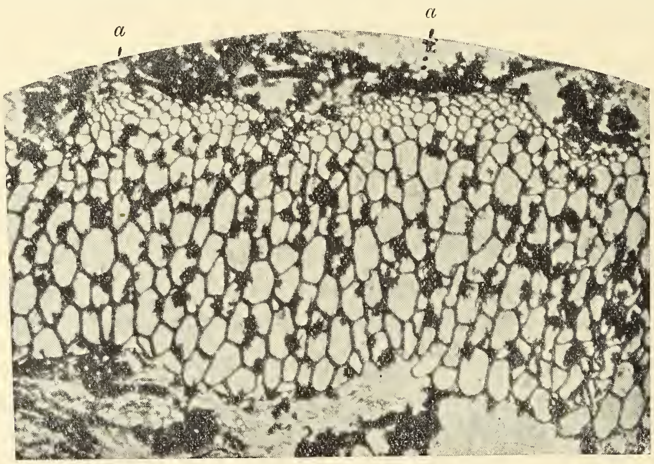


FIG. 1.—*Sigillaria mamillaris*, Brongt. Portion of primary xylem. *a*, protoxylem groups. ×30.

fewer elements and do not form such prominent projections as in *Sigillaria elegans*. Notwithstanding this great similarity of internal organisation in these two plants, the external form and arrangement of the leaf scars precludes the possibility of regarding them as having close specific relationship.

In the other example which I refer to *Sigillaria mamillaris*, but of which the outer surface was not seen, the matrix surrounding the specimen shows several transverse sections of Sigillarian leaves, some of which show a single vascular strand, whilst others mixed with them show the double strand, and are, in fact, the *Sigillariopsis* of Renault *; but it will be seen

* Renault, "Structure comparée de quelques tiges de la flore carbonifère," *Nouv. Archives du Muséum*, vol. ii., 2e sér., p. 270. Pl. xii. figs. 15–19 (*Sigillariopsis Decaisnei*); Pl. xiii. figs. 1–4 (*Sigillariopsis*, sp.), 1879. See also Renault, *Bassin houil. et perm. d'Autun et d'Épinac*, Flore foss., fasc. iv., deux. part, 1896, p. 245; Roche, "Biographie de Bernard Renault, avec extrait de ses notices scientifiques," *Mém. Soc. d'hist. nat. d'Autun*, vol. xviii., 1905, Pl. ii. figs. 11–15.

when describing the next species that these can only be regarded as the leaves of *Sigillaria*.

SIGILLARIA SCUTELLATA, Brongniart.

The specimen of this species is more perfect than that of *Sigillaria mamillaris*, the complete cylinder of the cortex being preserved, and the vascular axis, though compressed, remaining unbroken.

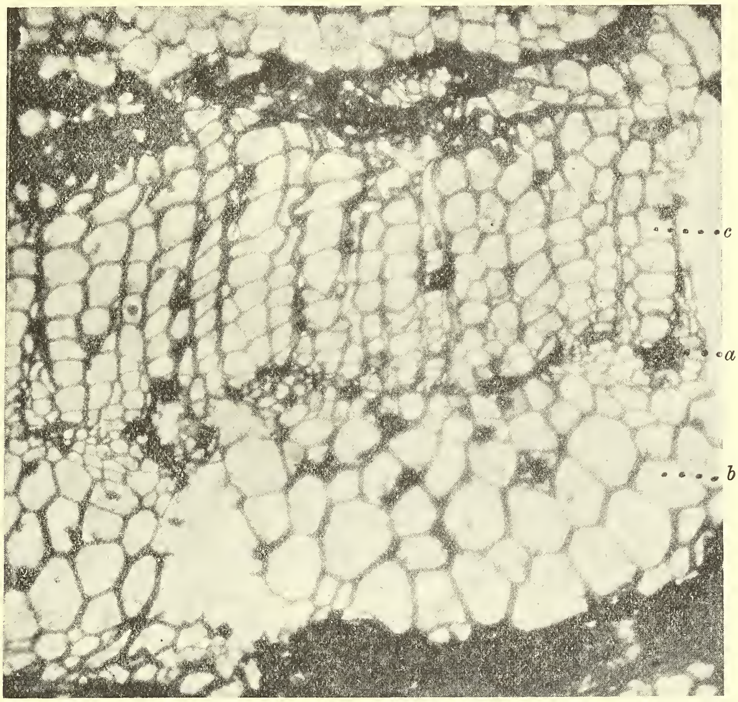


FIG. 2.—*Sigillaria scutellata*, Brongt. Portion of xylem ring. *a*, protoxylem of primary xylem *b*, secondary xylem *c* × 90.

The stem has a circumference of 15·50 cm., and contains twenty-two ribs, but the stele has only a circumference of about 2 cm. This specimen brings out very prominently the small size of the vascular axis when compared with the circumference of the stem. The same characteristic has been already observed in *Sigillaria elegans* and in the *Lepidodendraceæ*.

The closed ring of xylem in *Sigillaria scutellata* has only attained a width of about 0·90 cm., of which the primary xylem forms rather less than half.

The outer margin of the primary xylem is very feebly crenulated, and in some places it is difficult to separate the various groups of protoxylem

elements and to refer them to a separate wedge of xylem, a point so easy to determine in the other described *Sagillariæ*. The protoxylem elements are small and few in number (fig. 2, *a*), but the tracheids of the centrapetal xylem are large and irregularly placed, with little or no tendency to become arranged in lines (fig. 2, *b*).

The tracheids of the secondary xylem are smaller, though they gradually increase in size from within outwards, and are arranged in definite rows with their associated medullary rays (fig 2, *c*).

The occasional absence of a prominent crenulate margin to the primary xylem connects this type of stele structure with those of the *Lepidodendreæ* which do not possess a *corona*.

Scattered throughout the matrix are several transverse sections of leaves which sometimes show the double vascular trace of *Sigillariopsis*. In both the specimens described here, some leaves with a single, and some with a double, vascular strand occur, and it is almost impossible to doubt that these leaves belong to the Sigillarian stems which accompany them. On the presence of a double vascular strand in the leaves, Renault founded his genus *Sigillariopsis*, but it is evident that this genus can no longer be regarded as distinct from *Sigillaria*, seeing that *Sigillaria scutellata*, with which they occur—and, I doubt not in this case, is the plant to which they belong—is the type of the genus *Sigillaria*, Brongniart.

The other characters of the genus *Sigillariopsis* are similar to those of the *Clathrate Sigillariæ*, to which group Renault's type specimen belongs.

When, therefore, it is seen that both the *Clathrate* and ribbed *Sigillariæ* possess leaves with the structure of *Sigillariopsis*, the latter genus loses all individuality. It must be remembered that the leaf strand is not double throughout the whole of its course, and that only certain sections will show this character.

My thanks are due to Mr G. H. Knott for his kindness in placing these interesting specimens in my hands for description.

(*Issued separately August* 28, 1907.)

XXII.—Note on a New Species of Lepidodendron from Pettycur (Lepidodendron Pettycurense). By R. Kidston, F.R.S. L. & E., F.G.S., *Foreign Mem. Kaiserl. Mineral. Gesell. zu St Petersburg.*

(MS. received July 2, 1907. Read July 15, 1907.)

Among the undescribed species of plants occurring in the well-known material of Calciferous sandstone age found near Pettycur, Fife, is an interesting specimen belonging to the older type of Lepidodendroid structure which has a solid primary xylem.

Only two examples of this species are known to me; the larger, of which a portion is shown in the text figure, has a central stele 1·10 cm. by 0·90 cm. in diameter, and of this the solid primary xylem is 0·25 cm. by 0·15 cm. in size.

The slightly elliptical form of the axis is due to lateral pressure, as seen by the bands of flattened tracheids which occur in the primary xylem.

The axis of the smaller example is about 2·50 mm. by 2 mm. in diameter, of which the primary xylem is 1 mm. by 0·50 mm., but this specimen has also suffered from pressure.

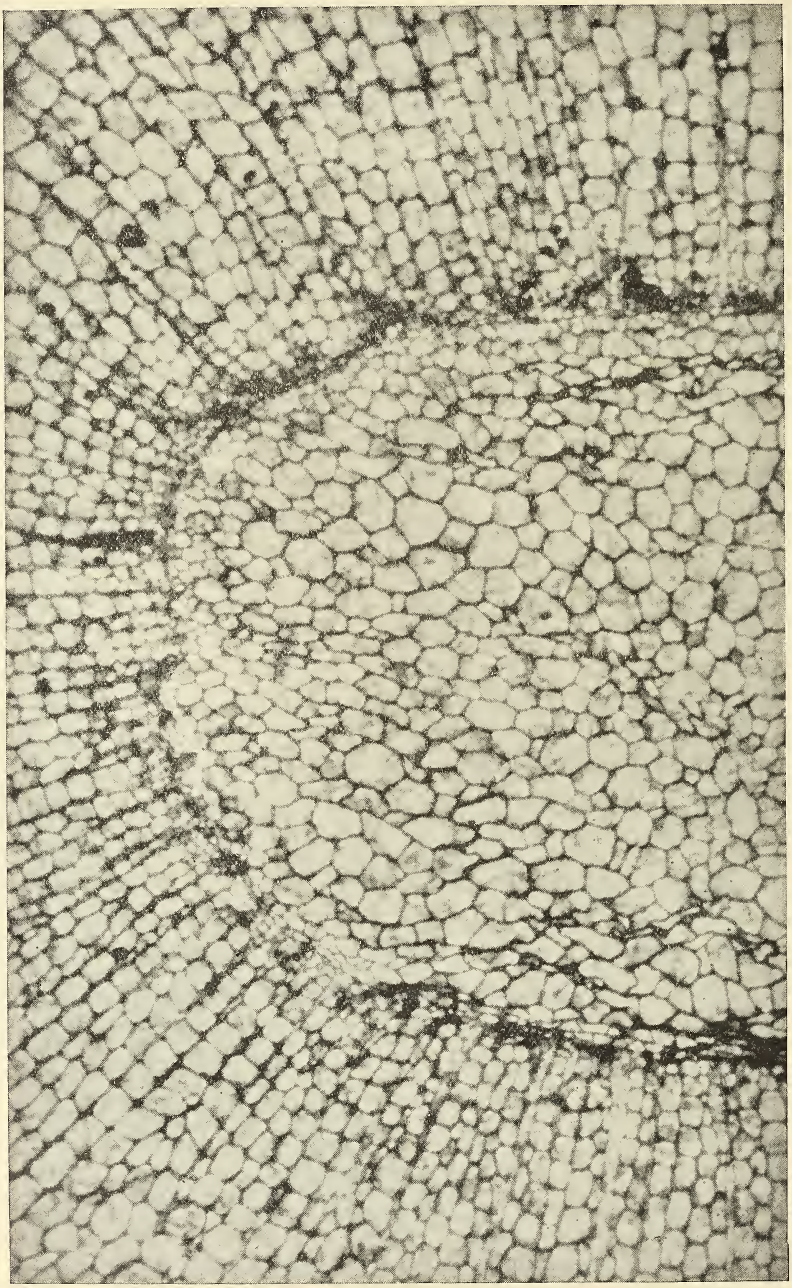
Returning to the specimen shown in the figure, as seen in transverse section, the primary xylem consists of a solid mass of tracheids, irregularly placed and varying in size, but the smaller tracheids are mixed with the larger without any order (*a*).

When close to the periphery of the primary xylem the tracheids become smaller and are surrounded by the protoxylem elements (*a′*), which form a narrow band, limiting the centripetal xylem, whose margin is smooth without the slightest trace of a corona.

The secondary or centrifugal xylem is formed of regular rows of tracheids which radiate outwards (*b*). Those next the protoxylem are of smaller size, but they increase regularly in diameter as they are traced outwards, until they equal or even exceed in size the largest tracheids of the primary xylem.

In longitudinal section the protoxylem elements and primary tracheids are very regular in their course, and bear scalariform thickenings. The tracheids of the secondary xylem which abut on the primary xylem are

slightly flexuous, but those immediately outside of them are very regular. They also show scalariform thickening.



Lepidodendron Pethycurense, Kidston. Transverse section of stele. *a*, primary xylem ; *a′*, protoxylem ; *b*, secondary xylem. × 60.

The leaf trace, as seen in transverse section in its passage through the secondary xylem, consists of a small oval mass of narrow tracheids, of

which a few are larger than the others, but these do not seem to hold a very definite position in the bundle.

Lepidodendron Pettycurense belongs to that section of Lepidodendron which possesses a solid stele, and which includes the culm species *Lepidodendron rhodumnense*, Renault, and *Lepidodendron saalfeldense*, Solms, though in neither of these has any secondary xylem been observed.

(*Issued separately September* 9, 1907.)

XXIII.—On the Application of a Differential Densimeter to the Study of some Mediterranean Waters. By John J. Manley, M.A., Daubeny Curator, Magdalen College, Oxford. *Communicated by* Sir John Murray, K.C.B. With an Appendix.

(MS. received June 6, 1907. Read June 24, 1907.)

From time to time various methods have been employed for the examination of samples of sea-water. Of these, the four which have been received most favourably by oceanographers are:—

(1) Estimation of the total combined chlorine per unit volume of water.

(2) Determination of the relative density.

(3) Measurement of the refractive index.

(4) Determination of the specific electric conductivity.

By any one of these methods it is possible to estimate variations in the relative proportions of the saline ingredients of any two samples of sea-water. In this paper it is, however, desired to deal chiefly with the practical application of the second of the methods enumerated above, to the study of some ninety samples of sea-water, collected during the outward and homeward bound voyages of the R.M.S. *Oruba*, under the supervision of Mr R. T. Günther, in the months of July and October 1905.

Hare's Apparatus.

An apparatus which is usually ascribed to Hare, but by Professor A. Grey to James Watt, embodies the principle elaborated in the several forms of densimeter used in this research.

Hare's apparatus may be described as a long-limbed inverted U-tube having a short side tube sealed into the uppermost portion of the bend; to this side tube a suitable length of rubber tubing, which can be closed by means of a pinchcock, is attached; the open ends of the U-tube dip into two glass cisterns.

To measure the density of a given liquid, one cistern must be charged with distilled water, the other with the liquid; by suction of the rubber tube, the liquids are raised to convenient heights within the limbs of the apparatus, and the pinchcock is then closed. Finally, the vertical heights

of the two columns of liquid are measured. As the densities of the two liquids are inversely as the heights of the two columns, we have—

$$\frac{h}{\mathrm{H}} = \frac{\mathrm{D}}{d},$$

in which h and H are the respective heights of the columns of distilled water and liquid, and d and D their corresponding densities.

In its primitive form, the apparatus suffers from several defects, and these render even an approximately accurate density-determination extremely difficult, if not an impossibility. We observe (1) that no attempt is made to keep the contents of the tubes at the same temperature. (2) The air enclosed in the upper part of the apparatus may, and generally does, vary in temperature during the course of measurement; any such variations cause the liquid columns to alter in height, thus leaving the observer in doubt as to their true values. (3) In order to measure the heights of the columns, readings must be taken (preferably by means of a telescope and vertical scale, the latter being set up between the two columns) for the surface of the liquid in each cistern, in addition to those for the meniscus in each tube, or four readings in all. Hence, in addition to a certain inexactness, the method becomes quite laborious when a number of measurements are undertaken. (4) Finally, we note that the limbs of the apparatus are generally too short for accurate work.

The Differential Densimeter.

In order to minimise the several errors inherent in Hare's original apparatus, and also to reduce the labour of experimenting, some six or seven modifications were constructed and tested. After numerous experiments, four of these proved unsuitable, and were consequently rejected. In each of the rejected forms, an attempt was made to introduce some kind of constant-level overflow cistern, but without success. A form of apparatus yielding values which were in close agreement with those obtained by means of pyknometers is shown in fig. 1.

The apparatus consists of two glass tubes S, K, each having an internal diameter of about 3 mm. and a length of 1200 mm.; these tubes, having obliquely perforated taps t, p, are sealed to the bulbs b, b; each bulb has a capacity of about 25 c.c., and their upper ends are joined by an inverted U-tube into the bend of which a three-way tap T is sealed. One of the two free branches of this tap communicates with an exhausted copper globe E, having a capacity of 4·2 litres; the other communicates with a suitable

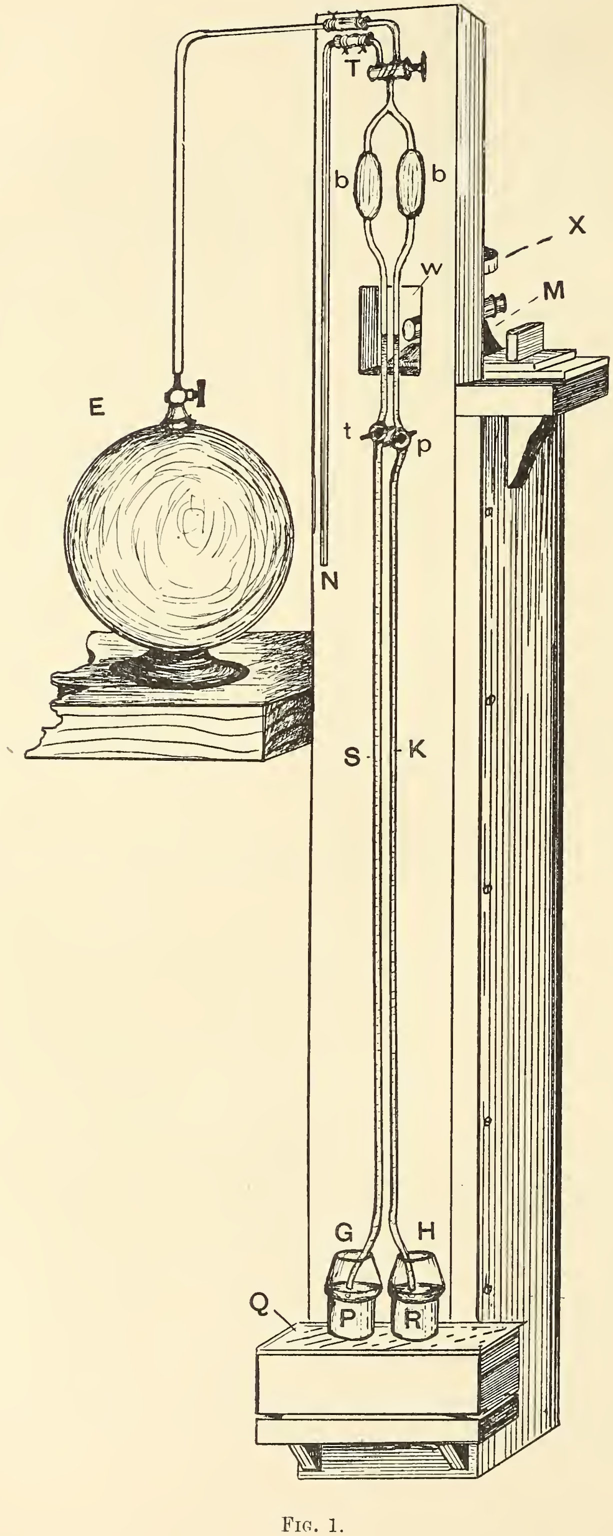


Fig. 1.

length of fine-bore thermometer tubing N. Finally, the open ends of the tubes enter the cisterns G, H, which are placed upon the supports P, R. These supports were made from two stout glass jars 8 cms. high; their bases and mouths were ground plane and parallel, and their heights adjusted until they were nearly equal. The mouths of the jars were closed by glass plates cemented on with marine glue. The use of blocks of wood or metal would be liable to introduce errors, since the former might become more or less distorted, and the latter corroded, by the unavoidable contact with sea-water; glass is free from these objections, The supports P, R occupy marked positions upon a slab Q, of plate-glass, cemented to the upper surface of a well-formed brick, which may be levelled by means of the set screws shewn below.

The two cisterns were cut from Erlenmeyer flasks of Jena glass, having a capacity of 200 c.c. These were chosen on account of the ease with which two flasks having the same diameter, to within one or two tenths of a millimetre, may be obtained.

The whole apparatus was secured to a stout and well-seasoned board of Kauri pine set up rigidly. A window, W, 10 cms. high, was cut in the board, and this had its centre approximately 110 cms. above the glass plate Q. Immediately behind and just below the window, a wooden bracket covered with plate-glass was fastened; this was used for supporting a travelling microscope M, to which a vertical movement could be imparted by a micrometer screw X, capable of indicating ·001 mm. The carriage of the microscope rested upon, and was cemented to, a glass plate as recommended by Quincke.

In order to secure uniformity of temperature for corresponding points, the tubes throughout their entire lengths were brought close together. For some experiments they were wrapped in cotton-wool, whilst for others they were left uncovered; but the best plan was found to be that in which the tubes are jacketed with water, the jacket itself being surrounded with a wooden cover (fig. 2).

The error due to any changes in the temperature of the air enclosed in the upper part of the apparatus is eliminated by the introduction of the taps t, p, which are simultaneously closed immediately before observations of the heights of the water columns are commenced.

The accuracy of the method is largely increased by the use of columns of ample length, the heights of which are determined with the micrometer.

In measuring the lengths of the two columns of water, two only, instead of four readings, are required. This desirable simplification was secured by the use of the conical cisterns above mentioned. The diameter of these at

their widest part was practically 7·4 cms.; from this we find the corresponding cross-sectional areas to be 43 square centimetres. By trial it was found that 70 c.c. of water were sufficient to fully charge one of the tubes and also its cistern to a level coincident with its greatest diameter. Now let the cisterns be charged, the one with 70 c.c. of distilled water and the other with an equal volume of sea-water; further, let the density of the sea-water be so great that the total difference in the lengths of the two columns is equal to 50 mm. A difference so large as this would, with tubes having an internal diameter of 5 mm., be equal to a volume of 1 c.c.; this would cause the waters in the cisterns to deviate slightly from a common level; and from the data given above it can be shown that the deviation would be 0·2 mm.; hence, to obtain the true value of the density, it would be necessary to allow for this. As, however, the instrument is chiefly intended to be used differentially, a standard sea-water of known density taking the place of the distilled water, the differences in the heights of the water columns will generally be quite small, and need never exceed 5 mm. The latter value would reduce the correction from 0·2 mm. to 0·02 mm., and this would only affect the density determination by 2 in the fifth decimal place. So small an error may be safely neglected.

After the apparatus had been set up, it was cleaned by successively charging the tubes with fuming nitric acid, distilled water, strong alcoholic solution of potash, and, lastly, with distilled water: each liquid was kept in the tube for a considerable time. It is believed that the glass was thus very thoroughly freed from foreign matter.

The tubes having thus been cleaned, one of the cisterns was charged with 70 c.c. of water, some of which was then drawn up into the attached tube and adjusted by trial until the column of water was 1000 mm. long, as determined by a cathetometer. The cross threads of the microscope M were then made to coincide with the meniscus, and a reading taken. In this way it was discovered that the micrometer reading corresponding to a column of water 1000 mm. long was 42·00 mm., this being the mean of a number of determinations. In order to ascertain the length of a column of water at any other time, it is only necessary to determine how much it differs from this standard reading.

Determination of the Density of a Standard Sea-Water.

In order to measure the density of a water which is subsequently to be accepted as a standard, the following method of procedure was adopted.

The cisterns were placed in their normal positions and charged, the

one with 70 c.c. of distilled water, and the other with an equal volume of the sea-water. By means of the tap T, communication is then established between the globe E and the tubes S, K. As soon as the waters have been raised within the tubes to a point somewhat above that desired, T is turned so as to allow air to enter the apparatus through the tube N; when the length of the column of distilled water is approximately equal to 1000 mm., T is closed and the apparatus left undisturbed for four or five minutes. After this the taps *t*, *p*, are simultaneously closed, and the height of each column of water measured with the micrometer.

With the view of acquiring confirmatory evidence, a second determination was effected as follows. The taps *t*, *p*, were first opened, and then air was admitted through N by turning T; when the water columns had suffered a depression of from 1 to 3 mm., T was closed; the tubes having been allowed to drain for about a minute, *t*, *p*, were also closed, and the new heights of the columns observed as before. Proceeding in this way, a series of three or four sets of readings was obtained.

At the conclusion of the first series, the lengths of the two water columns were increased to approximately their former values by suitably turning T; after this, a new series of observations was undertaken in precisely the same manner as that just described.

If we may assume that the water surfaces in the two cisterns are strictly equidistant from the cross threads of the micrometer, we possess all the necessary data for calculating the density of the sea-water. In practice it is somewhat difficult to adjust the apparatus so that this assumption may be sufficiently true. To overcome this defect, and at the same time to eliminate other slight incidental errors, the two waters were interchanged, sea-water filling the tube which formerly contained distilled water. New readings were then made, from which independent values for the density of the water were calculated. The final mean of all the series should be the true value of the density sought.

It may be observed that this method of using the densimeter is analogous to that of reversed weighing introduced by Gauss.

The data given in Table I. will serve to illustrate the preceding remarks.

[TABLE

Table I.—Density of a Water by the Method of Reversal.

Series.	Set of Obsers.	Micrometer Readings. Tube S. Distilled Water. Height = H.	Micrometer Readings. Tube K. Sea-Water. Height = h.	Difference.	Relative Density. $\Delta = H/h$.
I.	No. 1.	1320·16 mm.	1283·18 mm.	− 36·98 mm.	1·0288
	" 2.	17·66 "	80·87 "	− 36·79 "	7
	" 3.	13·84 "	77·11 "	− 36·73 "	8
II.	" 4.	1315·81 "	1278·87 "	− 36·94 "	9
	" 5.	14·79 "	77·92 "	− 36·87 "	9
	" 6.	13·83 "	77·06 "	− 36·77 "	8
				Mean =	1·0288
After interchanging waters.		Sea-Water. Height = h.	Distilled Water. Height = H.		
III.	" 7.	1283·79 mm.	1318·90 mm.	− 35·11 mm.	1·0274
	" 8.	80·52 "	15·68 "	− 35·16 "	4
	" 9.	78·82 "	13·94 "	− 35·12 "	4
IV.	" 10.	1284·01 "	1319·20 "	− 35·19 "	4
	" 11.	81·46 "	16·64 "	− 35·18 "	5
	" 12.	76·51 "	11·55 "	− 35·04 "	5
				Mean =	1·0274
				Mean of means =	1·0281
				By pyknometer Δ_{18} =	1·0279
				Diff. =	·0002

The results tabulated above were among the first obtained with this particular form of the densimeter. With a fuller knowledge of the instrument, it was found possible to increase the standard of accuracy considerably, the means of the density values as determined by the densimeter and pyknometer differing by not more than $\pm$ ·00005, or 1 in 20,000 (*vide infra*).

Now, although the density of a sea-water may thus be measured with a high degree of accuracy, it is quite obvious that the method is both slow and irksome—not only because some 60 or 70 revolutions must be imparted to the micrometer screw during each set of observations, but also because of the time required for reversing the waters in the cisterns and tubes. Hence, for routine work, the ordinary process of using the instrument should be discarded for the method of differences, as set forth in the next section.

The Densimeter used Differentially.

Before the instrument can be used differentially, its error must be determined. This may be done by introducing into each cistern 70 c.c. of any given sea-water, and charging the tubes; the lengths of the water columns are then adjusted, and micrometer readings taken as already described. If, now, the two cisterns have the same form and capacity, and rest upon the same horizontal plane, and if, also, the two tubes are of the same diameter, then the micrometer readings for both tubes will be identical. In practice, the readings will differ somewhat, because it is so difficult to attain to strict equality. We therefore content ourselves by approximating to the ideal conditions, and then determine any difference as outlined above, and allow for it subsequently.

We are now in a position to measure the density of a water by the differential method. A sea-water is selected, and its density determined by the method of reversal (page 216); this water then becomes the standard with which others may be compared. The cisterns are then charged, the one with the normal volume of the standard water, and the other with an equal volume of the water to be examined. After observing the usual preliminaries, micrometer readings are taken, and from these the difference in the lengths of the two water columns is found, due allowance being at the same time made for the densimeter error; finally, the density of the water is calculated.

The following examples will, it is hoped, make this clear:—

Table II.—Determination of Densimeter Error.

70 c.c. of the same sea-water in each cistern.

Exper.	Micrometer Readings.		Diff. for Tube K.
	Standard Tube S.	Comparison Tube K.	
1.	42·95 mm.	43·14 mm.	+0·19 mm.
2.	39·95 "	40·14 "	+ ·19 "
3.	36·92 "	37·13 "	+ ·21 "
4.	33·78 "	33·98 "	+ ·20 "
5.	30·85 "	31·04 "	+ ·19 "
6.	27·86 "	28·06 "	+ ·20 "
		Mean =	+ ·20 mm.

Table III.—To illustrate the Method of Determining the Density of a Water, R.

Density, Δ_{18}, of standard sea-water χ = 1·02922.

Exper.	Microm. Readings. Water χ in Tube S.	Height of Column = H.	Microm. Readings. Water R in Tube K.	Height of Column (corrected) = h.	Density = H/h × Δ_{18}.	Diff. from Mean.
1.	42·80 mm.	1000·80 mm.	44·32 mm.	1002·12 mm.	1·02786	− ·00004
2.	39·80 "	997·80 "	41·25 "	999·05 "	93	+ 3
3.	36·87 "	994·87 "	38·35 "	996·15 "	90	± 0
				Mean =	1·02790	− 7
				By pyknometer Δ_{18} =	1·02797	

In general practice, a simplified method of calculating the density from the data may be adopted. For, if the difference between the lengths of the two columns of water does not exceed 2 or 3 mm., on adding it to, or subtracting it from, the density value of the standard water, according as the water under examination has a greater or less density than the other, we at once obtain a very close approximation to the value of the density sought. In other words, when the densities of the two waters are nearly the same, the difference in the lengths of the columns is practically equal to the difference in their respective densities. We may here observe that a difference of 1 mm. in the micrometer reading indicates a change of ·001 in the value for the density.

By applying this method to the data obtained for the water described in Table III., we obtain the results shown in Table IV.

Table IV.—Example illustrating the Differential Method.

Density of standard water χ = 1·02922 at 18° C.

xper.	Water χ in Tube S.	Water R in Tube K.	Apparent Difference.	Error for Tube K.	Corrected Difference.	Density Found.
1.	42·80 mm.	44·32 mm.	+1·52 mm.	+0·20 mm.	+1·32 mm.	1·02790 (1·02922 − ·00132)
2.	39·80 "	41·25 "	+1·45 "	"	+1·25 "	7 (" ·00125)
3.	36·87 "	38·35 "	+1·48 "	"	+1·28 "	4 (" ·00128)
						1·02794
					By pyknometer =	1·02797
					Diff. =	·00003

If we compare the values set forth in Tables III. and IV., it will be at once apparent that the simpler method of calculating the density yields results which, in point of accuracy, are probably amply sufficient to meet the demands made by oceanographers. All the values for $\Delta_{18^\circ \mathrm{C.}}$ given in Tables V. and VI. were calculated by this method.

In Table V. are given the densities of thirty samples of sea-water as determined with the aid of the densimeter having the form described above; side by side with these, the corresponding pyknometer values are also noted.

Table V.—Densities, etc., of Waters collected between Cape St Vincent and Cape Palos, July 1905.

No. of Water.	Δ_{18} by the pyknometer.	Δ_{18} by the densimeter.	Difference.	Grams of Cl per litre.	4th July.	Position of Ship.
1.	1·0276	1·0276	± ·0000	20·651	5 p.m.	
2.	7	7	± 0	·651	6 "	
3.	8	7	− 1	·756	7 "	
4.	9	6	− 3	·686	8 "	
5.	8	8	± 0	·756	9 "	
6.	9	8	− 1	·756	10 "	
7.	8	8	± 0	·756	11 "	
8.	7	4	− 3	·615	Midnight.	Off Trafalgar.
					5th July.	
9.	7	6	− 1	·615	1 a.m.	
10.	8	6	− 2	·721	2 "	Off Tarifa Pt., 5th July, 4.42 a.m.
11.	9	7	− 2	·721	3 "	
12.	9	7	− 2	·721	4 "	Off Carnero Light, 5.27 a.m.
13.	1·0285	1·0285	± 0	21·215	11 "	
14.	4	3	− 1	·144	Noon.	
15.	1·0279	1·0278	− 1	20·792	1 p.m.	
16.	9	9	± 0	·792	2 "	
17.	8	7	− 1	·756	3 "	
18.	1·0282	1·0280	− 2	·968	4 "	
19.	2	2	± 0	21·004	5 "	
20.	7	Omitted.	...	·604	6 "	
21.	9	1·0286	− 3	·533	7 "	
22.	9	8	− 1	·533	8 "	
23.	9	9	± 0	·533	9 "	
24.	1·0299	1·0298	− 1	22·310	10 "	
25.	93	93	± 0	21·780	11 "	
26.	92	89	− 3	·674	Midnight.	
					6th July	
27.	91	91	± 0	·674	1 a.m.	
28.	1·0309	1·0307	− 2	22·857	2 "	
29.	1·0317	Omitted.	...	23·686	3 "	
30.	9	"	...	·686	4 "	Off Cape Palos, 6th July, 4 a.m.
		Mean diff.	= − ·0001			

On the Occasional Abnormal Behaviour of the Densimeter.

After the apparatus had been in use for some time, the density values, when compared with those derived from pyknometer measurements, became incorrect. The discrepancies, though small at first, would frequently increase quite suddenly and cause a maximum difference of about ±·002 in the density value. Numerous experiments proved that this error could not be due to any defect in the design of the instrument, but was the result of some change in the condition of the interior surface of one or both of the glass tubes. A certain oily appearance could sometimes be detected. At first it was thought that the cause might be some growth, similar to that which has been observed by Dr Veley in burettes which have been in use for some time. This theory received some support from the fact that the densimeter gave perfectly normal values after it had been recleaned. The improvement was not, however, maintained by allowing any air that passed into the apparatus to filter through cotton-wool; we may therefore safely conclude that the abnormal condition of the tubes was not due to a growth of bacteria introduced from this source. Further, the tubes occasionally assumed a decided oiliness after they had been cleaned, so rapidly as to suggest that the trouble was due not to any organic growth, but to some volatile constituent of the vaseline used for lubricating the taps.*

With the object of verifying this, all vaseline was removed, the apparatus cleaned as already described, and finally steamed out. In some experiments honey, and in others glycerine, was employed as a lubricant; these, as a rule, produced a decided improvement, but sooner or later the glass tubes again became affected. The last possibility was that the oiliness might be due to some volatile emanation from the black vulcanised rubber connections at the top. This possibility was put to the following test. A flask was made chemically clean by treating it with a mixture of ethyl-alcohol and fuming nitric acid, and repeatedly rinsing with distilled water; it was then partly filled with distilled water; whenever the flask was shaken, the water wetted the upper parts and then drained in a perfectly normal manner. A short length of the rubber tubing was now introduced; a very

* Mr Günther suggested that bacteria were already present in the waters. This, however, appears improbable; for it was found that, after the india-rubber tubes had been purified and the use of vaseline discarded, another portion of the water with which the tubes were charged when much trouble had been experienced, could be kept in the apparatus for three or four weeks without producing any ill effects; micrometer readings taken at various times during that period were always normal.

It may also be added that, although the author has examined many hundreds of samples of sea-water, he has never observed the development of any similar oiliness within the containing bottles.

marked oiliness developed inside the flask almost immediately. Hence the rubber tubing was obviously the chief cause of the abnormal condition of the instrument.

The rubber tube connections were now removed and treated with moist chlorine gas, then boiled with potash solution to which a little hydrogen peroxide had been added, and finally soaked for some hours in water, dried without heating, and replaced in their former positions. The densimeter then behaved quite normally for a considerable period, and then became a little unsatisfactory; but after re-treating the rubber tubes no further trouble was experienced.

It was found that the rubber tubes might be freed from the volatile matter by boiling them with potash and hydrogen peroxide alone, or even by soaking them for some weeks in distilled water. Experiments were also made with grey rubber pressure tubing; these showed that such tubing might be safely used without subjecting it to treatment.

These observations are also of importance in connection with various forms of apparatus used for gas analysis.

Portable Form of Densimeter.

The densimeter already described is well adapted for laboratory use, but for work on board ship certain modifications are desirable.

The somewhat rough usage to which the instrument would be exposed at sea might lead to injury. Again, the arrangement of the cisterns leaves something to be desired in the matter of convenience, though nothing in point of accuracy. Chiefly for these reasons a new apparatus, having the form indicated in fig. 2, was constructed.

In this instrument the long glass tubes are not fused to the headpiece, but are attached by means of grey pressure tubing. The portions of the tubes above the taps t, p, have an internal diameter of 5 mm., and their ends are bent a little outwards, so as to enable an observer to scrub the interior of the wider portions of the tubes with a small brush. Below t, p, the tubes are quite narrow, the internal diameter being about 2 mm. only. The cisterns have the same size as those used in the other form of the apparatus, but side tubes are sealed into them at the lowest possible point, and these are joined to the vertical tubes by purified black rubber tubing.

When *in situ*, the cisterns rest side by side upon one and the same levelled plate-glass shelf S. To prevent evaporation of the water, the mouths of the flasks are closed by loosely fitting glass stoppers with flat tops.

This apparatus is both portable and convenient; it possesses the

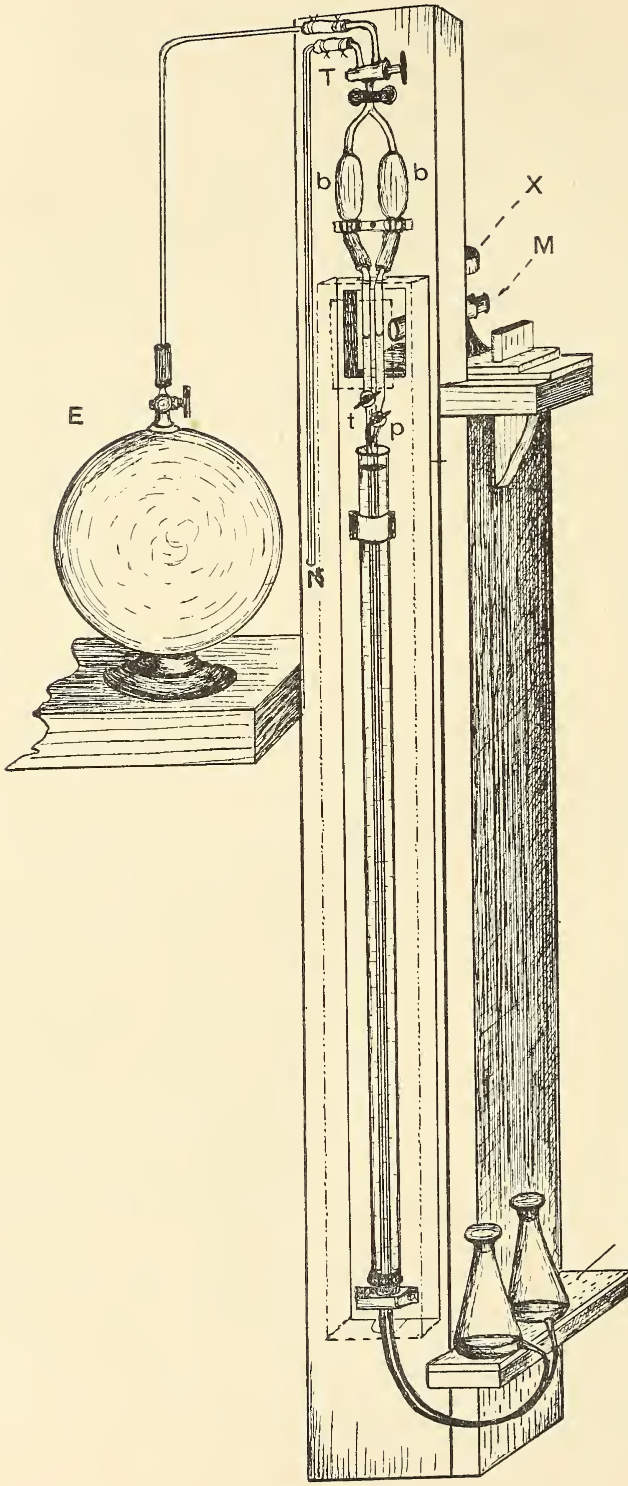


Fig. 2.

requisite flexibility, and any damage it may suffer can be at once repaired by the substitution of a duplicate for the broken part. The cisterns can be emptied, rinsed, and drained quite easily without detaching them from the vertical tubes.

In order that the instrument might be capable of rendering the fullest service on board ship, it would probably be necessary to suspend it in gimbals. The plane containing the knife-edges of the gimbals should be as near as may be convenient to the horizontal plane passing through the axis of the reading microscope. It would also be necessary to load the lowest extremity with, say, a moderately large block of lead.

The instrument, having been calibrated in the manner previously described, was used to measure the densities of the remaining sixty samples of sea-water.

In Table VI. are set forth the values obtained with (*a*) the pyknometer at 18° C., and (*b*) the densimeter, with their differences; the corresponding amounts of combined chlorine per litre of water are also tabulated.

Table VI.—Densities, etc., of Waters collected between Marseilles and Cape Finisterre, October 1905.

No. of Water.	Δ_{18} by the pyknometer.	Δ_{18} by the densimeter.	Differences.	Grams of Cl per litre.	Date.	Time.	Temp.	Position of Ship.
31.	1·02919	1·02914	− ·00005	21·710	2nd Oct. 1905.	8 a.m.	62°	
32.	10	02	− 8	·639		Noon	64°	
33.	45	34	− 11	·921		4 p.m.	63°	
34.	1·03050	1·03044	− 6	22·663		8 "	64°	
35.	·02889	·02885	− 4	21·533		Midnight	64°	
36.	·03133	·03123	− 10	23·333	3rd Oct.	4 a.m.	63°	6.44 a.m., 5 miles off C. Palos.
37.	·02873	·02870	− 3	21·374		8 "	70°	
38.	821	817	− 4	20·933		Noon	63°	1.41 p.m., 5½ miles off C. Gata.
39.	819	809	− 10	·933		4 p.m.	66°	3.52 " 9 " C. Sabinal.
40.	800	799	− 1	·792		8 "	64°	
41.	·02933	930	− 3	21·745		Midnight	63°	
42.	·02821	822	+ 1	20·968	4th Oct.	4 a.m.	60°	5.35 a.m., 1 mile off C. Europa.
43.	·02773	774	+ 1	·615		8.30 "	64°	8.13 " off Carnero Light.
44.	73	773	± 0	·580		8.45 "	63°	
45.	73	766	− 7	·580		9.0 "	62°	
46.	79	Insufficient water.		·580		9.15 "	62°	9.21 a.m., 2 miles off Tarifa.
47.	72	1·02775	+ ·00003	·580		9.30 "	63°	
48.	74	79	+ 5	·615		9.45 "	66°	
49.	72	83	+ 11	·615		10.0 "	67°	
50.	73	81	+ 8	·615		10.15 "	66°	10.40 a.m., course altered; 35° 58′ N. lat.
51.	68	72	+ 4	·615		10.30 "	66°	5° 52′ W. long.
52.	67	71	+ 4	·615		10.45 "	66°	
53.	74	77	+ 3	·615		11.0 "	67°	
54.	70	75	+ 5	·615		11.15 "	66°	11.24 a.m., off Trafalgar.
55.	68	71	+ 3	·580		11.30 "	66°	
56.	72	72	± 0	·615		11.45 "	68°	

Table VI.—*Continued.*

No. of Water.	Δ_{18} by the pyknometer.	Δ_{18} by the densimeter.	Differences.	Grams of Cl per litre.		Time.	Temp.	Position of Ship.
57.	1·02770	1·02769	− ·00001	20·608		Noon	68°	
58.	69	772	+ 3	·580		12.15 p.m.	68°	
59.	69	774	+ 5	·580		12.30 "	68°	
60.	66	769	+ 3	·580		12.45 "	68°	
61.	70	775	+ 5	·580		1.0 "	68°	
62.	73	770	− 3	·651		1.15 "	68°	
63.	97	794	− 3	·792		1.30 "	68°	
64.	68	769	+ 1	·615		1.45 "	69°	
65.	98	792	− 6	·825		2.0 "	69°	
66.	97	802	+ 5	·825		2.15 "	70°	
67.	1·02802	797	− 5	·825	4th Oct.	2.30 "	70°	
68.	06	803	− 3	·862		2.45 "	70°	36° 20′ N. lat. 7° 5′ W. long.
69.	02	795	− 7	·825		3.0 "	71°	
70.	06	797	− 9	·862		4.0 "	72°	
71.	07	801	− 6	·862		5.0 "	71°	
72.	1·02792	793	+ 1	·756		6.0 "	68°	
73.	80	772	− 8	·668		7.0 "	66°	
74.	93	787	− 6	·756		8.0 "	66°	
75.	67	761	− 6	·545		9.0 "	62°	3 miles S. of Sagres (C. St. Vincent).
76.	48	747	− 1	·439		10.0 "	59°	
77.	53	750	− 3	·545		11.0 "	60°	
78.	54	750	− 4	·456		Midnight	62°	
79.	60	760	± 0	·545		1.0 a.m.	63°	
80.	54	751	− 3	·474		2.0 "	62°	
81.	58	753	− 5	·509		3.0 "	62°	
82.	47	738	− 9	·439		4.0 "	59°	
83.	55	749	− 6	·474		5.0 "	58°	
84.	57	755	− 2	·509	5th Oct.	6.0 "	61°	
85.	43	732	− 11	·403		7.0 "	59°	
86.	44	734	− 10	·439		8.0 "	60°	
87.	48	741	− 7	·439		9.0 "	62°	
88.	52	749	− 3	·474		10.0 "	62°	
89.	1·02807	798	− 9	·862		11.0 "	63°	
90.	1·02742	729	− 13	·403		Noon	62°	

On the Effect of Variations in Temperature.

(*a*) *Normal Ocean Waters.*

We have now to consider an important point, namely, the effect of variations in the temperature of the two waters in the densimeter upon their relative density.

To maintain tubes of more than 1 metre in length at some standard temperature is a matter of no little difficulty, and probably on account of this, Hare's apparatus has not been used more extensively. When, how-

ever, the instrument is used differentially, and the two waters are of almost the same salinity and density, this difficulty is obviated; their coefficients of expansion will be practically identical, and consequently the ratio of the two densities will remain almost the same for all ordinary changes of temperature. Therefore, if we so arrange the apparatus that any two adjacent points along the tubes may assume the same temperature, all the necessary requirements will be fulfilled. Further, the temperature of one pair of adjacent points may differ within fairly wide limits from that of any other pair, without appreciably affecting the values obtained. In support of these statements the following experimental evidence may be adduced.

The relative densities of two sea-waters, marked θ_1, θ_2, were determined by means of a pyknometer at the temperatures of 12°, 16°, and 20° C., and their ratios computed. The results obtained are set forth in Table VII.

Table VII.—Showing Effects of Variations in Temperature upon the Relative Densities of Normal Waters.

Temperature.	θ_1.	θ_2.	Ratio δ/Δ
12° C.	δ = 1·02712	Δ = 1·02945	0·99774
16° C.	" 1·02684	" 1·02915	6
20° C.	" 1·02661	" 1·02889	8
Cl per litre	19·994 grms.	21·734 grms.	

Thus, for a difference of each degree within the limits of the temperatures named, the change in the measured value of the density would not be greater than 1 in 200,000. We also observe that, should the temperature of the laboratory differ from any standard temperature by ± 8° C., the value obtained for the density will still be true to within ± ·00005; in other words, for an extreme range of 16° C. above or below a given standard temperature the maximum error introduced from this source would amount to only ± ·0001.

In his report upon the specific gravity of ocean water, Mr Buchanan drew attention to the fact that for all true ocean waters the densities vary within the extreme limits of 1·0278 and 1·0240.* The difference between these two extremes is nearly twice as great as that in the case of the

* *Challenger Reports*, vol. i., part 2, p. 1.

two waters cited above; from which it follows that for the greatest possible difference both in density and temperature, the maximum error introduced by neglecting the temperature correction ($\pm$ ·000006 per 1° C.) would be equal to $\pm$ ·0002. It would, however, seldom happen that the two maximum differences would conspire to produce this effect; and in such a case the observer could hardly fail to be aware of the fact, and would apply the necessary correction.

(*b*) *Abnormal Ocean Waters.*

To investigate the effects produced by fluctuations in the temperature of a water which is either exceptionally rich or poor in chloride, supplemental experiments were made. Five waters marked $K_1 \ldots K_5$ were chosen; of these, two were diluted, two others were concentrated over a water-bath, whilst the fifth was kept in its natural condition. Their approximate chlorine values were respectively 10, 14, 20, 25, and 30 grms. per litre (the precise values are given in Table VIII.). The densities of all five waters were then determined at the temperatures of 10°, 20°, and 30° C.; and in order to eliminate as far as possible slight incidental errors, all the measurements were made with the same pyknometer. In calculating densities, that of pure water at 4° C. was taken as unity.

From the data thus obtained, the coefficient of expansion for each water within the above-named limits of temperature was found, and expressed in the form

$$V_t = V_0\{1 + \alpha t + \beta t^2\},$$

V_0 being the volume at 10° C., and V_t the volume at t° C. above 10.

The various values for the densities of the several waters, the corresponding values for the terms α and β in the above formula, and the weights of chlorine per litre, are set forth below:—

Table VIII.—Data from which α and β in the Expression $V_t = V_0\{1 + \alpha t + \beta t^2\}$ were Obtained.

Water.	$\Delta_{10°}$	$\Delta_{20°}$	$\Delta_{30°}$	$\alpha \times 10^6$.	$\beta \times 10^8$.	Grms. Cl per litre.
K_1	1·01359	1·01170	1·00889	+ 134	+ 500	9·997
K_2	·01948	·01743	·01450	+ 153	+ 465	14·322
K_3	·02712	·02489	·02187	+ 179	+ 390	20·065 { Natural water.
K_4	·03369	·03132	·02815	+ 193	+ 380	24·957
K_5	·04031	·03777	·03450	+ 212	+ 343	29·991

Accepting the density of the natural water, K_3, as a standard, and taking the differences between it and those of the other four waters, we can show what possible changes may be produced in the value for the relative density of an abnormal water, when the temperature is allowed to fluctuate within somewhat exceptionally large limits. By adopting this plan, and denoting the density of the water K_3 at any temperature by ρ, and the density of any of the others at the same temperature by Δ, the results given in Table IX. are obtained.

Table IX.—Showing Effects Produced upon the Relative Densities of Abnormal Waters by Extreme Variations in Temperature.

Water.	Differences between			Deviations from Δ_{20}.	
	ρ_{10} and Δ_{10}.	ρ_{20} and Δ_{20}.	ρ_{30} and Δ_{30}.	At 10° C.	At 30° C.
K_1	−·0135	−·0132	−·0130	+·0003	−·0002
K_2	−·0076	−·0075	−·0074	+ 1	− 1
K_4	+·0066	+·0064	+·0063	+ 2	− 1
K_5	+·0132	+·0129	+·0126	+ 3	− 3

This table brings out very clearly the somewhat singular but important fact that, notwithstanding the considerable variations both in the temperature and chlorine values of the several waters, the error introduced by determining a relative density by means of the differential densimeter would, even in the most extreme cases, have amounted to no more than ± ·0003 for a range of 10° C. on either side of the standard temperature of 20° C.

By referring to the temperature coefficients in Table VIII., we may discern that the reason for the insignificance of these deviations, except in the case of the waters K_1 and K_5, is that the temperature coefficients are not greatly dissimilar; hence any increase or decrease in the volume of the waters, consequent upon a change in temperature, will be nearly identical for all.

The close similarity between the coefficients of expansion for the five waters is well shown by the curves given in fig. 3. The various volumes of the several waters are taken as ordinates and plotted against the corresponding temperatures as abscissæ, the unit volume being in all cases taken at 10° C. For the sake of comparison, a similar curve for pure water has also been drawn.

It may be observed that the curvature of the graphs becomes less and less as we pass from pure water towards those waters which are richest in chloride. This suggests that for a highly concentrated sea-water the graph would approximate to a straight line; that is to say, the term β in the expression for the temperature coefficient would become an almost negligible quantity.

In conclusion, we would draw attention to an interesting connection

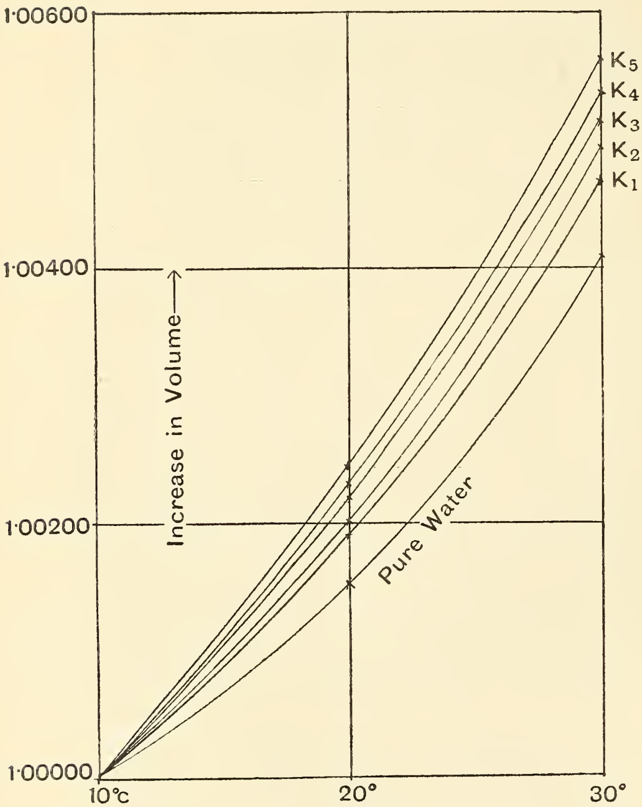


Fig. 3.

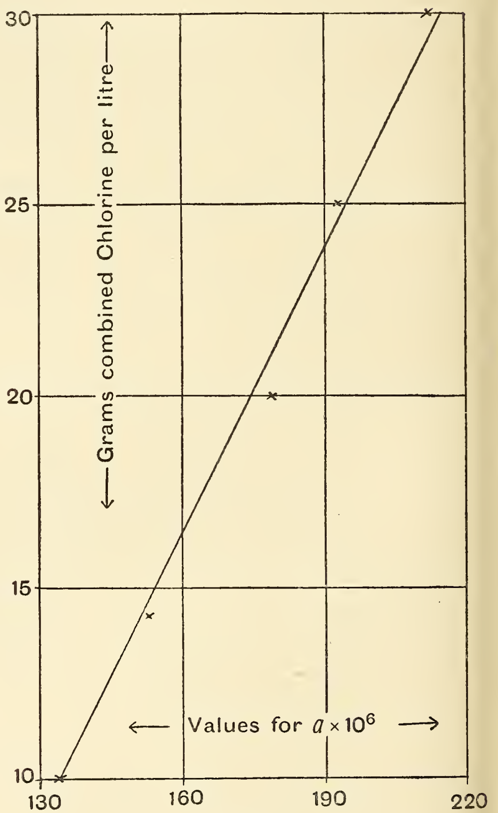


Fig. 4.

which appears to exist between the total chloride present and the temperature coefficient of the solution.

In fig. 4 the different chlorine values are taken as ordinates and plotted against the first term a of the temperature coefficients as abscissæ; the resultant curve indicates that equal increments in the total chloride produce certain other equal increments in the magnitude of a. The author hopes that he may be able to investigate this relationship more fully in the near future.

Graphic Representation of the Results.

Following the method adopted in the *Challenger Reports*, the results obtained from the density measurements are set forth graphically in figs. 5*a* and 5*b*.

The author would venture to express the hope that the methods described and the data contained in this communication may prove helpful to those who are interested in the various problems connected with oceanography.

I desire to express my best thanks not only to Mr R. T. Günther, who has given me so much valuable assistance, but also to the President and Fellows of Magdalen College for the continually increasing facilities which they offer for research work.

I would also take this opportunity of expressing my high appreciation of the courtesy of Captain Plunket of the *Oruba* and of the Managers of the Orient Steam Navigation Company, who not only made the necessary arrangements for collecting and packing the samples of water, but also delivered the same in London free of charge. Without their willing and generous co-operation this research would, in all probability, not have been attempted.

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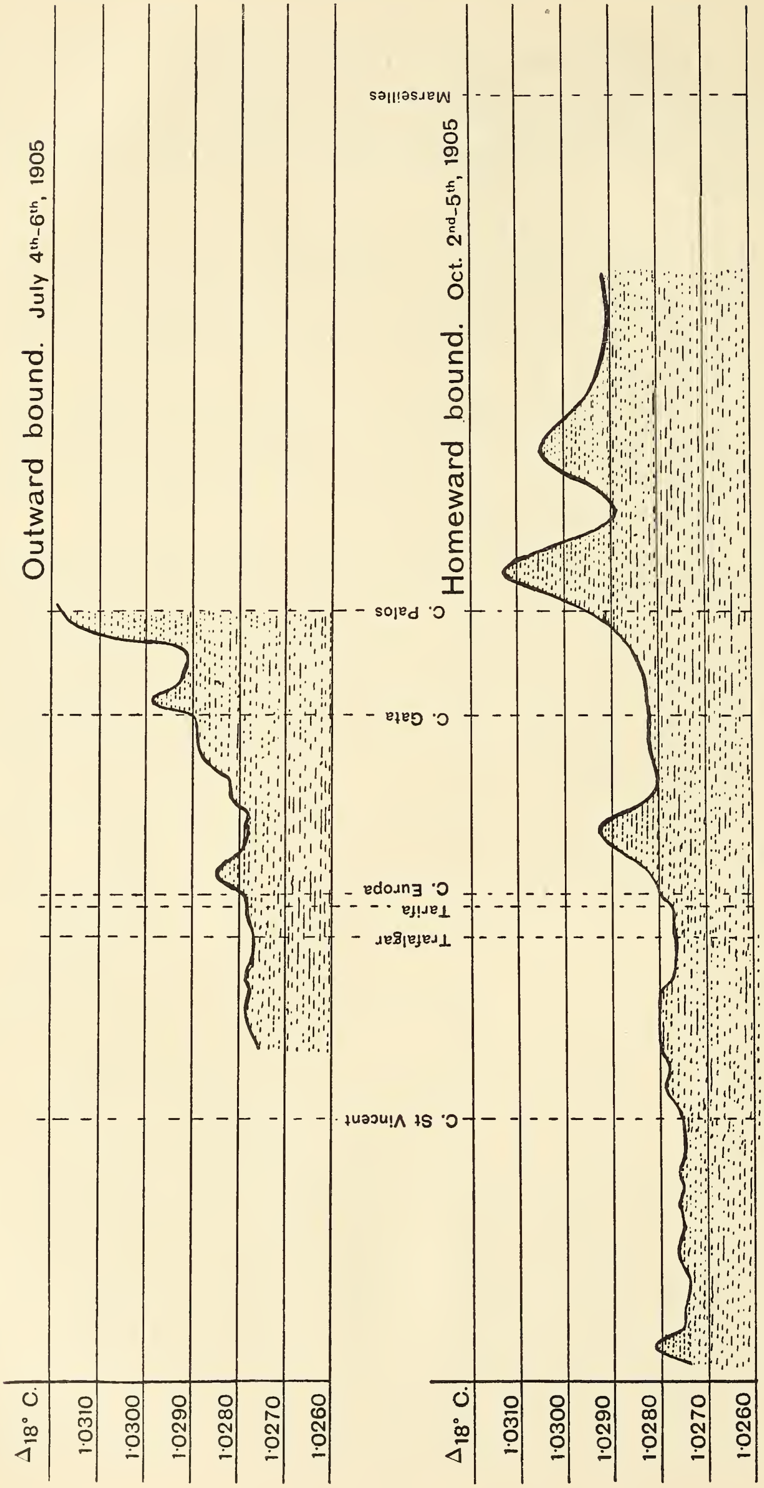
APPENDIX.

Relative Density Determinations with Pyknometers.

In order to test the degree of accuracy attainable by means of the various types of densimeter described, the relative densities of all the samples of water were first measured at 18° C. in the usual manner; and for this purpose five Sprengel pyknometers were employed. These were constructed from ordinary test-tubes, and proved highly satisfactory on account of the rapidity with which the charged instruments assumed the standard temperature of the bath in which they were placed. This was doubtless due to the thinness of their walls and to their low capacities, the latter ranging from 15 to 19 c.c. A series of experiments showed that a charged pyknometer acquired the temperature of the bath so readily



Figs. 5*a*, 5*b*.—Illustrating the variations in the density of the sea-water between Marseilles and Cape Finisterre.

that after five or six minutes' immersion it was practically impossible to detect any change in the volume of the contained water.

The temperature of the water-bath was indicated by a Baudin thermometer graduated to $\frac{1}{50}$ of a degree Centigrade, and previously compared with a standard thermometer from Kew. The water-bath was large, and was maintained at a uniform temperature throughout by a stirrer of the type devised by Griffiths for his re-determination of the mechanical equivalent of heat;* this stirrer, driven by a small electro-motor, proved both convenient and efficient.

The method of procedure adopted for the determination of the relative densities of the waters has already been described in a former communication;† it may, however, be stated that all the weighings of any pyknometer and its contents were effected with the instrument placed first in one pan of the balance and then in the other, suitably adjusted counterpoises being employed, as first advocated by Regnault. The weighings were performed with a short-beam Bunge balance sensitive to $\frac{1}{10}$ of a milligram, and a set of re-standardised weights.

After the constants of the various pyknometers had been ascertained, it was deemed advisable to test their accuracy by a determination of the relative density of some one sample of water with all five instruments; and for this purpose the sample marked No. 30 was chosen, because it contained the largest amount of chloride, and would therefore ensure the detection of any discrepancy that might exist, with a greater degree of certainty.

The following are the results obtained :—

Pyknometer.	Approximate Capacity.	Relative Density Δ_{18}.	Difference from Mean Value.
α.	15·92 c.c.	1·03188	± ·00000
β.	15·60 "	89	+ 1
γ.	16·19 "	85	− 3
δ.	18·85 "	91	+ 3
ϵ.	19·64 "	87	− 1
	Mean =	1·03188	

From the above it was inferred that the constants of the pyknometers were as nearly as possible correctly determined.

* *Phil. Trans.*, vol. A, 1893, p. 435.

† *Proc. Roy. Soc. Edin.*, 1903, p. 356.

Estimation of the Total Chlorine.

The amount of combined chlorine per litre of each sample was found by the usual method, in which a given volume of the water is titrated by standard silver nitrate, with potassium chromate as the indicator.

The necessary measuring instruments consisted of a 10 c.c. pipette* furnished with a tap and fine point, and with fiducial marks above and below the bulb, and a 50 c.c. burette which could be read direct to 0·1 c.c.; the burette was standardised at the Reichsanstalt, Charlottenburg, and the pipette subsequently compared with it.

The silver nitrate was supplied by Messrs Johnston, Matthey & Company, and was labelled "triple crystallised." The crystals were reduced to a powder and then dried in a water oven at a temperature of 95°–98° C. A standard solution was then prepared by weighing the salt direct, dissolving in water, and diluting to an appropriate volume. This solution was then compared with (1) a $\frac{N}{10}$ solution of highly purified sodium chloride, and (2) a standard sea-water received from Dr Nansen; the latter standard had been used by me in connection with the analyses of waters brought home in 1904 by the Antarctic Expedition. The chlorine values obtained for the silver nitrate by these methods did not differ by more than 1 in 600, and their mean value was accepted as the true one.

In making a determination of the combined chlorine, the water was first drawn into the burette direct from the bottle containing the sample; about 9·7 c.c. of it was then allowed to run into a porcelain dish and approximately titrated with the $\frac{N}{10}$ silver nitrate solution; the remaining portion was then delivered into the dish and the titration completed. By this method of procedure a very complete and uniform draining both of burette and pipette was in every case secured.

The results obtained are set forth in Tables V. and VI. on pp. 219 and 223.

* For the continued loan of this pipette, I have to thank Dr Dickson.

(*Issued separately September* 11, 1907.)

XXIV.—The Electric Conductivity and Refracting Power of ninety samples of Sea-Water, and a comparison of these with the Salinity and Density.

By **E. G. Hill**, B.A. Oxon, D.Sc. Dublin, *Professor of Chemistry, Muir College, Allahabad.* *Communicated by* Sir John Murray, K.C.B.

(MS. received June 6, 1907. Read June 24, 1907.)

There has been considerable discussion as to whether any physical constant of sea-water can be used to give an accurate measure of its total salinity, and from theoretical reasoning it would appear obvious that, in so complex a mixture, no physical constant can give an accurate measure of either the total amount of dissolved salts or of the chlorine present. In a paper communicated to this Society,* Mr J. J. Manley showed that the optical method for distinguishing between various samples was as delicate as the relative density method, but the refractometer used was expensive and not fitted for use on board ship. In a second paper † the same author compares the electric conductivities of a few samples of sea-water with the relative densities and the chlorine per litre. He comes to the conclusion that the conductivity bears no simple relation to either of these properties, and disagrees with Knudsen, ‡ who considers that the conductivity of a water may be taken as a measure of its salinity.

The present author gives, therefore, the measurements of the conductivity and deviation made by himself on ninety samples of water which were collected in the Mediterranean for Mr Manley,§ and compares these with the values obtained by Mr Manley for the total chlorine and relative densities.

The Water.—The samples of water were taken by a pipette from the bottles in which they had been collected. The water was part of the samples used by Mr Manley, and was used before he made the majority of his measurements.

The Electrical Conductivity. The Wheatstone's Bridge.

The bridge used consisted of a calibrated platinum-iridium wire about a metre long. This was highly insulated, and stretched by a heavy weight at one end. The ends of the bridge had a resistance equal to that of 21 mm. of the bridge wire, and the total length of the bridge was 1047 mm.

* *Proc. Roy. Soc. Edin.*, Jan. 1900.

† *Proc. Roy. Soc. Edin.*, Nov. 1902.

‡ *Berichte der Kommission zur wissenschaftlichen Untersuchung der dänischen Fahrwasser* (Copenhagen, 1900).

§ *Proc. Roy. Soc. Edin.*, 1907, p. 210.

The slider was also highly insulated, and the index line could be read to one-fifth of a millimetre; consecutive and independent readings agreed to half a millimetre.

The Resistance Coils.—It was found convenient to work with only one coil, and the coil selected as fitting the needs of the cell was one marked 400 ohms in a box of standard coils made by Groves. These had been recently standardised by Mr Manley, and the value of the coil used was 396·224 legal ohms at 15° C. The temperature of the coils was kept as constant as possible by placing the box in a larger box, the space between the two being packed with a non-conducting material. The temperature varied, however, by several degrees, and the necessary corrections were made. The temperature coefficient of the coils was ·00015 per ohm per degree.

The Leads and Switchboard.—The switchboard consisted of eight highly insulated mercury cups, and the connecting leads used for joining the coils with the bridge had a resistance of 0·006 ohms, while the leads to the cell had a resistance of 0·0669 ohms. The variation in the temperature of the room would not affect the resistance of these leads within the limits of experimental error.

The Current and Telephone.—The current was derived from a single large bichromate cell, and the alternating current from a small Köhler's induction coil. The telephone was a French one.

The Cell.—This was made from a U-tube whose diameter at the bend was about 6 mm. and much wider at the ends. The electrodes were about 10 mm. in diameter and were just above the constricted portions of the tube, coming close to the glass but not touching it. One was fixed in position and never moved; the second was sealed into a piece of glass tubing which passed through a sound selected cork. The latter just fitted one limb of the cell. Into the cork three pins were firmly fixed, so that each of these rested on the rim of the cell. The cork entered the cell far enough and tightly enough for no play to be possible, and the cell was marked so that each pin came into exactly the same position when the cork was removed and replaced. This cell was found to answer admirably. Comparison tests were made with sea-water No. 1 at intervals during the measurements of the other waters, and the constant of the cell was found not to change at all. The latter was determined by means of a solution of potassium chloride of normal strength at 25° C. and was found to be 22·8714. The electrodes were platinised in the usual manner.

Thermometers.—These read directly to 0·1, and each of those used were compared with a Kew standard and the correction applied.

The Thermostat.—The tank used for keeping the sea-water at 25° during the measurement was a thermostat of several gallons' capacity of the Ostwald-Luther pattern. As a matter of fact, the temperature of the bath was found to vary by about one-fifth of a degree, rising slightly and slowly during the day, because the temperature of the room was comparatively high; consequently, the temperature of the bath was always read for each observation and the necessary correction applied. It was found that over small ranges of temperature (2°–4°), one division of the bridge wire corresponded to 0·2° rise or fall in temperature. The correction was the same for seven samples which included one each of high and low salinity. The correction applied was thus taken as 0·5 mm. of bridge wire for each tenth of a degree above or below 25° C. The cell was held in position by a clamp. The sea-water was not placed directly into the cell, but was first pipetted into a Jena glass tube fitted with a rubber cork, which was suspended in the thermostat. This assumed the temperature required and was then placed in the cell, which was first thoroughly rinsed with it. The cell was then allowed to stand in the thermostat for a few minutes, and meanwhile the Jena tube was filled with the next sample of water. No trouble was experienced from the formation of air-bubbles in the cell after this treatment. The cell assumed the temperature of the thermostat extremely quickly owing to the thinness of the glass and the smallness of its contents.

Method of Determining the Conductivities.—The cell was rinsed three or four times with the sea-water to be examined and the removable electrode fixed in position. The cell was then clamped in the thermostat, and in a few minutes the battery connection made and the induction coil started. The slider was then adjusted without any alteration of the standard resistance, and a reading of the bridge taken. The thermometer in the thermostat was read, and a second reading of the bridge made after re-adjusting the slider. A third balance was similarly made and the mean of these taken as the true value. (The bridge readings were taken with the coils and cell in both the left and right gaps of the bridge respectively.) The temperature of the coils was then read. The observations were recorded as follows:—

Number.	Bridge Readings.		Temperature of Cell.	Temperature of Coils.	Mean Readings of Bridge corrected to 25°.
	Cell in Left Gap.	Cell in Right Gap.			
1	521·5 521·5 522·0	495·0 495·5 495·0	25·35° " "	20·5° " "	521·5 495·4 diff. = 26·1

The resistance was then calculated from the formula

$$R=\frac{1047+d}{1047-d}\times S$$

where $S=394{\cdot}224\{1+0{\cdot}00015\,(t\ 15^\circ)\}+0{\cdot}006$ (the standard coil and its leads), and R = the resistance of the water and its lead. Hence the actual resistance of the water is $R-0{\cdot}0669$ legal ohms, the reciprocal of which multiplied by the constant of the cell (*vide supra*) gives the true specific conductivity.

Errors of Experiment.—So far as the author is aware, the only errors not common to all the ninety measurements are those incidental to correct balancing and reading of the bridge. And the extreme limit would be half a millimetre of the wire. As the mean of several readings was taken, it would probably be much less than this; but allowing 0·25 mm. of wire as a possible error, we find that this affects the conductivity by 2 in the fifth figure, an amount which corresponds to the effect produced by an error of 0·01 grams in the estimation of chlorine.

The Optical Measurements.

These were made by means of a refractometer reading directly to 30″ and which could be easily estimated to 15″. The prism was a glass one with movable sides made by Hilger, and when set up had an angle of 60° 6′ 10″. The prism was not broken till No. 87, when it was reset and the angle found to be 60° 3′ 54″. The values for the last four measurements were calculated for a prism of 60° 6′ 10″, and are so given.

Methods of Experiment.—The bottles containing the water were kept near the refractometer, and were thus at the temperature of the room. The actual temperature of the water in the prism was determined by a thermometer reading to tenths of a degree, which had been standardised by comparison with a Kew standard. The left-hand edge of the image of the slit was made to coincide with the point of intersection of the cross wires. Sodium light was used as illuminant. The prism was rinsed twice with the water to be examined, and the thermometer inserted. The prism was placed in position on the refractometer and the position (1) of minimum deviation found and read; the temperature of the water was then read, the prism was turned round through 180°, and the position (2) of minimum deviation again read. The whole operation was then repeated.

Half the difference between 1 and 2 gives the angle of minimum deviation at the temperature noted. Corrections for temperature were applied as below, and the mean of the two values taken as correct. The difference in no case exceeded 30″. In two cases only it exceeded 15″.

Temperature Corrections.—Several of the waters were examined at different temperatures in the above way, and it was found that all of them behaved similarly. For a mean temperature difference of 1° C. it was found that the mean difference in the minimum deviation was 30″ of arc. The individual measurements agreed well among themselves, and the temperature correction is practically identical with that obtained by Mr Manley, whose value was 31″ for 1° C. This correction was accordingly applied. It would have taken time to work out the refractive indices of the samples, and the writer agrees with Mr Manley * that, for the end in view, the minimum deviation gives exactly the same information as the refractive index does. The angles of minimum deviation are, therefore, compared in place of the refractive indices.

Errors of Experiment.—The error of observation on a single measurement might be as much as 30″; but since at least two independent measurements were made with each sample of water, the probable error is not more than 10″ to 15″, and the agreement of the values obtained from different measurements of the same water showed that 10″ may be considered a reasonable approximation to the probable error.

The following table gives the values for the various constants and the chlorine in grams per litre. The latter, together with the relative density Δ, at 18° (by the pyknometer), were determined by Mr Manley for his own work and kindly supplied by him.

TABLE I.

Serial Number.	Chlorine, Grams per Litre.	Relative Density Δ_{18}.	Specific Conductivity $1/\rho$ at 25° in terms of the Legal Ohm.	Angle of Minimum Deviation at 24° by a Prism having $\angle = 60° 6' 10''$.
1	20·651	1·02764	0·054858	86925″
2	·651	74	4787	913
3	·756	83	5018	931
4	·686	85	5333	925
5	·756	83	5137	931
6	·756	91	5137	934
7	·756	84	5137	926
8	·615	70	4922	919
9	·615	74	4964	922
10	·721	81	4922	919
11	·721	87	5069	932
12	·721	86	5175	940
13	21·215	1·02849	6117	967

* *Proc. Roy. Soc. Edin.*, 1900.

Table I.—*continued.*

Serial Number.	Chlorine, Grams per Litre.	Relative Density Δ_{18}.	Specific Conductivity $1/\rho$ at 25° in terms of the Legal Ohm.	Angle of Minimum Deviation at 24° by a Prism having $\angle = 60° 6' 10''$.
14	21·144	1·02841	0·056027	86970″
15	20·792	1·02788	5281	945
16	·792	94	5122	935
17	·756	80	5069	910
18	·968	1·02817	5758	949
19	21·004	23	5875	952
20	·604	67	7189	966
21	·533	90	6916	976
22	·533	88	6970	981
23	·533	93	7080	985
24	22·310	1·02989	8459	87055
25	21·780	34	7737	019
26	·674	15	7352	010
27	·674	14	7408	007
28	22·857	1·03093	·060448	123
29	23·686	174	1203	175
30	Not examined.			
31	21·710	1·02919	·057390	016
32	·639	10	321	86985
33	·921	45	765	87027
34	22·663	1·03050	·059510	086
35	21·533	1·02889	·056889	86968
36	23·333	1·03133	·060949	87138
37	21·374	1·02873	·056785	010
38	20·933	21	·055758	86961
39	·933	19	526	953
40	·792	00	314	931
41	21·745	1·02933	·057691	87021
42	20·968	1·02821	·055739	86965
43	·615	1·02773	·054861	946
44	·580	73	787	946
45	·580	73	787	915
46	·580	79	787	917
47	·580	72	787	917
48	·615	74	787	919
49	·615	72	787	917
50	·615	73	839	916
51	·615	68	903	901
52	·615	67	903	911
53	·615	74	903	901
54	·615	70	787	907
55	·580	68	668	917
56	·615	72	668	910
57	·608	70	772	913
58	·580	69	815	908
59	·580	69	815	911
60	·580	66	867	906
61	·580	70	782	913
62	·651	73	...	934
63	·792	97	·055050	936
64	·615	68	·054704	928

Table I.—*continued.*

Serial Number.	Chlorine, Grams per Litre.	Relative Density Δ_{18}.	Specific Conductivity $1/\rho$ at 25° in terms of the Legal Ohm.	Angle of Minimum Deviation at 24° by a Prism having $\angle = 60° 6' 10''$.
65	20·825	1·02798	·055256	86931″
66	·825	97	256	938
67	·825	1·02802	468	951
68	·862	06	468	948
69	·825	02	468	946
70	·862	06	468	952
71	·862	07	468	939
72	·756	1·02792	345	949
73	·668	80	·054861	929
74	·756	93	·055186	944
75	·545	67	·054641	919
76	·439	48	401	913
77	·545	53	464	907
78	·456	54	420	892
79	·545	60	483	907
80	·474	54	440	905
81	·309	58	650	894
82	·439	47	440	904
83	·474	55	514	890
84	·509	57	514	913
85	·403	43	317	890
86	·439	44	317	893
87	·439	48	512	887
88	·474	52	301	896
89	·862	1·02807	·055351	923
90	·403	1·02742	·054344	890

For the sake of comparison, some of the above figures have been rearranged in ascending order of the chlorine values. The mean of the values for the various physical constants corresponding to the chlorine has been struck, and the variation of the physical constants from their respective means is given.

TABLE II.

Serial Number.	Chlorine per Litre.	Relative Density Δ_{18}.	Variation from Mean.	Specific Conductivity $1/\rho_{25}$.	Variation from Mean.	Minimum Deviation.	Variation from Mean.
76	20·439	1·02748	+·00001	·054401	−·00002	86913″	+14″
82	"	47	0	440	+ 02	904	+ 5
86	"	44	− 3	317	− 10	893	− 6
87	"	48	+ 1	512	+ 10	889	−12
	Mean	1·02747	±·00001	·054418	±·00006	86899	9

Table II.—*continued.*

Serial Number.	Chlorine per Litre.	Relative Density Δ_{18}.	Variation from Mean.	Specific Conductivity $1/\rho_{25}$.	Variation from Mean.	Minimum Deviation.	Variation from Mean.
44	20·580	1·02773	+ ·00002	·054787	·00000	86946″	+29″
45	„	73	+ 2	787	00	915	−02
46	„	79	+ 8	787	00	917	00
47	„	72	+ 1	787	00	917	00
55	„	68	− 3	668	− 12	917	00
58	„	69	− 2	815	+ 03	908	−09
59	„	69	− 2	815	+ 03	911	−06
60	„	66	− 5	867	+ 08	906	−11
61	„	70	− 1	782	00	913	−04
		1·02771	± ·00003	·054788	± ·00003	86917	±06
8	20·615	1·02770	− ·00001	·054922	+ ·00008	86949	+03
9	„	74	+ 3	964	+ 12	922	+06
43	„	73	+ 2	861	+ 02	946	+30
48	„	74	+ 3	787	− 06	919	+03
49	„	72	+ 1	787	− 06	917	+01
50	„	73	+ 2	839	− 01	916	00
51	„	68	− 3	903	+ 06	901	−15
52	„	67	− 4	903	+ 06	911	−05
53	„	74	+ 3	903	+ 06	901	−15
54	„	70	− 1	787	− 06	907	−09
56	„	72	+ 1	668	− 18	910	−06
		1·027716	± ·00002	·054848	± ·00007	86916	±08

In Table III. are given the mean values for density, conductivity, and deviation for the various values for total chlorine. Those cases only are considered where there were more than four samples of the same chlorine value.

TABLE III.

	Chlorine.	Mean Relative Density Δ_{18}.	Average Variation.	Mean Specific Conductivity $1/\rho_{25}$.	Average Variation.	Mean Deviation.	Average Variation.
(*a*)	20·439	1·02747	·00001	·054418	·00006	86899″	9″
(*b*)	·580	771	3	788	3	917	6
(*c*)	·615	772	2	848	7	916	8
(*d*)	·756	787	5	·055149	6	932	8
(*e*)	·792	795	4	192	10	937	4
(*f*)	·825	800	2	362	10	941	7
(*g*)	21·533	806	0	439	2	941	7
(*h*)	·533	890	2	·056964	7	977	5

The average variation means the mean of the individual differences between each recorded observation and the mean of all the observations for each value.

In the case of all the other values for total chlorine we have too few samples of water with the same value, and no means can be taken. For the sake of comparison, however, the means of *all* values of all observations of waters whose chlorine value is above 22 grams per litre is given. It is

(*i*) 22·970 1·03088 ·060104 87115″

In Table IV. are given the differences between the values of the various physical constants for variations in the total chlorine.

TABLE IV.

	Difference in Chlorine.	Difference in Relative Density.	Difference in Specific Conductivity.	Difference in Minimum Deviation.
$b-a$	0·141	0·00024	0·000370	18″
$c-a$	·176	025	0430	17
$c-b$	·035	001	0060	...
$d-c$	·141	015	0301	16
$e-c$	·177	023	0344	21
$e-d$	·036	008	0043	5
$f-e$	·033	005	0170	4
$g-f$	·037	006	0077	0
$f-c$	·210	028	0514	25
$h-g$	·671	084	1525	36
$h-a$	1·094	143	2546	78
$l-a$	2·531	341	5686	216
Total	5·286	0·00703	0·012966	436
Average for	0·100	0·00013	0·000238	8

From a consideration of the above it is seen that neither the densities, conductivities, nor refractive indices (deviations) give values which vary exactly with the chlorine values. On the other hand, the variations in the physical constants do agree with astonishing accuracy for all variations in total chlorine above 0·100 grams per litre.

In the above tables and remarks the assumption has been made that the titrated values for chlorine were absolutely correct. Of course this is not warrantable, and since 1 cc. of the silver solution used in titrating was equivalent to 0·343 grams of chlorine per litre, the possible error is about 0·02 grams per litre. If this is taken into consideration, a difference of between 2 and 3 in the fifth figure for densities, and of between 4 and 5 in

the same figure for conductivities, would be quite possible for waters which are taken as having the same chlorine value. It will be seen from Table III. that this difference is frequently doubled; but, as stated above, errors in measurement of the conductivity would account for a further difference of 2 in the fifth figure, and the error in density determinations would account for a difference of the same order of magnitude; so that, as a whole, the conclusion is reached that either density or conductivity measurements give a chlorine value sufficiently near the true one for all practical purposes in oceanography.

This may be shown by taking the density or conductivity values farthest from the mean for any given chlorine values and calculating the chlorine value corresponding to the density or conductivity from Table IV. Thus in Nos. 46 and 60 we get the relative densities $\Delta_{18°} = 1·02779$ and $1·02766$. These differ from the mean density for all the waters of the same salinity by $+0·00008$ and $-0·00005$. These differences correspond to differences in chlorine of 0·06 gr. and 0·04 gr. respectively. If we allow a titration error in these extreme cases of 0·02 grams, it will be evident that the approximation to accuracy is quite sufficient when the values corresponding to the densities are taken in place of the titration values, the actual errors being about 0·04 gr. and 0·02 gr. in the two extreme cases, or less than 0·2 and 0·1 per cent. of the total chlorine.

Taken as a whole, the values obtained from the conductivities are slightly less accurate, the average variations being a little more than double those of the densities; while for 0·100 of chlorine the difference in density should be 0·00013 and in conductivity 0·00024, or slightly less than double. In the most extreme case, No. 56, the variation from the mean is $-0·00018$, which corresponds to a difference in chlorine of $-0·075$. The admitted possible error is 0·02 (titration) + 0·01 (for specific conductivity) = $\pm 0·03$. Thus the difference between the value from the corrected conductivity would differ from a corrected titration value by 0·045 grams of chlorine in the most extreme case. This, again, is little over 0·2 per cent.

It is apparent at once that the angles of minimum deviation obtained above are of little value in comparison with the other methods. This is probably due to the fact that the instrument used was not sufficiently delicate. It would be necessary to use a scale reading to at least 5″ of arc to obtain the same degree of accuracy as is given by the other physical methods. As stated above, that used by the author read directly to 30″. The optical measurements are, in consequence, not further discussed.

The conclusion that may be drawn from these experiments is that certain

physical properties are not always exact measures of the chlorine in the sea-waters, but that the difference between the value for chlorine calculated from the physical constants, and that measured by titration, is sometimes so small, that for purposes of oceanography it may be assumed that the two values are identical.

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(*Issued separately September* 11, 1907.)

XXV.—The Influence of Twist on the Strength of a Thread. By Thomas Oliver, B.Sc. (Lond. and Edin.), Carnegie Research Fellow. *Communicated by* Dr C. G. Knott.

(MS. received May 20, 1907. Read May 20, 1907.)

On this very important subject I have not been able to discover much published work of a quantitative description. Professor Hartig, a writer in *Dingler's Polytechnic Journal*, 1879, appears to have made a special study on the strengths of threads, and was able to express the results of his investigation in the algebraic formula,

$$y = \frac{n\mu}{\lambda}x^2 - \frac{nk}{\lambda}x + nk\,,$$

where y = the resistance of the thread to rupture, *i.e.* the strength of the thread.
x = the length of the thread under test.
λ = the length of each fibre composing the thread.
(The fibres are evidently assumed of equal lengths.)
n = the number of fibres contained in a cross section of the thread.
k = the resistance of a fibre to rupture, *i.e.* the strength of the fibre.
μ = the coefficient of resistance to slipping of the fibres on each other.

Professor Hartig evidently used μ in a different sense from its general meaning as the coefficient of friction. In his formula μ is the absolute resistance to slipping per unit length of fibre, and not a ratio. He stated further that μ varied between the limits ·00005 gram and 9·015 grams per millimetre, the former limit applying to threads composed of silk filaments without twist, the latter limit to woollen threads with an abnormally "hard" twist.

I have not the full details of Professor Hartig's analysis at my disposal. I am indebted to a French work, *Essai des Matières Textiles*, by J. Persoz, Director of the Conditioning House at Paris, for the above information. However, it is not difficult to see how the formula has been arrived at.

Suppose for simplicity that a thread is composed of parallel lines of fibres broken at intervals of λ, as shown in fig. 1. The adhesion necessary to form a thread could be afforded by a size or glue.

Let M and N represent the jaws of the testing machine, distant x units of length from each other. A and B represent lines of fibres which are continuous; C and D, lines of fibres which are discontinuous between the

points M and N. When tension is applied to the thread, A and B will not slip, while the fibres forming C and D will.

In λ millimetres, all the lines of fibres, *i.e.* n, will on the average have become discontinuous.

Therefore, in x millimetres, $n\frac{x}{\lambda}$ fibres will be discontinuous.

$\therefore$ $n\left(1-\frac{x}{\lambda}\right)$ fibres will be continuous throughout the range of x mm.

Each of these latter fibres offers k grams resistance to rupture.

$\therefore$ $n\left(1-\frac{x}{\lambda}\right)$ of these latter fibres offer $nk\left(1-\frac{x}{\lambda}\right)$ grams resistance to rupture.

The discontinuous fibres do not offer any resistance in this respect, because they are already broken within the limits M and N.

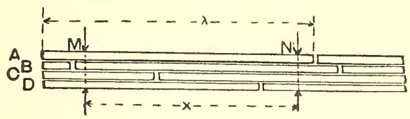

Fig. 1.

Each fibre offers $\mu\lambda$ grams resistance to slipping on its entire length, and therefore μx grams resistance to slipping on x millimetres of its length.

$\therefore$ $n\frac{x}{\lambda}$ fibres offer $\frac{n\mu}{\lambda}x^2$ grams resistance to slipping.

$\therefore$ The total resistance of the thread to rupture

$$=\frac{n\mu x^2}{\lambda}+nk\left(1-\frac{x}{\lambda}\right),$$

as shown by Professor Hartig.

Now, it will be instructive to examine the matter a little more closely, and to see if this simple formula can represent the actual conditions, and if not, wherein it fails.

In the first place, yarn-testing is only useful in so far as it enables us to ascertain the weaving qualities of a yarn. Therefore the conditions of test should approximate as closely as possible to those conditions which shall hold during weaving. The warp threads are subjected to considerable tensile stress in the loom by "weighting" the warp beam. This stress, however, is not nearly up to the limit of elasticity of the threads. It is in the forming of the "shed" that breakages occur. Fig. 2 shows diagrammatically the conditions of shedding the warp in the loom.

E represents the breast beam of the loom, over which the cloth passes after being woven. F, the back beam, over which the warp passes.

D is the position of the lease rods, which determine the position of the threads in the warp relative to each other, and also the length of warp thread which shall be under increased strain during shedding. A is the fell of the cloth.

The dotted line AD shows the position of the threads when the "shed" is closed. In order to form an opening or "shed" through which to pass

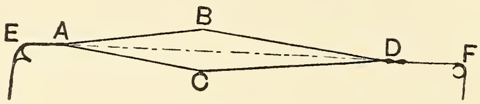

Fig. 2.

the shuttle carrying the weft, part of the threads are raised into position ABD, while the remainder are depressed into position ACD.

$$AB + BD > AD .$$

Therefore a length of thread AD must be able to stretch AB+BD−AD without the stress reaching the limit of elasticity, or breakage will occur during the continued repetitions of stress in shedding. The length AD is never much less than 27 inches, and in the broad looms of the Scotch woollen trade will be more. Therefore the length tested should be about 27 inches.

Now, in all threads, except those of thrown silk, the fibres are much shorter than this: *e.g.* in crossbred worsted, 4-inch fibres; in Botany worsted, $2\frac{1}{2}$-inch fibres; in carded merino threads, 1-inch or even shorter fibres, are very common.

Professor Hartig's formula is only applicable when x is less than λ; but we have just seen that testing is only of value to the manufacturer when x is about ten times greater than λ.

Let us now consider the limiting case when $x=0$, *i.e.* when the length of the thread under test is infinitesimal.

The formula becomes

$$y = nk ,$$

i.e. the strength of the thread is equal to the sum of the strengths of its component fibres.

Practically, it would not be possible to test infinitesimal lengths of thread. Probably 1 millimetre would be the minimum length which could be tested on the best-designed machines. Experimental results show that the breaking strength of 1 millimetre of thread is never anything near equal to the sum of the breaking strengths of its fibres. There are several reasons why this should be so.

(1) "That the strength of the whole should equal the sum of the strengths of its parts" appears plausible enough, and it would be substantially correct if the fibres were rigid and did not admit of stretching under tension, or if all the fibres stretched an equal amount for the same stress, and if it could be ensured that all the fibres in the thread were at the same tension when the load was applied to the thread. None of these conditions can be realised in the actual thread. The first condition, "absolute rigidity," is far wrong. Every textile fibre is more or less plastic, wool and silk especially so. The second condition, "equal degrees of plasticity amongst the fibres of a thread," is not far wrong up to the limit of elasticity, because the modulus of elasticity is approximately the same for all fibres of the same quality. But threads are often made by blending different qualities of the same material, and even different materials, *e.g.* cotton and wool blends. In such there would be a marked difference in the degrees of stretching under the same stress. After the elastic limit is passed, however, there is no uniformity in the stretching of the fibres even of the same quality. The fibres which stretch least would take up the greater part of the stress, and would quickly reach their ultimate breaking strengths before the average load has reached, say, two-thirds the strength of the weakest fibre. After the "harder" fibres have been ruptured, a correspondingly smaller number of fibres would be left to carry the load. So that the strength of the thread may be equal to the sum of the strengths of its fibres, each fibre must take up its due share of the load, and this can only occur if the fibres are all at equal tension to begin with, and if they continue so throughout the pull. This brings us to another reason for earlier rupture.

(2) Let us suppose that a thread contains 90 fibres, the average tenacity of which = 10 grams, and that the following table shows the variation of strength amongst the fibres:—

TABLE I.

5	fibres	break separately at	8·5	grams.	
10	,,	,,	9·0	,,	
20	,,	,,	9·5	,,	
20	,,	,,	10·0	,,	
20	,,	,,	10·5	,,	
10	,,	,,	11·0	,,	
5	,,	,,	11·5	,,	
90	,,	,,	10	,,	average.

Assuming that the fibres are all at the same tension at the beginning, and stretch equally throughout the pull, then, when the total load on the thread

is 765 grams, the average load on each fibre is 8·5 grams. At this point the five weakest fibres break; there are now only 85 fibres to carry the load of 765 grams, which gives an average of 9 grams per fibre. But this is the load at which the second weakest group of fibres break. Therefore these break immediately after the first group, before the total load has increased. Then 75 fibres are required to carry 765 grams, *i.e.* an average of 10·2 grams; but this is a higher load than 40 of these fibres can carry. The result is that the thread tears across almost simultaneously with the rupture of the weakest fibres. If the average strength of the fibres were deduced from this result, it would be set down as 8·5 grams, but we have seen that this is merely the strength of the weakest group of fibres.

In general, if there are n fibres in a thread, the strength of the weakest fibre is a, and the strength of the second weakest is $a+d$.

Then, the condition that the total load will increase between the times of rupture of the weakest and second weakest fibres is that

$$a+d > \frac{na}{n-1}$$

$$\text{or} \qquad d > \frac{a}{n-1}.$$

(3) Another discrepancy is introduced, because the fibres are not all continuous throughout even 1 millimetre of thread under test. This would not, however, be an objection against the hypothetical testing length of infinitesimal magnitude. If we take a carded yarn made from a Botany clothing wool, 1 inch is a common length of fibre. With 100 fibres in a cross section, there are 100 fibre ends in 1 inch on the average, and therefore four ends on 1 millimetre; *i.e.* there are only 96 fibres continuous throughout the length of 1 millimetre under test.

Therefore the common practice of setting down the breaking strength of 1 millimetre, and sometimes even of 1 centimetre, of thread as the sum of the strengths of its constituent fibres is misleading.

The further consideration of Professor Hartig's formula raises several very interesting questions, but these do not come properly into the scope of this paper, and will be reserved for discussion in another communication which deals with the influence of the length of the test sample on the breaking strength observed. The weakest point in the formula is that μ varies between such wide limits. Although noting that the variation of μ is due to variation in twist, he apparently made no attempt to ascertain if a relation held between μ and the degree of twist. Twist is by far the most important factor in determining the strength of a thread of a given weight

per unit length, and no formula from which the degree of twist is absent can make any pretension to representing the actual conditions which obtain in a twisted thread. Probably the most important experimental evidence that has ever been published on the strength of threads will be found in a paper entitled, "Influence of Certain Reagents on the Tensile Strength and on the Dyeing Properties of Cotton Yarn," by Professors Julius Hübner and William J. Pope, F.R.S., of Manchester, read before the Manchester section of the Society of Chemical Industry on January 9, 1903. In each of their tests they broke from 100 to 120 threads, each of 10 cm. length, on a Schopper testing machine, so as to obtain a mean result with a satisfactory degree of accuracy. The Schopper testing machine is a product of the Berlin Charlottenburg. It is one of the most elaborate and is the most accurate of testing machines at present available for testing single threads. Further, the limits of accuracy to which the mean results could be relied upon were carefully determined by applying the method of least squares to the observations. They were thus enabled not only to give the mean tensile strengths for cotton yarn in its natural state and after treatment with boiling water, sodium carbonate, sodium hydroxide, potassium iodide, and other chemical reagents, but to state that the limits of error did not exceed $\frac{1}{2}$ per cent. Unfortunately, these gentlemen did not confine their conclusions to the problems which they ostensibly set out to solve. They committed themselves to the statement "that for one and the same yarn the tensile strength is directly proportional to the twist." They founded this statement on what seems to me to be insufficient experimental evidence.

After making the strength tests referred to above, a series of fifty-eight determinations of the twist were made on 10-inch lengths of the same two-fold 50's raw Egyptian cotton yarn as had been used for the strength tests. From the results of these determinations it appeared that the twist varied from 22·0 turns to 30·3 turns per inch, the mean being 25·66 turns per inch. The twist being the main factor in determining the tensile strength of a cotton yarn, the paper went on to show that the above three twist numbers, viz. 22·0, 30·3, and 25·66, are in the ratio 324 : 446 : 378, which is practically the same as the ratio 330 : 440 : 378, the lowest, highest, and mean values of the tensile strength in grams taken from the table of results. The conclusion above mentioned was then drawn. That this conclusion is not altogether logical in several particulars will be evident if we consider the following points:—

(1) If it had been possible to ensure that the same number of fibres were present in every cross section of the thread, there would not have

been any variations in the twist. Twist distributes itself along the length of the thread according to the number of fibres present in each section. [The precise relation which connects these quantities will be discussed in a future paper on the distribution of twist in a thread.] It is sufficient for our present purpose to state that in drawing the sliver or roving, the fibres do not as a rule distribute themselves uniformly along the thread. When twist is subsequently put into the thread during the spinning operation, it runs to an excessive degree in the sections containing the smaller numbers of fibres. When the sliver is very uneven or "pointed," this action of the twist forms local hard, small parts in the thread, which are technically called "twits." However, the fact must never be overlooked that these are always present in the best-spun yarns, but to a smaller extent in number and degree. We have then at least two causes of the phenomenon observed by the Manchester investigators, viz.: higher degree of twist and lower number of fibres, or *vice versa*; lower degree of twist and higher number of fibres. The strength increases with the twist, and decreases as the number of fibres decreases, but not necessarily in the same proportion. Indeed, it is possible that the twitty parts are stronger than the average, but only if the "twit" or unevenness is not excessive; *i.e.* the strengthening of the thread due to twist may be greater than the weakening due to decrease in the number of fibres.* But the point at issue remains, that there were at least two causes of the phenomenon, and unless these were isolated and their effects determined separately, any assumption as to their single effects on the strength of a thread is unwarranted. The observations taken were not due to the effect of twist alone, but to the superposition of the effects of twist and at least one other cause operative at the same time.

(2) The second flaw in their argument appears to be that the yarn that was tested for twist was not the same yarn that was tested for strength. Fifty-eight threads were tested for twist, after testing ninety-nine threads for strength. The probable reason for this procedure might be that it was not possible to vary the twist on the machine during the test; therefore two operations would have been necessary on different machines for the same thread. The Schopper testing machine, although the best for the main purpose that the experimenters had in view, is not the best for this side question of the relation between twist and strength of the thread. This difficulty, however, put them in the position to some extent of assuming what they had to prove.

(3) Thirdly, the limits of variation of twist were too close together to

* This question will be more fully discussed on p. 259.

make observations on which to found a general law. The law, if true at all, could only be so within the narrow limits of observations made. The twist on the thread should have been gradually increased in a series of threads from zero to a high degree, and the corresponding breaking strengths observed. It is evident that the law cannot be true much beyond the higher limit, because the statement involves a constantly increasing strength of thread as the twist increases. That a thread cannot go on getting stronger indefinitely with twist is so obvious as to require no demonstration; there must be a change in the character of the relation as the torsion proceeds.

In justice, it ought to be stated that the defects noted in no way invalidate the main results of the above interesting paper. The care manifested in working out the details of the chief subject of inquiry contrasts strangely with the treatment accorded to the twist problem—so much so, that one is led to the conclusion that the authors did not give it their best attention. This question of twist is not one in which an investigator can indulge in a little by-play as a prelude to what he may consider more serious business. Twisted threads really present a group of the most complicated problems which a physicist can set himself to solve.

There are two methods of experimental investigation which appear commendable:—

(1) The direct method, which is as follows. Take out all the twist from a thread and test its strength. This can only be done satisfactorily on a machine which admits of the twist being varied, and also of the test for strength being made. Then take out all the twist from another thread of the same hank and put on one turn per inch, and test the strength of the thread in this condition. Perform the same operation for two, three, and more turns per inch of twist. Then repeat the whole sequence of operations a sufficiently large number of times to ensure a satisfactory mean being determined.

(2) The indirect method, in which the thread is loaded to some convenient point short of breaking. When tensile stress is applied to a thread, the thread stretches, but the elongation is never quite proportional to the stress. The increments of length corresponding to any fixed increment of stress gradually increase as the stress rises. But there is no sudden change until the stress is considerable when the yield point is reached, after which the increments of length for any given increment of stress become very much larger, and go on increasing as the stress increases up to the breaking point. The yield point may be termed the limit of

elasticity; but there is no real limit of elasticity in testing threads, similar to what is met with in testing materials like steel. This difference arises from the constitution of a thread differing greatly from that of a continuous substance. A thread is composed of fibres arranged spirally or, as in carded woollen, lacking any well-defined arrangement. Therefore much slipping occurs amongst the fibres from the beginning of the pull.

The most valuable quantity to be determined relative to the strength of a thread is not its ultimate breaking strength but the yield point; because, if the stress exceed that corresponding to the yield point and be continued sufficiently long, the thread will ultimately be ruptured. This result will be much accelerated if the stress be periodically applied and withdrawn, as in the process of shedding the warp in the loom. This determination of the yield point provides us with a much more accurate method of determining the relation between the degree of twist and the strength of a thread. It is then possible to apply an increasing twist to the same thread while under tensile stress, and, if great care be exercised in the application of the load, quite a long series of observations of the yield point may be obtained on the same thread before the thread breaks. Theoretically, it should be possible to perform the whole range of experiments with one thread, because the stretch is not very great up to the yield point. In practice, this limit is exceeded at each observation, as it is often not well defined, and the accumulation of these increments soon wears the thread out at some weak point in its length.

The great advantage of the second method over the first is that we are eliminating the effect of variation in the uniformity of the thread, and thus a satisfactory mean can be struck from a much smaller number of observations. Yarn-testing has not received the attention from manufacturers which its importance demands. Many firms never test their yarns at all. The majority (possibly all) of those who do only ascertain the breaking strength and the ultimate stretch of their yarn. The ultimate stretch is spoken of as the elasticity of the thread. This statement is quite wrong; the ultimate stretch and the elasticity are quantities not necessarily connected. The elasticity of a thread may be measured by its stretch for a given stress well within the limit of elasticity, this stretch being distributed throughout the length of the thread. The stretch from the yield point to the ultimate breaking point, on the contrary, is largely confined to one or more "soft" or slackly twisted parts in the thread. This local stretch is of no importance to the manufacturer, because the thread is useless long before it takes place. Further, the local stretch, being dependent on so many accidental circumstances, does not bear any definite

relation to the strength, although in a general way a strong thread stretches more than a weak one of the same quality.

There is only one machine at present in the market adapted for testing the relation between twist and yield-point strength by the second method which has been described. This apparatus was designed by Mr G. R. Smith of Bradford, and is shown in fig. 3. A thread A is stretched between two clamps B and C. C forms the end of one arm of a bell-crank lever pivoted at G, whose other arm F carries a can E. Water may be run into can E from a reservoir D, and thus the thread may be twisted under any tension. The can is provided with a gauge. H is an adjustable counterpoise to balance F when E is empty. The amount which the thread stretches under tension is taken up by turning wheel I, which communicates motion by a train of wheels and special pinion to a cylindrical rack K. K moves the clamp B so that the lever F always remains horizontal. The twist in the thread may be varied by turning wheel M, the turns being indicated on dial T, to which motion is communicated by a worm W carried on bar K. Arm F carries at the end a rubber pad P, which drops on the water nozzle and stops the flow when the thread yields or breaks.

The method of testing the relation in question is as follows:—Take out all the twist from the thread. If it is a single, it will probably not carry any appreciable load at all after untwisting. Then put on one turn per inch and run in water very slowly to E from reservoir D, and take up the stretch by moving B. As soon as the yield point is noticed, read off the amount of water in E by the gauge. Proceed in the same way for two turns per inch and more until the thread wears out. Suppose this occurs at four turns per inch. Start a new series with one turn per inch, and the thread will probably break at five or six turns. Start a third series with two turns, and so on. The observations were restricted to 10-inch lengths of thread, because in the machine used rack K did not admit of greater movement than 4 inches. With excessive twist the stretch sometimes amounts to 35 or 40 per cent. of the original length. But as woollen, worsted, and cotton fibres are much less than 10 inches, it is probable that the results do not differ greatly from those obtained on lengths of 20 to 30 inches.

In the later experiments, the above apparatus was modified. The reservoir D was removed, arm F was graduated, and the tension produced by a sliding weight on arm F. When one weight was not sufficient to cause the thread to yield, it was dropped gently into E, and a similar weight was started on its outward journey from pivot G along the arm F. This modification was found to give much better results, because the hydraulic

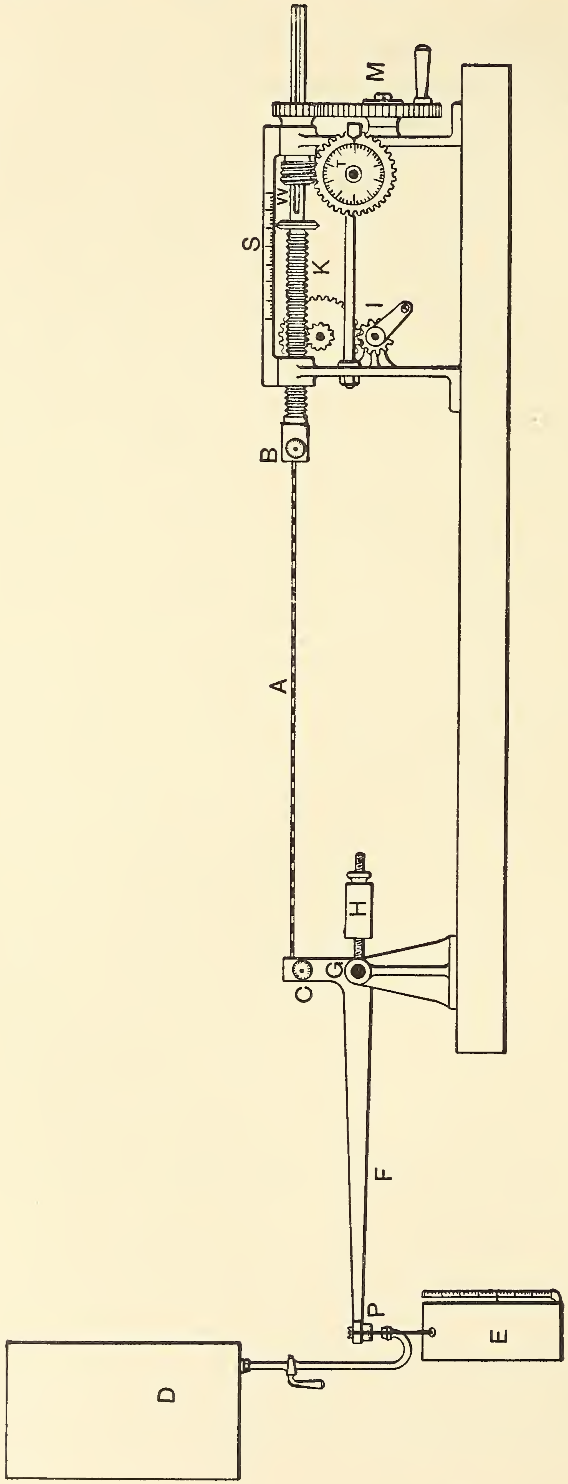

Fig. 3.

apparatus was not always quite steady in its action, and the can E, being of considerable cross section, did not lend itself to accurate reading of the volume of water at each yield point.

The following Table (II.) shows the results for the following single threads: 6's crossbred worsted; 7-cut woollen, spun from crossbred lamb's wool; 10-cut and 16-cut woollen. The worsted thread was composed of very long, straight fibres, while the three Cheviot woollen threads were of the soundest and most uniform quality. The cut system of numbering used in this paper is that of Galashiels, viz. the number of cuts of 300 yards each which weigh $1\frac{1}{2}$ lbs. is the yarn number. This system is now used largely in Scotland and Ireland for woollen yarns.

TABLE II.

Relation of Strength to Twist in Single Threads.

Turns per inch of Twist.	Load in lbs. on the Thread at the Yield Point.			
	6's worsted.	7-cut.	10-cut.	16-cut.
0	·04	·08	·04	·06
1	·06	·15	·05	·12
2	·26	·56	·08	·26
3	·60	1·67	·17	·40
4	·99	2·28	·36	·51
5	1·21	2·66	·60	·61
6	1·36	2·82	·82	·68
7	1·46	2·92	1·05	·72
8	1·51	2·98	1·11	·76
9	1·54	3·02	1·15	·80
10	1·62	...	1·20	·84
11	1·65	...	1·26	·88
12	1·69	...	1·32	·91
13	1·72	...	1·35	·94
14	1·75	...	...	·96

The results of Table II. are shown graphically in fig. 4. The loads at the yield points are plotted as ordinates, the corresponding turns of twist as abscissæ.

Table III. shows the results for the following two-ply threads: 2/16's Botany worsted, 2/12's crossbred worsted, 30-cut two-ply Saxony woollen (scoured), 56-cut two-ply Saxony woollen (greasy).

The results of Table III. are shown graphically in fig. 5. The loads at the yield points are plotted as ordinates, the corresponding turns of twist as abscissæ.

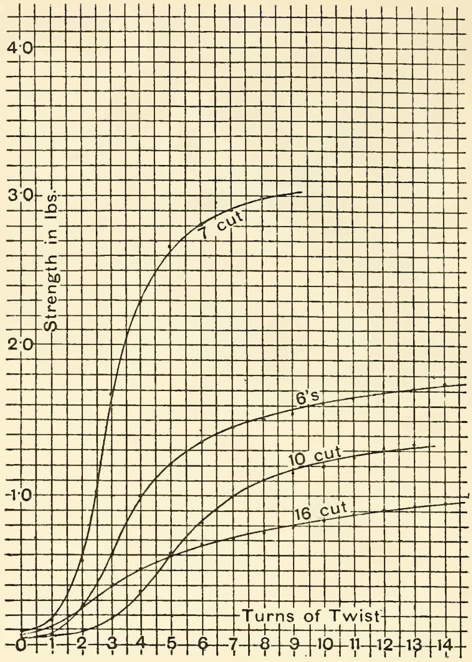
Strength in lbs.
Turns of Twist
7 cut
6's
10 cut
16 cut
4·0
3·0
2·0
·1·0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

Fig. 4.

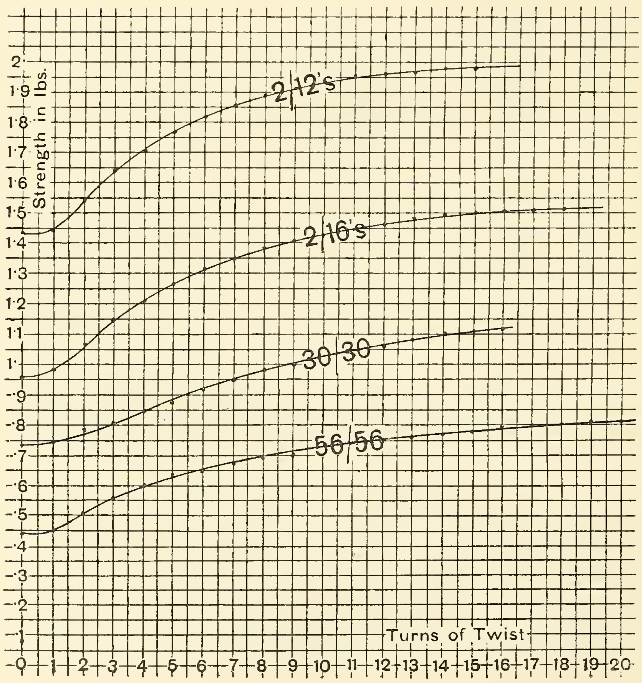
Strength in lbs.
Turns of Twist
2/12's
2/16's
30/30
56/56
2·
1·9
1·8
1·7
1·6
1·5
1·4
1·3
1·2
1·1
1·
·9
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Fig. 5.

TABLE III.

RELATION OF STRENGTH TO TWIST IN TWO-PLY THREADS.

Turns per inch of Two-ply Twist.	Load in lbs. on the Thread at the Yield Point.			
	2/16's Botany.	2/12's Crossbred.	30/30 Saxony.	56/56 Saxony.
0	·96	1·43	·73	·44
1	·98	1·44	·75	·45
2	1·07	1·54	·78	·51
3	1·15	1·64	·81	·56
4	1·21	1·71	·85	·60
5	1·27	1·77	·87	·63
6	1·32	1·82	·92	·65
7	1·35	1·86	·95	·67
8	1·38	1·89	·98	·69
9	1·41	1·92	1·00	·70
10	1·43	1·94	1·02	·72
11	1·44	1·96	1·05	·74
12	1·47	1·97	1·06	·75
13	1·48	1·97	1·08	·76
14	1·49	1·98	1·10	·77
15	1·50	1·98	1·11	·78
16	1·51	...	1·12	·79
17	1·51	...	...	·80
18	1·52	...	...	·80
19	...	...	...	·81
20	...	...	...	·81
21	...	...	...	·82
22	...	...	...	·82

One reason that the inflection in the graph for the 30/30 yarn is less pronounced than in the other graphs is that the thread was very elastic and stretched considerably up to the yield point. Thus the method used is not very accurate for yarns of this description.

Table IV. shows the ultimate breaking loads of the smallest possible length of each yarn which can be placed between the jaws of the testing machine. This length may be taken as 1 millimetre.

TABLE IV.

BREAKING LOADS OF 1 MM. OF YARN.

Single Threads.		Two-ply Threads.	
6's Worsted	2·32 lbs.	2/16's Botany	1·79 lbs.
7-cut Cheviot	4·15 „	2/12's Crossbred	2·66 „
10 „ „	2·42 „	30/30 Saxony	1·36 „
16 „ „	1·26 „	56/56 „	1·05 „

The characteristic features of the relations between twist and strength shown in the above tables and diagrams are:—In single threads the strength of the thread without twist is very small; there is not much of an increase in strength during the first turn; immediately afterwards the thread commences to get stronger at a rapidly increasing rate, but after a few turns the rate of change of strength begins to decrease, the curve approaching an asymptote. This asymptotic strength is much less than the ultimate breaking strength of 1 millimetre of thread.

The relations for two-ply threads have similar characteristics to those for single threads, but the initial strength in each case is considerable. During the first one or two turns of twist, two processes are in operation affecting the strength of the thread. The two-ply twist strengthens the thread, but simultaneously with this action the twist in the single threads is being opened out to some extent; the latter process weakens the thread. After the first turn per inch, the opening-out action becomes of little account. It is probable that the thread may be actually weakened in the initial stages of the second twisting. The above experiments were not sufficiently extensive and refined in measurement in the initial stages to settle this point. The question is of no practical importance, as the twist impressed on two-ply threads is always greatly in excess of the limit of twist within which the strength of the thread is affected by the opening out of the singles. For a similar reason the direct twist side of the diagram was not investigated; two-ply yarn is never twisted for commercial purposes in the same direction as the twist in the singles. The investigation therefore deals with the inverse twist in two-ply threads. Previous to the spinning operation, single yarn comes from the condenser in a thread-like sliver, but possessing no twist. The relation between strength and twist will not be changed by the direction of the twist, whether "crossband" or "openband."

When the twist becomes excessive the determination of the yield point is beset with great difficulties. Frequently the thread breaks with a slightly lower load than it carried with a lesser degree of twist. It does not follow, however, that this is due to a weakening effect of increased twist at this point. The reason is rather that the thread becomes weaker by the continued application of tensile stress. Again, very "hard" threads break suddenly without giving much evidence of a yield point. As such excessively twisted threads are too "hard" for use in any textile fabric, the exact character of the relation holding between strength and twist in these did not appear to be of sufficient importance to demand very careful investigation. But the striking fact remains that, although twist is the most

important factor in determining the strength of a given thread within the limits of useful requirements, a point is ultimately reached beyond which increased twist makes no material addition to the strength of the thread. The point raised by Professors Hübner and Pope, that twist is the controlling factor in the strength of a yarn, is not necessarily true under all conditions. Indeed, it is only true throughout the length of a fairly uniform thread. If we experiment with "pointed" or "twitty" thread, such as is frequently met with in woollen spinning, a very different result will be obtained. The thread under test will be found to have broken at the "twit." To see how this may come about, let us suppose that the 30/30-cut Saxony of Table III. had been a "pointed" or uneven yarn instead of exceptionally uniform as it was, and that one lumpy section contained 150 fibres and had six turns per inch, also that a second twitty section contained 100 fibres and had fourteen turns per inch. These are average results taken from an actual experiment on the distribution of twist. The twitty part is probably weaker than the lumpy part in the ratio of 2 : 3, owing to the reduction in fibres; and from Table III. we see that the increase of strength due to greater twist is in the ratio ·92 : 1·10.

Therefore, if W is the strength of the lumpy part, the strength of the "twit" is probably

$$= W \times \frac{2}{3} \times \frac{1{\cdot}10}{{\cdot}92}$$
$$= {\cdot}8W.$$

The "twit" would thus break with 80 per cent. of the load required to break the lumpy part.

The Manchester investigators inferred that the variation in twist in the various threads of their sample yarn was responsible for the variation in strength so well recorded in their tests.

Taking the above example, and even allowing for decrease in the number of fibres, which they did not do, the strength of the twit would

$$= W \times \frac{2}{3} \times \frac{14}{6}$$
$$= 1{\cdot}56W,$$

a very improbable result. Still less probable would their own statement work out as it stands, that the strength of the twit would

$$= W \times \frac{14}{6}$$
$$= 2{\cdot}33\,W,$$

on the assumption that the strength of the thread is directly proportional to the twist. If their inference had been correct, it would have placed a premium on bad spinning. The careless spinner and unskilful blender might equally cease from troubling, and rest in the assurance that the twist would effectually cover up the flaws in their work. Now, every spinner is well aware of the fact that, the more uniformly he can distribute the fibres along the axis of the thread, the better value will he give to his employer from every pound of wool or other material committed to his care. The uniform thread, although containing exactly the same number of fibres as the uneven thread, can stand a higher stress, and fewer breakages will result in weaving.

That the analytical problem of this paper is not the easy deduction performed by Professor Hartig will be readily admitted by anyone who has followed at all closely the various points discussed. The author has attempted a mathematical synthesis from the fundamental constants of the fibres constituting the thread and the degree of twist impressed in its formation, but he does not venture to present the results of the attempt with this communication. As each fibre is arranged in a spiral, the helices of which vary in diameter, increased torsion introduces both a bending moment and a twisting moment. The measurement of these involves the tension modulus and the torsional rigidity of the fibre. The strengthening influence of increased twist on the thread is due mainly to the resultant increase in the normal pressure of the fibres on each other, and therefore the resistance to slipping must be greater. Lesser causes are the increase of the surface of contact of the fibres and the interference of the twisted fibres with each other. It will not be a case of pure slipping, because the convolutions of a twisted fibre may prevent the motion of its neighbour by simply blocking the way. Here again, the extent to which the fibre can offer such resistance will depend upon its flexural and torsional rigidities. Taking these and other difficulties into account, it is probable that no satisfactory mathematical synthesis of the results obtained by experiment can be effected.

While the aforesaid conclusion may be true, an investigation of the graphical pictures of the relations between strength and twist in the various threads experimented on leads to very interesting results. The graphs are evidently of the family of curves to which mathematicians apply the name of "witch."

The general equation of a "witch" curve, as shown in fig. 6, is

$$y = b - \frac{(b-a)^3}{kx^2 + (b-a)^2},$$

where y is the ordinate of any point on the curve.
x ,, abscissa of the same point on the curve.
a ,, height of the point where the curve cuts the y-axis.
b ,, ,, ,, asymptote to which the curve approaches.
k is a constant number.

In the graphs for single threads a practically $=0$. The general equation then simplifies for such to the following form:—

$$y=b-\frac{b^3}{kx^2+b^2}$$
$$=\frac{kbx^2}{kx^2+b^2}$$
$$=\frac{bx^2}{x^2+\frac{b^2}{k}}$$

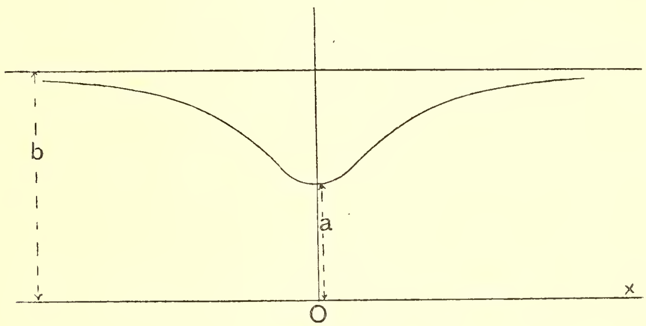

Fig. 6.

Table V. shows the values of $y=2-\frac{\cdot185}{\cdot02x^2+\cdot325}$ compared with the experimental values obtained for 2/12's crossbred worsted. This equation is obtained by substituting $b=2\cdot$, $a=1\cdot43$, $k=\cdot02$ in the general equation. Table V. also shows the values of $y=\frac{1\cdot9x^2}{x^2+14\cdot4}$ compared with the experimental values obtained for 6's single crossbred worsted. This latter equation is obtained by substituting $b=1\cdot9$, $k=\cdot25$ in the second equation, *i.e.* the one deduced for single threads from the general equation,

$$y=\frac{bx^2}{x^2+\frac{b^2}{k}}.$$

Fig. 7 shows graphically the relation between the curve for the two-ply thread 2/12's worsted and the graph to $y=2-\frac{\cdot185}{\cdot02x^2+\cdot325}$.

TABLE V.

x.	$y=2-\dfrac{\cdot185}{\cdot02x^2+\cdot325}$.	2/12's Worsted.	$y=\dfrac{1\cdot9x^2}{x^2+14\cdot4}$.	6's Worsted.
0	1·43	1·43	0	·04
1	1·46	1·44	·12	·06
2	1·54	1·54	·40	·26
3	1·63	1·64	·73	·60
4	1·71	1·71	1·00	·99
5	1·78	1·77	1·20	1·21
6	1·82	1·82	1·36	1·36
7	1·86	1·86	1·46	1·46
8	1·89	1·89	1·55	1·51
9	1·91	1·92	1·61	1·54
10	1·92	1·94	1·66	1·62
11	1·93	1·96	1·70	1·65
12	1·94	1·97	1·73	1·69
13	1·95	1·97	1·75	1·72
14	1·96	1·98	1·77	1·75

The asymptote to which the latter graph approaches as x increases is $y=2$. This asymptotic strength is much less than the breaking strength of 1 millimetre of thread, which, as shown in Table IV., is 2·66 lbs. There-

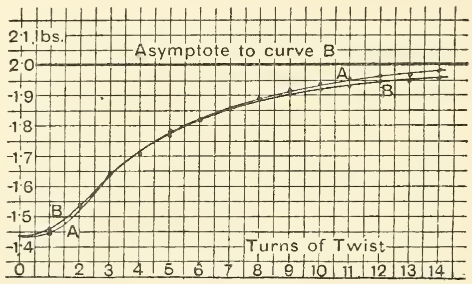

FIG. 7.

fore it would appear not to be possible to construct a thread stronger than about 75 per cent. of the strength of 1 millimetre of thread. The graph of the experimental results practically coincides with the graph of the assumed "witch" curve.

Fig. 8 shows similarly the relation between the curve for the 6's single thread and the graph to $y=\dfrac{1\cdot9x^2}{x^2+14\cdot4}$; the asymptote to which the latter approximates when x is large is $y=1\cdot9$. This maximum strength is 82 per cent. of the strength of 1 millimetre of the 6's worsted thread. The

curves practically coincide within the limits $3\frac{1}{2}$ and 8 turns. These limits would include all yarns of commercial value for textile purposes.

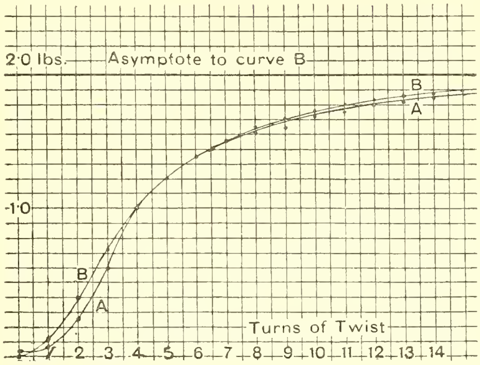

Fig. 8.

The author has pleasure in acknowledging his indebtedness to the Carnegie Trust for the Universities of Scotland for the financial assistance which has enabled him to prosecute this research.

(*Issued separately October* 2, 1907.)

XXVI.—A New Method of determining the Degree of Twist in Single Threads. By Thomas Oliver, B.Sc. (Lond. and Edin.), Carnegie Research Fellow. *Communicated by* Dr C. G. Knott.

(Read May 20, 1907.)

The determination of the degree of twist in a folded or ply thread composed of two or more strands is an easy matter. It is merely necessary to stretch the thread between two clamps, one being fixed and the other forming the end of a bar which can be rotated about its axis. A counter is attached to indicate the number of rotations. Rotating the thread in the opposite direction to its twist ultimately brings the singles parallel to each other. This point is easily observed. If we apply this method to single yarns we are at once confronted with the difficulty that the fibres constituting the yarn cannot be reduced to exact parallelism with each other. A reason for this is that the fibres in the thread before twist is put in are not always parallel. In the formation of the worsted "top" in the combing process, the aim is to arrange the fibres parallel to each other: this object is attained within narrow limits of error. If, in the spinning operation, it were possible to fix the fibres, with respect to their neighbours, so that there would not be any relative motion amongst the fibres in the thread, then, on untwisting the thread again, the fibres would be reduced to a parallel condition. But this end cannot be attained; the fibres in the thread are free in the initial stages of twisting to slip on each other, both longitudinally and circumferentially, within limits. This motion is, however, not so great as to preclude possibility of the degree of twist being determined in combed yarns with a satisfactory degree of accuracy.

Several pieces of apparatus have been devised from time to time, similar to that described above for ply threads, but more accurately constructed. It is usual to place one inch of thread between clamps at the focus of a magnifying lens, or the clamps and torsion apparatus are mounted on a frame so as to slide the thread across the field of a compound microscope fitted with a low-power objective.

When the afore-mentioned methods are applied to carded woollen threads the results are very unsatisfactory. The reason is that, in the processes preliminary to spinning, the fibres of a woollen thread are never reduced to parallelism. The aim in forming the woollen sliver is to make the fibres include a maximum number of air spaces. The distinctive mossy

handle of high-quality woollen goods depends upon this. Many fibre ends project from the surface of the thread. Therefore if, to begin with, in twisting, the fibres are not parallel, by untwisting we cannot arrive at the parallelism which it is the object of microscopic examination to observe.

In the preceding papers * on "Take-up in Twisted Threads" I have shown that the relation which exists between length and twist in a thread is expressible in the form

$$L = l(1 - kx^2),$$

where L is the length of twisted thread.
l „ „ untwisted thread.
x „ turns of twist per inch.
k is a constant.

Evidently the thread is longest when the twist is zero. This is true with every description of single thread, irrespective of its process of manufacture. To devise some form of apparatus which will utilise this property is all that remains to be done.

The apparatus which I have designed for the purpose is shown in fig. 1. A represents ten inches of single thread, which is stretched between clamps B and C. B is carried on the end of a bar, which can be rotated on its axis through toothed wheel N by turning wheel M.

The number of turns are registered on dial T, to which motion is communicated by worm W carried on the rotary bar. The other clamp C is carried on the shorter arm of a light brass bell-crank lever pivoted at G. The pointer of the long arm F can move along scale S, thus indicating any change in the length of the thread. The method of conducting the experiment is as follows:—Take the length of thread such as will cause the pointer to come opposite the top of the scale; read the twist dial. Turn wheel M so that clamp B rotates in the opposite direction to the twist in the thread. Observe the lowest point on scale S to which lever F dips; this point will be reached when the thread is untwisted. Read the twist dial again; the difference of the readings will give the turns of twist in the length of thread under test.

It will be seen from the graph in fig. 2 that the point at which the thread is untwisted gives a stationary value for the length.

There is scarcely any difference on the length for a number of turns on

* "The Relation between Take-up and Degree of Twist in Twisted Threads," *Proc. Roy. Soc. Edin.*, 1906, vol. xxvi., part iii., pp. 182–206. "The Relation between Take-up and Yarn Number for constant Twist in Twisted Threads," *Proc. Roy. Soc. Edin.*, 1906, vol. xxvii., part ii., pp. 93–106. "The Relation between Take-up and Degree of Twist in Twisted Threads with Singles of unequal Sizes," *Proc. Roy. Soc. Edin.*, 1906, vol. xxvii., part ii., pp. 107–116.

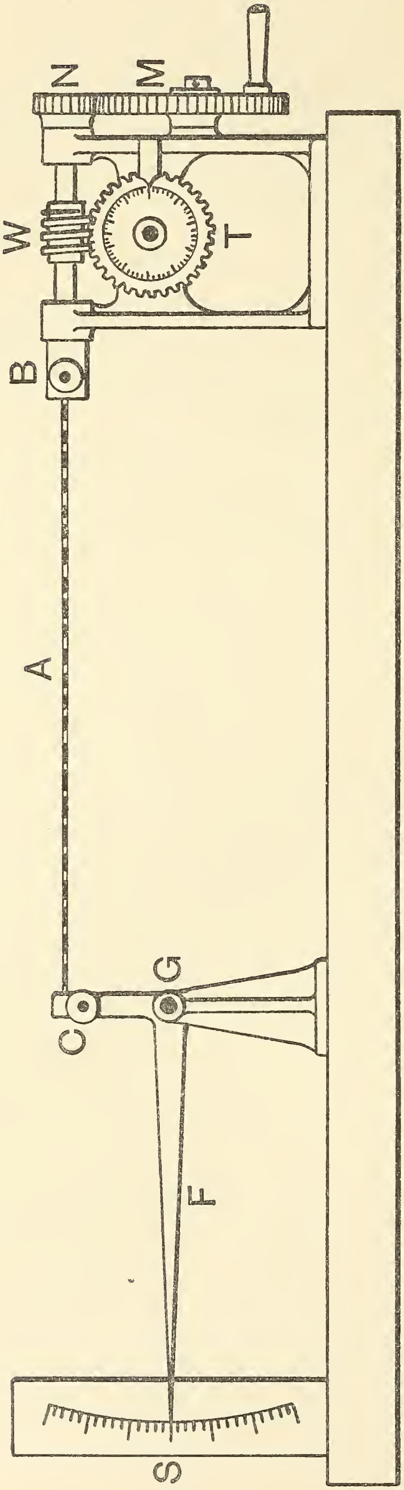

Fig. 1.—A New Method of ascertaining the Twist in Threads.

both sides of the zero twist. It would therefore be impossible to determine the point of maximum length with sufficient accuracy. The following modification of the experiment will, however, meet all requirements.

In fig. 2 the lengths of thread are plotted as ordinates, and corresponding degrees of twist as abscissæ. The zero position O is unknown. Let A on the axis of x represent x, the turns of twist in the thread, an unknown number. Take off a convenient number of turns from the twist of the thread, say a, so that the twist can now be represented by point B in the diagram. Read the position of the pointer on scale S (fig. 1), and also the twist dial. Then turn wheel M quickly in the same direction as before, the observer keeping his eye fixed on scale S. The pointer will dip down, remain practically stationary for a little, then rise up again,—when the

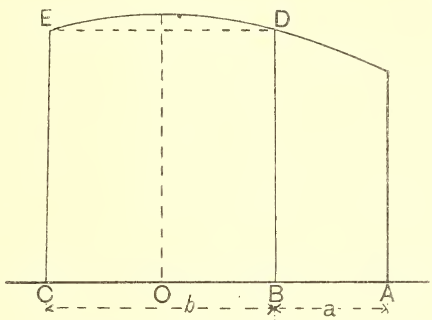

Fig. 2.—New Method of ascertaining Twist in Threads.

pointer comes back to the position on the scale which indicates that the thread has the same length as it had with $x-a$ turns. Let the degree of twist be now represented by point C in fig. 2, and that $BC=b$ turns. Then the zero position O lies half-way between B and C and $\therefore x=a+\frac{b}{2}$.

This method is founded on the principle that x turns of inverse twist produce the same contraction in a single thread as x turns of direct twist.

The reason that the twist must be changed quickly through the zero position is that the untwisted thread can stand very little stress, and the prolonged action of even the light lever F on the thread would produce an elongation due to tension which would falsify the reading on the scale. The number of turns to bring the pointer back to the same position on the scale would be greater than if there had been no stretch due to tension in the thread.

Another method of determining the twist may be arranged with the same form of apparatus provided with a heavier lever F. The twist is thrown out until the thread breaks.

I have shown in a paper on the "Influence of Twist on the Strength of Threads" * that a thread has much the same strength throughout the first turn per inch or so of twist, and afterwards becomes rapidly stronger. The difference of the readings on the twist dial, with probably a turn per inch added, would give the degree of twist in the thread. This method is not nearly so accurate as the first described, because the period of approximately constant strength may vary from a half to even two turns per inch, depending on the various properties of the yarn. The weight of lever F would require to be adjusted so as to produce the breaking tension for the untwisted thread, and no more.

The author has pleasure in acknowledging his indebtedness to the Carnegie Trust for the Universities of Scotland for financial assistance in this research work.

* *Proc. Roy. Soc. Edin.*, 1907, vol. xxvii., part. iii.

(*Issued separately October* 2, 1907.)

XXVII.—The Composition of Terrigenous Deposits. By F. W. Clarke, D.Sc., LL.D., Chief Chemist, U.S. Geological Survey. *Communicated by* Sir John Murray, K.C.B., F.R.S., etc.

(Read July 8, 1907. MS. received July 10, 1907.)

For purposes of comparison with the analysis of a composite sample of Red Clay recently published in the *Proceedings*,* I have received from Sir John Murray a composite of fifty-two "terrigenous" clays, dredged up from oceanic depths ranging from 140 to 2120 fathoms. In the nomenclature of the *Challenger* Expedition, forty-eight of the individual samples are classified as "blue muds," and four as "green muds." Twenty-three of the clays were collected by the *Challenger*; the others were brought in from voyages of the *Buccaneer*, *Dart*, *Egeria*, and *Rambler*. The range of collection, as in the case of the "red clay," was world-wide, and all of the great oceans are represented in the composite sample. The results of analysis appear in the following table :—

	A.	B.	C.
SiO_2,	46·64		57·09
TiO_2,	1·04		1·27
Al_2O_3,	14·08	·14	17 24
Cr_2O_3,	·044		·05
Fe_2O_3,	4·14		5·07
FeO,	1·88		2·30
MnO_2,	·10	...	·12
MgO,	1 95	·18	2·17
CaO,	7·20	·38	2·04
SrO,	·025	...	·03
BaO,	·05	...	·06
K_2O,	1·84	...	2·25
Na_2O,	2·98	2·12	1·05
V_2O_3,	·028	...	·03
P_2O_5,	·17	...	·21
CO_2,	4·05	...	...
SO_3,	·32	·32	...
S,	·11	...	13
Cl,	2·25	2·25	...
CuO,	·016	...	·02
C,	1·38	...	1·69
H_2O at 105°,	4·73	...	...
H_2O above 105°,	5·86	...	7·18
	100·883	...	100·00
Less O = Cl,	·56	...	...
	100·323	...	...

* *Proc. Roy. Soc. Edin.*, vol. xxvii., pp. 167–171.

A. General analysis by Mr Steiger.

B. Portion soluble in water.

C. Analysis reduced to standard form by rejecting soluble salts, calcium carbonate, and hygroscopic water, and recalculation of the remainder to 100 per cent.

Molybdenum and zirconium were not detected. Nickel, cobalt, lead, zinc, and arsenic, which were reported in the red clay, were not looked for. Apart from these trivial omissions, the red and terrigenous clays are fairly comparable. The red clay is lower in silica and alumina, but higher in iron than the muds, and other minor differences appear. The high manganese of the red clay may be correlated with the abundance of manganese nodules in the greater oceanic depths.

(*Issued separately October* 2, 1907.)

XXVIII.—A Preliminary Note on the Optical Rotations (throughout the Spectrum), the Electrical Conductivities, and the Densities of Mixtures of Sodium-Potassium-Tartarate and Ammonium-Molybdate in Aqueous Solution. By James Robert Milne, D.Sc., Carnegie Research Fellow in Physics.

(MS. received June 14, 1907. Read July 8, 1907.)

I. Introduction.

It has long been known that the rotative power of certain optically active salts in aqueous solution is greatly, and in some cases enormously, changed by the addition of certain non-active salts. Of late years a good deal of work has been done on this subject,* and a considerable amount of data has been collected; but, generally speaking, little explanation has been obtained of the precise causes of the observed phenomena. In some cases at least the reason for this no doubt lies in the fact that in such solutions there exist complicated molecular combinations, about which it may not be easy to obtain ordinary chemical evidence. This, however, only makes it the more important that we should utilise to the fullest extent such means of investigation as are readily available, and of these none is so good as that of optical activity.

It seemed to me, therefore, that it would be well to try what could be done to extend the use of this method of investigation, by measuring the rotations of a few of these solutions not only for D light, but also for light of a number of other wave lengths chosen at intervals throughout the spectrum. In this way a great deal of additional optical information would be obtained; for the usual method only, as it were, runs a single traverse over the unknown country whose topography is to be explored.

Of course in some cases it has been found that this procedure is sufficient, and that, given the rotation of a substance for one wave length of the spectrum, the rotation for any other wave length can be deduced from that of the first by the use of a mathematical formula. Quartz is a very good example of this, for its observed rotations all fit very accurately into Boltzmann's formula $a=\frac{A}{\lambda^2}+\frac{B}{\lambda^4}$. On the other hand, however, there were early discovered cases of strikingly abnormal rotation-dispersion; for

* References, and a good summary of the chief results, may be found in H. Landolt's *Das Optische Drehungsvermögen* (second edition).

instance, the case of aqueous solutions of tartaric acid, in which the maximum rotation is found as usual at the violet end of the spectrum when the solutions are weak, but becomes displaced at greater concentrations, and, travelling along the spectrum, finally arrives at the red end. Such abnormal cases show that the information to be obtained from spectro-polarimetry will by no means be necessarily a repetition of, or mere variant on, that which can be obtained by polarimetry alone.

The present note is only of a preliminary nature, for the purpose of giving a first instalment of data; for it has been found that already certain interesting conclusions may be established.

II. Description of the Experiments.

The active and the inactive salts employed were respectively sodium-potassium - tartarate and ammonium - molybdate, and both were of Kahlbaum's manufacture. The solutions were prepared by weighing out the required quantities of the two salts, and placing these quantities in a graduated flask, which was then filled up to the mark with distilled water at 25° C. The flasks employed were specially graduated by myself, so as to give true indications at the temperature mentioned, which is that at which all the measurements were carried out.

The method of experiment was as follows. Each solution was made up to always be a third normal as regards the tartarate, and some simple fraction of a third normal as regards the molybdate. Each solution was measured not only for its optical rotation at various parts of the spectrum, but also for density and for electrical conductivity, separate portions being always set aside for each purpose. The object of the two latter kinds of measurement was to obtain if possible other indications of the changes in the solution, with a view to comparing them with the optical indications.

In carrying out the work the following order of solutions was observed. First of all a $\frac{1}{3}$ N solution of the tartarate alone was made up and measured. Then a solution was made up containing $\frac{1}{3}$ N tartarate and $\frac{1}{8}\cdot\frac{1}{3}$ N molybdate and similarly measured. The next solution contained $\frac{1}{3}$ N tartarate and $\frac{1}{4}\cdot\frac{1}{3}$ N molybdate, and so on; the successive concentrations of molybdate in terms of $\frac{1}{3}$ N being 0, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1.

1. *The Density Measurements.*

Little requires to be said about these. Ostwald's form of Sprengel pyknometer was employed. It was filled with the solution and placed in a thermostat bath at 25° C. until no further movement was observed on the

part of the liquid in the stem. It was then adjusted, wiped dry, and weighed. The resulting densities were found to be—

Concentration of the Molybdate in the Solution.	Density of the Solution at 25° C.
0	1·0260
$\frac{1}{8}$	1·0318
$\frac{1}{2}$	1·0414
$\frac{3}{4}$	1·0598

These are plotted in fig. 1.

A test of the accuracy of the measurements is afforded by comparing the figures obtained in two completely independent experiments which were made on one of the solutions. The first experiment gave the density as 1·03178, the second as 1·03180, a result which seems very satisfactory.

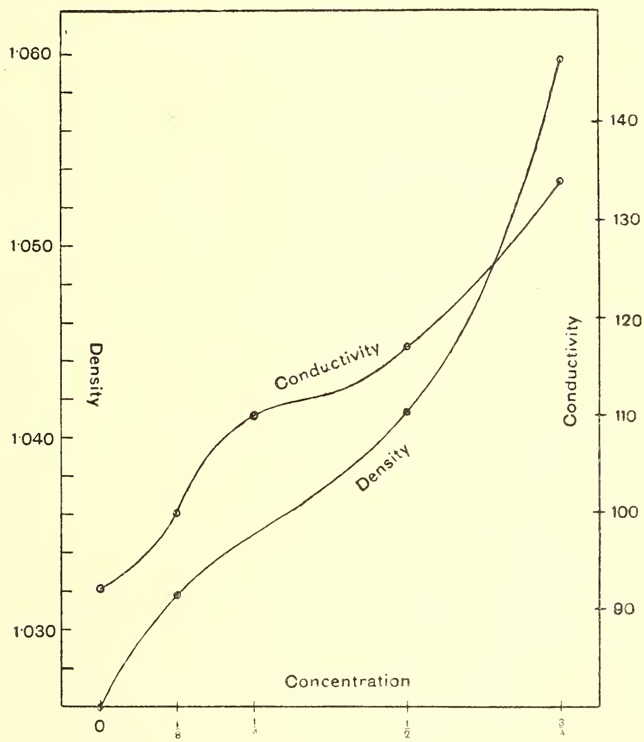

Fig. 1.

2. *The Conductivity Measurements.*

These were carried out by Kohlrausch's method. The solution was contained in a U tube also placed in a thermostat bath. The metre wire

bridge was carefully calibrated beforehand against a standard box of coils, as also were the comparison resistances. The latter took the form of three coils of 100, 200, and 400 ohms resistance respectively. In every case three different combinations of these coils were made—by means of a mercury key—and the three measurements so obtained were never found to differ by more than a unit in the third significant figure. The connections between bridge, coils, and U tube were made with heavy copper wires whose resistances had been found to be negligible on measurement. To ensure correctness of temperature in the case of the solution, the final readings were not made until the U tube had stood for half an hour or so in the bath, and had thereafter given identical readings on two occasions separated by a considerable interval of time. The resistance constant of the cell was measured by means of a N/50 KCl solution. The electrical measurements gave the following results:—

Concentration of the Molybdate present in the Solution terms of N/3.	Molecular Conductivity at 25° C.
0	92·2
$\frac{1}{8}$	100
$\frac{1}{4}$	110
$\frac{1}{2}$	117
$\frac{3}{4}$	134

These are plotted in fig. 1.

3. *Rotation Measurements.*

The instrument used for the rotation measurements was a special form of spectro-polarimeter invented by myself, the cost of which was defrayed by a grant from the Carnegie Trustees. The solutions were contained in a jacketed glass tube 49·86 cm. long, through the jacket of which a constant stream of water was passed, that had previously been made to traverse long coils of tubing immersed in a special thermostat bath. The temperature regulation of the tube was not quite so satisfactory as could have been wished. A difference of about 0°·4 C. existed between the two ends of the polarimeter tube, as measured by thermometers let down branch tubes into the solution. It was also found difficult to keep the mean temperature exactly the same in successive experiments, as will be seen from the figures attached to each set of measurements.

The dispersion prism of the instrument was calibrated by the employ-

ment of H and He vacuum tubes, and by the use of Na, Sr, Th, and Li salts in a Bunsen flame. The results were as follows:—

Reading of telescope scale,	154·5	156	157	158	159	160
Mean wave length,	455	487	515	550·5	598·5	666
Longest wave length in the transmitted light,	463	496·5	525·3	565	628	745
Shortest wave length in the transmitted light,	447	476·5	500	530	573	637
The consequent impurity,	16	20	25·3	35	55	108

The settings of the Nicol prism of the instrument were automatically recorded by the author's photographic recorder, to be subsequently described,

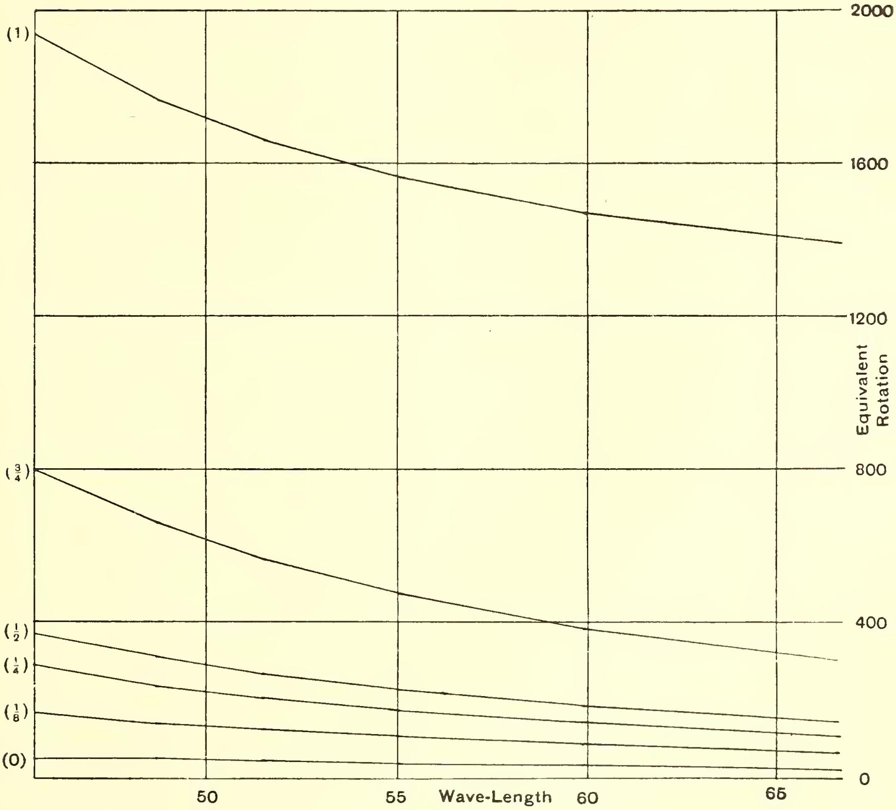

Fig. 2.

the expense of which was met by a grant from the Carnegie Trustees: a plan which was found to greatly facilitate the work. The zero positions of the Nicol were ascertained with the polarimeter tube in place and filled with distilled water at 25° C., and thereafter the end discs of the tube were

never unscrewed, so as to ensure that the zeros would remain constant during the subsequent measurements.

At each selected point in the spectrum nine settings of the Nicol were made, and the mean taken. As regards the accuracy of these measurements two considerations come in.

A large rotation makes the observer's error in setting the Nicol proportionally of less account. But then, on the other hand, a large rotation is

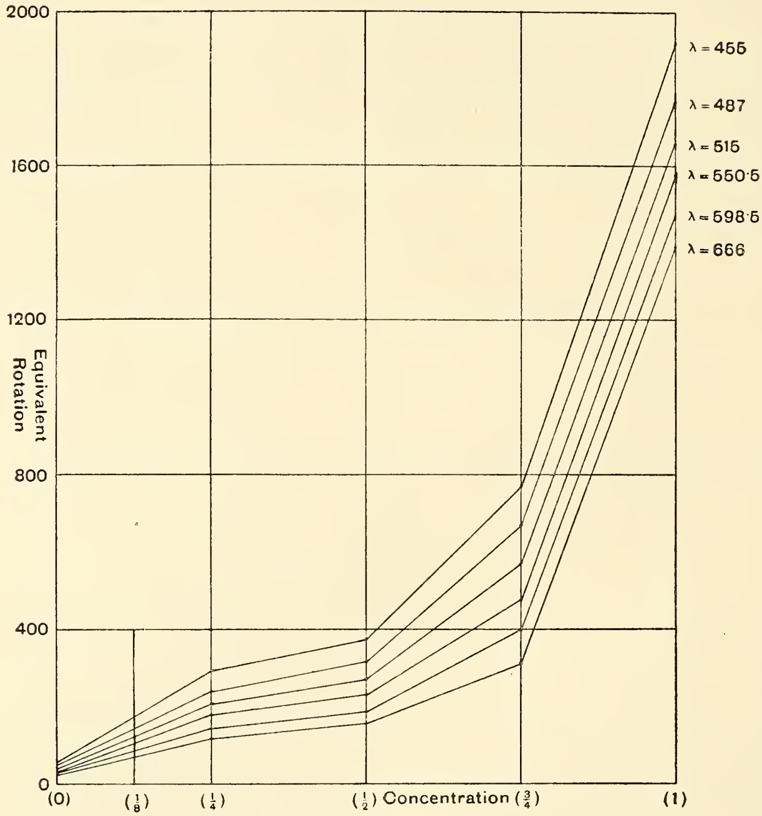

Fig. 3.

usually accompanied by a large dispersion, which has the effect of somewhat increasing the observer's error.

I have accordingly calculated the probable error of the settings of the Nicol in two different cases. In the first of these the rotation dispersion was zero, the polarimeter tube containing water only; in the second it was large. The results are respectively 0′·458 and 2′·46. The latter probable error is that obtained from the nine settings made at $\lambda = 515$ on the strongest of the solutions. As the measured rotation in this case was

277°·42, it follows that the percentage probable error of the measurement in this case is equal to $\frac{2'{\cdot}46 \times 100}{277^\circ{\cdot}42} = 0{\cdot}0148$ per cent.

The rotation results are as follows (see also figs. 2 and 3):—

λ	Solution (0). E.	Solution ($\frac{1}{8}$). E.	Solution ($\frac{1}{4}$). E.
666	25°·05	69°·6	114°·9
598·5	30·35	85·6	143·4
550·5	36·15	104·0	173·0
515	41·15	121·3	203·5
487	46·50	138·1	235·6
455	51·40	166·7	289·0

λ	Solution ($\frac{1}{2}$). E.	Solution ($\frac{3}{4}$). E	Solution (1). E.
666	151°·1	310°·4	1404°
598·5	186·2	392·7	1482
550·5	228·0	481·0	1575
515	269·1	571·0	1668
487	314·3	672·0	1772
455	370·0	769·0	c. 1944

Temperatures.

Concentration.	(0) c.		($\frac{1}{8}$) c.		($\frac{1}{4}$) c.		($\frac{1}{2}$) c.		($\frac{3}{4}$) c.		(1) c.	
	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.
I.	24°·8	25°·4	25°·1	25°·1	25°·0	25°·1	25°·0	25°·7	25°·2	25°·6	25°·1	25°·6
O.	24·3	24·8	24·6	24·6	24·2	24·4	24·4	25·1	24·4	25·0	24·6	24·8

Where "A" is the temperature of the solution *after* the rotation measurements.

"B" is the temperature of the solution *before* the rotation measurements.

"I" is the temperature of the solution at the end of the tube at which the heating water entered.

"O" is the temperature of the solution at the end of the tube at which the heating water left.

Note.—The measurements were always made in regular order through the spectrum from red to violet.

The values of the rotation-dispersion were obtained by taking the differences of the "Es" and the "λs" in the above table: the "λs" given in the table below are the means of the former "λs" for an obvious mathematical reason.

λ	ΔE/Δλ					
	(0) c.	($\frac{1}{8}$) c.	($\frac{1}{4}$) c.	($\frac{1}{2}$) c.	($\frac{3}{4}$) c.	(1) c.
632	0·0785	0·237	0·422	0·520	1·22	1·16
574·5	0·121	0·383	0·617	0·871	1·84	1·94
533	0·141	0·487	0·859	1·16	2·54	2·62
501	0·191	0·600	1·15	1·61	3·61	3·71

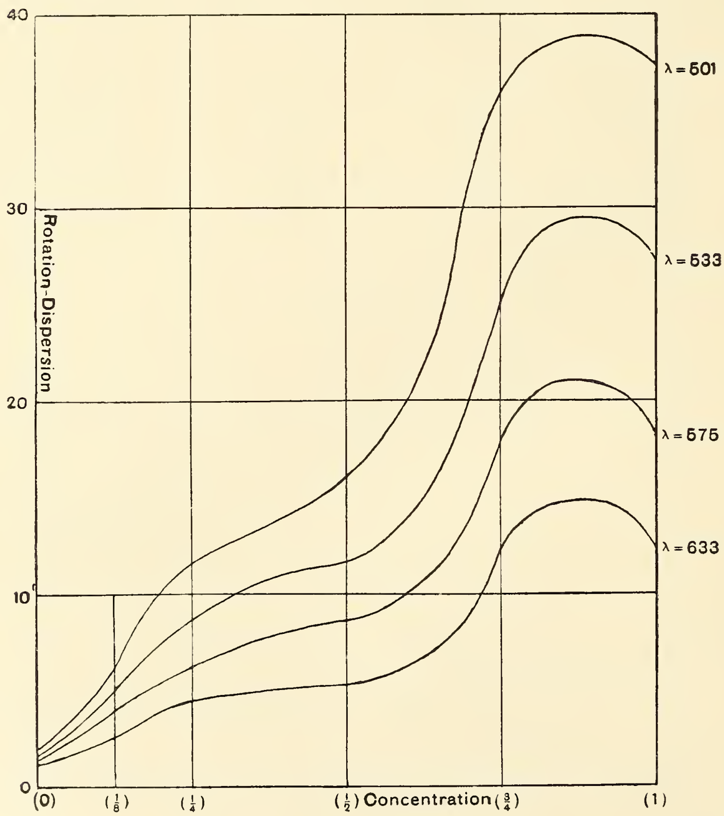

Fig. 4.

III. Discussion of the Observations.

On examining figures 1, 3, and 4 it will be seen that an increase in the concentration of the molybdate salt produces an increase in the density, conductivity, rotation, and rotation-dispersion of the $\frac{1}{3}$ normal tartarate solution. Further, it will be noted that whereas these curves are in general concave to the concentration axis, for a short distance in the neighbourhood of concentration $\frac{1}{2}$ they are in every case convex. Such unanimity is no doubt significant.

Another point of interest is as follows. The original tartarate solution has its rotation-dispersion governed approximately by the simple law, (Rotation) $\times$ (Wave-length)^2 a constant. This is brought out in the subjoined table. There is also given there, for the sake of comparison, the figures obtained in a similar way for the other solutions, and it will be seen that the addition of the molybdate in general causes departure from the above law. It will be noted, however, that whereas the value of $a\lambda^2$ decreases with increase of λ in the case of solutions ($\frac{1}{8}$), ($\frac{1}{4}$), and ($\frac{3}{4}$), it increases for solution (1). Hence there is probably some concentration lying between ($\frac{3}{4}$) and (1) for which the law once more holds.

λ	Solution (0). E λ^2.	Solution ($\frac{1}{8}$). E λ^2.	Solution ($\frac{1}{4}$). E λ^2.
666	1111	3088	5097
598·5	1087	3066	5136
550·5	1096	3152	5243
515	1091	3217	5398
487	1103	3276	5589
455	1064	3451	5983

λ	Solution ($\frac{1}{2}$). E λ^2.	Solution ($\frac{3}{4}$). E λ^2	Solution (1). E λ^2.
666	6704	1377	6229
598·5	6670	1407	5308
550·5	6910	1458	4774
515	7137	1514	4424
487	7454	1594	4203
455	7660	1592	c. 4024

As regards the rotation dispersions plotted in figure 4, the convexity of all the curves near concentration ($\frac{1}{2}$) has already been pointed out. It is possible that the existence of a maximum in all the curves between ($\frac{3}{4}$) and

(1) may be connected with the fact that between these two concentrations we probably have $a\lambda^2$ = const.

Any attempt at explanation of the underlying causes of these very interesting phenomena is probably best postponed until further data become available.

It remains to express my best thanks to Professor Clifton for his kindness in granting me facilities for carrying out the above research in the Clarendon Laboratory, Oxford.

(*Issued separately October* 3, 1907.)

XXIX.—**Degenerations following Experimental Lesions in the Motor Cortex of the Monkey.** By **Sutherland Simpson**, M.D., D.Sc., and **W. A. Jolly**, M.B. (*From the Physiological Laboratory, University of Edinburgh.*) *Communicated by* Professor E. A. Schäfer, F.R.S.

(MS. received July 5, 1907. Read July 15, 1907.)

The object of the present research was to follow, by the degeneration method, the course of the fibres proceeding from definite and limited areas of the motor cortex, and to determine to what extent there is a grouping or localisation of the fibres of the pyramidal tract at different levels in the brain and spinal cord.

The statement given in most text-books of anatomy and physiology that such a grouping exists in the internal capsule is based mainly upon experimental evidence obtained by Beevor and Horsley * in the macaque monkey and in the orang-outang. These observers found that on electrically exciting the capsule in transverse section no motor response could be obtained from the anterior limb or lenticulo-striate portion, but that the genu and the thalamo-lenticular portion of the posterior limb were excitable, and that definite movements were called forth by the stimulation of definite areas within these parts. From before backwards they found the fibres to be arranged in the following order:—farthest forward are situated those which control the movements of the eyes, then come those for the opening of the mouth, next those governing the movements of the head and eyes, and behind these the fibres for the tongue and the angle of the mouth. Just posterior to these lie the fibres for the anterior limb in the following order,—shoulder, wrist, fingers and thumb; behind these come the trunk fibres, and most posterior of all those for the hind limb in the order of hip, ankle, knee, hallux, toes from before backwards. According to these observers, therefore, the arrangement of the fibres in the internal capsule is a reproduction of that of the areas on the cortex from which they arise, as determined by them.† The fibres situated at the genu of the capsule, *i.e.* farthest forward, come from the lowermost region of the motor cortex, behind them those from the middle region, and most posteriorly those from the uppermost portions of the motor area.

* Beevor and Horsley, *Phil. Trans. Roy. Soc. Lond.*, vol. clxxxi. B. (1890), p. 129.

† Beevor and Horsley, *loc. cit.* (see fig. 7 in their paper).

By the degeneration method Mellus * found that fibres from the hallux and thumb centres, after passing through the corona radiata, enter the internal capsule towards its posterior extremity, while those from the face area enter it at its anterior extremity. As they pass downwards the leg and arm fibres become displaced forwards, while those from the face area are displaced backwards, until in the lower levels of the capsule the fibres from all three areas are crowded together in the middle third of the posterior limb. In the movement of the facial fibres backwards between the upper and lower planes of the capsule at a certain level they will be found at the genu, the position which they are usually said to occupy, but above this particular level they are in front of, and below it they are behind, the genu.†

In the cerebral peduncle the area occupied by the fibres of the pyramidal tract ‡ in transverse sections is variously given by different authorities, and also the relative positions of the fibres amongst themselves. They are often described as occupying the middle third of the crusta, the leg fibres lying most external, the face fibres most internal, and the arm fibres between. Van Gehuchten § ascribes to them the middle three-fifths of the crusta. Stanley Barnes, ‖ in several cases where the whole of the fibres from the Rolandic area had been interrupted, found that the fibres in the middle region of the crusta were all degenerated; the inner two-fifths (the fronto-pontine region) and the outer one-sixth (the temporo-pontine region) were in the main free, but the distinction between the degenerated and the undegenerated regions was not sharp, the two sets of fibres intermingling at the junction.

In the monkey, Mellus ¶ found that the pyramidal fibres take up the middle third of the crusta, and that the face fibres are mixed up with those from the leg and arm areas and do not occupy a space to themselves mesial to the latter.

Regarding the arrangement of fibres in conducting tracts generally in the spinal cord Sherrington ** in 1893 pointed out that for the ascending

* Mellus, *Proc. Roy. Soc. Lond.* (1894), vol. lv. p. 208, and (1895) vol. lviii. p. 206.

† In the literature the term "internal capsule" is very loosely applied. In every case where a lesion or degeneration of the capsule is described its exact horizontal level should be given.

‡ Strictly speaking, the term "pyramidal tract" should be applied only to those fibres which pass to the spinal cord and govern the movements of the trunk, arm and leg, but by most writers it is used in the most comprehensive sense, and includes all the projection fibres arising from the motor cortex.

§ Van Gehuchten, *Système nerveux de l'homme*, 4th ed. (1906), p. 907.

‖ Stanley Barnes, *Brain*, vol. xxiv. (1901), p. 464.

¶ Mellus, *loc. cit.*

** Sherrington, *Jour. of Physiol.*, vol. xxiv. (1893), p. 298.

tracts the longer fibres tend to occupy a more peripheral position, the shorter fibres to lie nearer to the grey matter. Flatau* (1897) and others have found that a similar arrangement obtains both for the spino-cephalic and for the longitudinal commissural fibres terminating in the grey matter and uniting the different segments of the cord to one another; those fibres which unite adjacent segments lie close to the grey matter, while those passing between more distant segments are situated nearer to the periphery of the cord. Sherrington and Laslett,† again, in 1903 showed that the fibres of the direct cerebellar tract of Flechsig are arranged in definite strata, those from the post-thoracic region of the cord lying outermost, those of most anterior origin lying innermost, and fibres of intermediate origin forming intermediate strata.

In view of the observations above recorded (and many more are to be found in the literature corroborative of these), it might naturally be supposed that the same arrangement would be found in the fibres of the descending tracts of the spinal cord, particularly in the pyramidal tract, the largest and most important of these, and it has been stated by Gad and Flatau ‡ that such is the case. These observers, adopting the direct excitation method, such as was employed by Beevor and Horsley in the case of the internal capsule, divided the cord transversely in the cervical region below the origin of the phrenics, and stimulated with the faradic current the freshly made cross section of the caudal portion within the area of the crossed pyramidal tract. They used in their experiments large dogs. They came to the general conclusion that the fibres innervating the forelimbs and upper segments of the body lie nearer to the grey matter than those which supply the hindlimbs and lower segments. This would appear to agree with the above-mentioned law for the ascending and internuncial fibres of the cord, viz., that the short fibres run nearer the grey matter, the long fibres nearer the periphery.

Ziehen § is quoted by them as having come to a somewhat different conclusion by the degeneration method. In dogs which had been operated on by H. Munk, he found that after extirpation of limited portions of the motor cortex the degenerated fibres from the forelimb area in the upper cervical region lay nearer the grey matter, while those from the neck area were placed outside them, nearer the periphery. There was differentia-

* Flatau, *Sitz. der königl. preuss. Akad. d. Wissen. zu Berlin*, 1897.

† Sherrington and Laslett, *Jour. of Physiol.*, vol. xxix. (1903), p. 191.

‡ Gad and Flatau, *Neur. Cent.*, 1897, p. 481.

§ Ziehen, *Archiv f. Psychiat.*, 1887, p. 300.

tion of the fibres, but the arrangement was different from that found by Gad and Flatau.

Sherrington,* on the other hand, in the monkey, found only slight evidence of any localisation of fibres within the pyramidal tract in the spinal cord. After extirpation of the more mesial and anterior parts of the motor cortex (leg and trunk areas presumably), the most marked degeneration of the crossed pyramidal tract was to be found bordering the dorsal cerebellar tract, while, on the other hand, this region was only slightly affected when the extirpation was in the more lateral and posterior parts of the motor cortex (arm and face areas). Otherwise the degeneration was scattered uniformly over the whole field of the transverse section of the tract.

Mellus † found, similarly, that after lesions in the hallux and thumb centres the degeneration in the crossed pyramidal tract in the cord was evenly scattered over its entire area. The thumb centre, however, he located behind the fissure of Rolando, in the post-central convolution,—a region which is now believed to lie outside the motor area.

Thus, according to those observers who have employed the physiological method of direct electrical excitation, there would appear to be a definite grouping of the fibres within the pyramidal tract in the internal capsule and in the spinal cord, corresponding to the grouping of nerve cells in the cortex from which these fibres arise. Tested by the anatomical or degeneration method, on the other hand, the statements are conflicting, and with regard to the spinal cord the majority of observers have found that no such grouping exists.

Methods employed.

In our experiments we have used monkeys—*Macacus rhœsus* chiefly—and a few specimens of *callithrix*. In each case the animal was completely anæsthetised with ether, the scalp, after being shaved and cleansed with carbolic acid and alcohol, was incised, a skin flap turned down, and a trephine opening made in the skull over the fissure of Rolando. This opening was then enlarged with bone forceps. After reflection of the dura and exposure of the cortex, the centre to be extirpated was located by stimulation with a weak faradic current, Sherrington's unipolar method being employed in the manner described by us in a former communication.‡ A small portion of the cortex embracing the centre located was

* Sherrington, *Jour. of Physiol.*, vol. x. (1889), p. 429.

† Mellus, *loc. cit.*

‡ *Proc. Roy. Soc. Edin.*, vol. xxvii., pt. i., p. 64.

destroyed by a thermo-cautery, the depth of the lesion being presumably sufficient to cauterise the grey matter entirely without damaging the underlying white matter to any appreciable extent. In some cases the lesions were more extensive, and involved the cortical area for the entire limb (arm or leg), and in one case the whole and in another almost the whole of the motor cortex, both on the lateral and mesial aspects. The extirpation was made on the left side in some instances, and on the right in others. After replacing the dura mater, the scalp wound was closed with horse-hair sutures and sealed with a collodion dressing. The operations were carried out with the strictest aseptic precautions, and in every case the wounds healed by first intention.

The animals were allowed to live for a period of three weeks after the operation, during which time the symptoms following the lesions were observed and recorded. At the end of that time they were killed by an overdose of chloroform, and the brain and spinal cord of each removed and placed in a large quantity of a 3 per cent. solution of potassium bichromate for three weeks, with frequent changing of the fluid. Before it was cut into slices to be stained by the Marchi method, the brain was photographed in order to show the position and the superficial extent and depth of the lesion.

In staining, Busch's modification of the Marchi method was used in a few of our earlier cases, but in our experience the results obtained were not always entirely satisfactory, and subsequently we employed Van Gehuchten's modification of this method.* The fluid used by him consists of a mixture of a 1 per cent. solution of osmic acid and a 3 per cent. solution of potassium bichromate, in the proportion of 1 part of the former to 4 parts of the latter. The tissue is allowed to remain in the fluid for three weeks; better penetration is claimed for the weaker solution of osmic acid acting for a longer time, and this we found to be the case. The segments, after staining, were imbedded and cut in either colloidin or paraffin, and sections were made at different levels extending from the corona radiata to the lower extremity of the spinal cord.

Results obtained.

As it has been shown by Sherrington † and by ourselves ‡ that the directly excitable motor cortex in the monkey is less extensive than was formerly supposed, and as in all previous extirpations, so far as we know,

* Van Gehuchten, *Système nerveux de l'homme*, 4th ed., 1906, p. 340.

† Sherrington, *The Integrative Action of the Nervous System*, 1906, p. 297.

‡ *Proc. Roy. Soc. Edin.*, vol. xxvii., pt. i., p. 64.

made with the object of involving the whole motor area, the ascending parietal convolution (which is now known to produce no motor response on direct excitation) had been included, we deemed it necessary, in the first place, to determine, in one animal, the extent of the degeneration following from destruction of the entire motor cortex on the lateral and mesial aspects of the cerebral hemisphere, without including the ascending parietal gyrus. This area includes practically the whole width of the precentral convolution, and extends from a line about 3 millimetres above the fissure of Sylvius over the border of the hemisphere on to the marginal convolution on its mesial aspect (fig. 1). The lesion was made on the

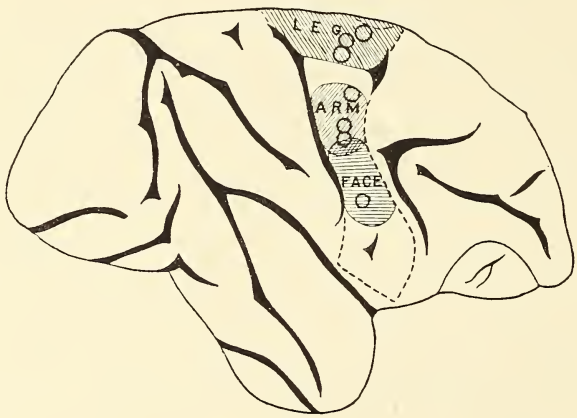

Fig. 1.—Lateral aspect of right cerebral hemisphere. The area of total extirpation (exclusive of "head and eyes") is included within the interrupted line. The shaded parts indicate the extent of the destructive lesions of the leg, arm, and face areas respectively, while the circles represent the limited lesions made within these areas.

right side. The animal was killed twenty-three days afterwards, and the brain and cord were treated in the manner already described. We shall first describe in detail the degeneration resulting from this total extirpation, and this will serve, in position and extent, as a standard of comparison for the localisation within the pyramidal tract at different levels from above downwards of the fibres proceeding from the limited lesions of the motor area of the cortex.

Degeneration following Extirpation of the entire Motor Cortex.

From the area of extirpation numerous fibres, coarse and fine, radiate into the subjacent white matter; many of the fine fibres (association fibres)

pass to the neighbouring convolutions of the frontal and parietal lobes. No attempt was made to trace them to the cortex in the occipital and temporal lobes. A considerable number of fibres, mainly fine, cross through the corpus callosum to the opposite hemisphere, and many of these can be followed to the precentral convolution of that side. The coarse fibres, and also many of the fine, after crossing, turn downwards in the corona radiata, and probably pass into the internal capsule of the opposite side.

Most of the coarse fibres, with a large intermixture of medium and

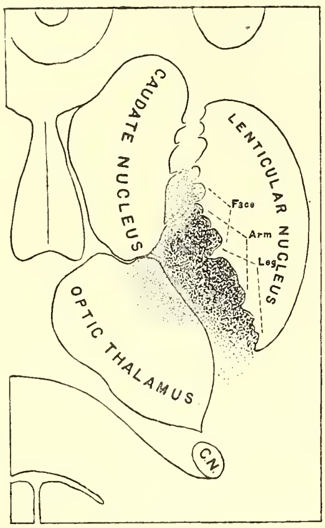

Fig. 2.—Horizontal section through right cerebral hemisphere cutting fibres of internal capsule transversely; upper level. This, and the subsequent figures, show the extent of the degeneration following total destruction of the right motor cortex exclusive of the "head and eyes" area.

fine fibres, pass into the corona radiata of the same side, and can be followed downwards into the internal capsule. *In the upper levels of the capsule* in horizontal sections* (fig. 2) the degeneration covers a wide area. This area extends from a point about the middle of the lenticulo-striate portion (corresponding to the anterior limb at a lower level) anteriorly, to a point a little behind the posterior extremity of the putamen posteriorly (fig. 2). The degenerated fibres are most numerous in the anterior half of the thalamo-lenticular portion of the capsule, gradually becoming fewer both behind and in front of this. There is a

* The section described corresponds to a horizontal plane passing through the lenticular nucleus a little below its upper surface. It includes the head and a large part of the body of the caudate nucleus.

narrow zone bordering the optic thalamus, corresponding to the external medullary lamina of the thalamus, which shows an abundant fine degeneration, but contains fewer coarse fibres. Fine fibres are very numerous in the grey matter of the optic thalamus ; they terminate chiefly in the lateral nucleus. Some are found in the anterior nucleus, but practically none in the mesial nucleus. The grey matter of the caudate and lenticular nuclei are quite free. Only a very few of the coarse fibres are to be found in the capsule behind the posterior end of the lenticular nucleus, but the fine degeneration extends beyond that point.

In the lower levels of the capsule the genu and the anterior limb are practically free from degeneration, both coarse and fine ; it is confined

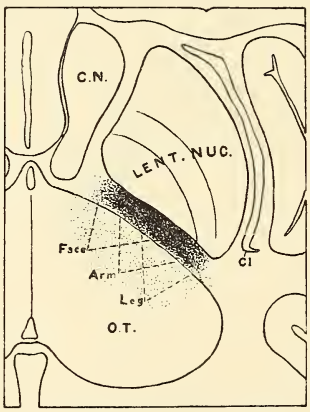

Fig. 3.—Internal capsule, lower level. C.N., head of caudate nucleus. O.T., optic thalamus. Cl., claustrum.

entirely to the posterior limb, and does not extend much behind the posterior extremity of the lenticular nucleus (fig. 3). The fine fibres are less numerous than in the higher levels of the capsule, but they are very abundant amongst the nerve cells in the lateral nucleus of the thalamus. In this section the posterior commissure is cut through longitudinally, but no fibres can be seen to pass across it to the other side. The degeneration shows the greatest concentration about the middle of the thalamo-lenticular portion of the posterior limb. On the external (lenticular) aspect of the capsule a few detached bundles of fibres radiate outwards, but they cannot be traced to their destination. No degeneration is visible in the grey matter of the lenticular nucleus towards which these bundles are directed.

In sections through the mid-brain at the level of the third nerve root

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(fig. 4) the degenerated fibres are found to be included for the most part in the middle three-fifths of the crusta, roughly speaking, but they are not limited to this region. Both in the mesial fifth (fronto-pontine) and in the external fifth (temporo-pontine) are to be found many scattered fibres; they become fewer and fewer as one passes away from the area of dense degeneration in the middle part of the crusta, and only disappear entirely in the very outer part of the external fifth. The degeneration is most concentrated in the middle portion (about the middle third) of the crusta, and gradually shades off laterally and mesially from this area. Many fibres pass backwards from the posterior aspect of the crusta; some end in the grey matter of the substantia nigra, others can be followed beyond this into the tegmentum. There is no evidence of any distinct

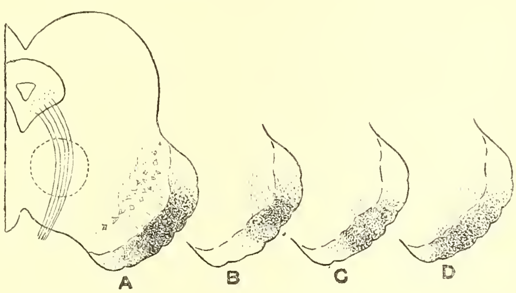


Fig. 4.—Transverse section of mesencephalon, showing degeneration in crusta following destruction—A, of entire motor cortex; B, of leg area; C, of arm area; D, of part of face and arm areas.

bundle of fibres leaving the crusta to join either the mesial or lateral fillet, as has been described by Hoche, Barnes, Spitzka, and others in the human subject.

In the uppermost regions of the pons, where the pyramidal tract begins to be subdivided into bundles of varying size by the transverse cerebellar fibres, the degeneration affects all the bundles, but not uniformly. There are several small bundles lying ventro-laterally to the main mass, and flattened antero-posteriorly; these and the lateral portions of the larger bundles show only slight degeneration. It is densest in the central and larger bundles, and fades away both in the mesial and in the lateral directions from these, but the degenerated fibres are more numerous in the mesial than in the lateral bundles. It is noteworthy that none of the bundles, even in the higher levels of the pons, are free from degeneration following a lesion strictly confined to the motor cortex, neither the mesial (fronto-pontine) nor the lateral (temporo-occipito-pontine) bundles. They contain a mixture of normal and degenerated fibres, in which the former predominate.

In the middle (fig. 5) *and lowermost regions of the pons* these smaller bundles on the lateral, ventro-lateral, and mesial aspects gradually disappear, and the degeneration becomes more uniformly scattered throughout the central and larger bundles, which at the lowest levels are reunited into a single fasciculus just before it passes into the medulla oblongata to form the anterior pyramid.

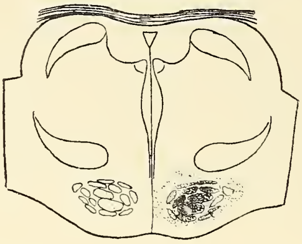


Fig. 5.—Transverse section through middle of pons varolii.

Throughout the pons, in all its levels, there is a very copious fine degeneration amongst the nerve cells of the nuclei pontis on all aspects of the pyramidal bundles. This fine degeneration is strictly confined to the side of the lesion, and does not extend beyond the median raphe. No degeneration is to be found in the fillet, and no fibres can be traced to any of the motor nuclei either in the pons or mesencephalon.

In the highest levels of the medulla oblongata the transverse section

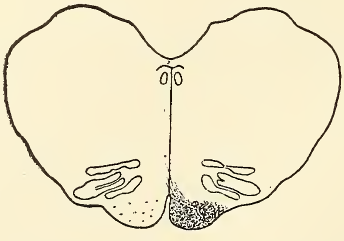


Fig. 6.—Transverse section through upper part of medulla oblongata.

of the anterior pyramid is oval in shape, being flattened antero-posteriorly, but slightly below this, where the plane of the section passes through the upper extremity of the inferior olivary nucleus, it is roughly triangular, and from the sharp postero-mesial angle a few fibres can be seen to come off (fig. 6); they are directed backwards and inwards, and after crossing in the ventral part of the median raphe they are lost in the formatio reticularis of the opposite side. These continue to come off, not in bundles, but as single fibres, and to take the course described, throughout the whole upper

half of the medulla oblongata until the true decussation of the pyramids begins. The degeneration in the pyramid is dense, and is scattered uniformly over its whole area. The fine fibres have to a large extent disappeared, and have evidently terminated in the grey matter of the optic thalamus, substantia nigra and pons.

The decussation of the pyramids is first observed in sections passing through a plane slightly below the level of the lower end of the inferior olivary nucleus, in the closed part of the medulla oblongata. The fibres leave the postero-mesial angle of the pyramid in bundles, and passing backwards, inwards and downwards, cross the raphe, interlacing with similar bundles of normal fibres from the sound side. They can be traced through successive sections, and are seen to pass obliquely downwards through the formatio reticularis until, at the junction of the bulb and spinal cord, they take up a position in front of the substantia gelatinosa

Fig. 7. — Transverse section through lower part of medulla oblongata; middle of pyramidal decussation showing homolateral fibres passing towards side of lesion.

of Rolando.* About midway between the beginning of the decussation and the junction of the medulla oblongata and cord (fig. 7) a few separate fibres can be seen to turn off from the bundles which are about to cross the raphe, and to pursue a course on the side of the lesion exactly similar to that just described for the crossed fibres. As the sections are followed downwards these homolateral fibres are given off from the degenerated pyramid in increasing numbers, and at the lower limit of the decussation the ratio of the crossed (heterolateral) to the uncrossed (homolateral) is about 40 to 1, roughly speaking. All that now remains of the degenerated anterior pyramid is a thin lamina of fibres lying along the border of the anterior median fissure in its posterior half. It can be traced well into the first cervical segment, where it disappears. It represents the direct (anterior) pyramidal tract in man and the anthropoid apes. In the first cervical segment there are thus three pyramidal tracts—crossed, direct

* The whole of the lower half of the medulla oblongata was cut and mounted serially, not a single section being missed.

lateral and direct anterior; below this there are only two. There is no evidence of any direct ventro-lateral pyramidal tract like that described by Barnes in the human subject, and nothing corresponding to Pick's bundle is to be seen.

In the 1*st cervical segment of the spinal cord* (fig. 8) the crossed pyramidal tract occupies an extensive area in the posterior part of the lateral column. It lies in front of, and in contact with, the substantia gelatinosa of Rolando, extending from the grey matter to the periphery of the cord. A few scattered degenerated fibres are seen in the dorsal

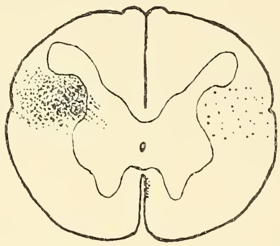


Fig. 8.—First cervical segment of spinal cord.

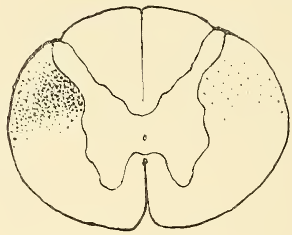


Fig. 9.—Fifth cervical.

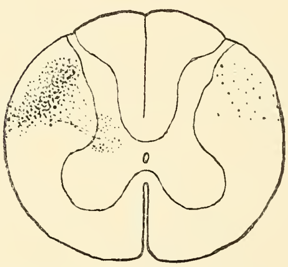


Fig. 10.—Seventh cervical.

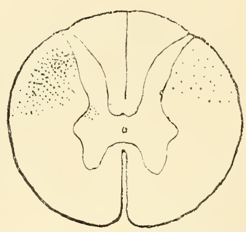


Fig. 11.—First thoracic.

cerebellar tract, which is at this level beginning to separate itself from the ventral cerebellar tract and to enter into the formation of the restiform body. The fibres are arranged in small bundles close to the grey matter, but elsewhere they are uniformly scattered over the whole area, and cannot be said to be more numerous at one part than at another, except anteriorly, where the degeneration shades off into the white matter.

In the 5th cervical segment (fig. 9) the position and area occupied by the degeneration is practically the same as in the first cervical segment. The dorsal cerebellar tract extends farther forwards along the margin of the lateral column, and comes in between the periphery of the cord and the area of degeneration in its posterior half, but in front of this the degeneration reaches quite to the margin of the lateral column.

In the 7th cervical segment (fig. 10), and more so in the 8th, the dorsal cerebellar tract extends farther forward still, and separates the crossed pyramidal tract from the periphery almost completely, but many fibres can be seen detached from the main mass invading the area of the cerebellar tract, particularly at its anterior extremity. In these segments the area of degeneration no longer abuts against the grey matter; a clear space intervenes between. Through this many degenerated fibres can be seen to extend forwards and inwards, and to enter the grey matter at the base of the posterior horn, and in this region of the grey matter an abundant fine degeneration is evident. The contrast is marked between this region and that of the opposite side, in which there is no fine degeneration. In transverse sections these fibres which enter the grey matter are cut obliquely, but in longitudinal sections they can be seen

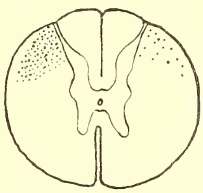


Fig. 12.—Sixth thoracic.

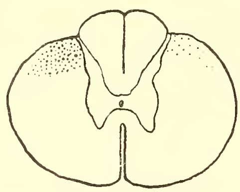


Fig. 13.—Tenth thoracic.

to come off from the main mass of fibres and to follow a slightly downward course before passing into the grey matter.

In the 1st thoracic segment (fig. 11) the area of degeneration has greatly diminished; it is almost completely cut off from the periphery by Flechsig's tract—more completely than in the cervical region—and fibres cut obliquely are seen to pass into the grey matter, to form there a copious fine degeneration, as in the cervical enlargement.

In the mid and lower thoracic segments—the 4th, 6th, and 10th segments were examined—(fig. 12), these fibres are not visible, and the amount of fine degeneration in the grey matter is very slight. The area of degeneration slowly diminishes in sections from above downwards, gradually coming nearer to the surface, until in the 10th thoracic segment (fig. 13) it touches the periphery of the cord in front of the postero-lateral fissure.

In the lumbar enlargement (figs. 14 and 15)—sections from the 3rd and 5th lumbar, and from all the sacral segments were examined—the degeneration lies close to the periphery at the posterior part of the lateral column, just in front of the entering posterior root; it rapidly diminishes in successive sections from above downwards. The homolateral fibres can be traced

as far as the 4th sacral segment (fig. 16) and the crossed (heterolateral) tract, to the first coccygeal segment. Fibres entering the grey matter can be seen in the lumbar region, as in the cervical enlargement, ending in fine degeneration in a similar position.

The ratio of the heterolateral to the homolateral fibres remains about the same in the different levels of the spinal cord from the upper cervical to the lower sacral region; it is approximately 40 to 1 in this particular animal. An eyepiece micrometer ruled in squares was used to estimate the number of degenerated fibres in the two respective areas, but no great degree of exactitude can be claimed for the enumeration. The area of degeneration and the number of degenerated fibres diminishes most rapidly from the 5th cervical to the 1st thoracic segment, and again from the 3rd lumbar to the 1st sacral segment. In all sections of the spinal cord numerous sound fibres are scattered amongst the degenerated ones.

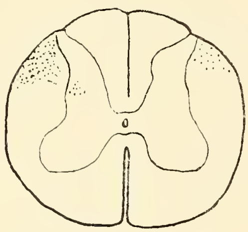


Fig. 14.—Third lumbar.

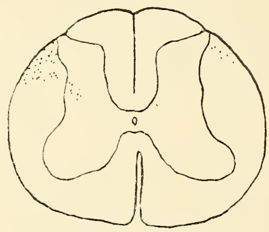


Fig. 15.—Fifth lumbar.

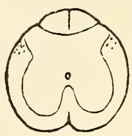


Fig. 16.—Fourth sacral.

In all sections examined above the spinal cord a few degenerated fibres are found in the area of the pyramidal tract on the side opposite to that of the lesion. They diminish from above downwards both absolutely and relatively to those of the other (homolateral) side, until in the anterior pyramid, just above the decussation in the medulla oblongata, only six or eight can be counted. In the internal capsule the proportion of fine to coarse fibres is greater than in the crusta and pontine bundles. The fine degeneration ends chiefly in the adjacent lateral nucleus of the optic thalamus.

In another animal an attempt was made to cauterise the whole motor area, but on the post-mortem examination it was found that the cortex

on the mesial aspect of the hemisphere had been only slightly involved in the lesion. The degeneration, however, was found to be almost as extensive as in the first case, and to show practically identical features, so that a description of this second case is not necessary.

Degeneration following Lesions in the Leg Area.

We first extirpated the whole leg area without encroaching on the arm and trunk areas. This included the upper portion of the precentral convolution above the superior genu of the fissure of Rolando on the lateral aspect, with a corresponding portion of the marginal convolution on the mesial aspect of the hemisphere (fig. 1). The lesion was made on the right side.

In *the internal capsule* the degeneration occupies the posterior half or three-fourths of the thalamo-lenticular portion of its posterior limb, its anterior extremity not reaching to the genu (figs. 2 and 3). The retro-lenticular portion of the posterior limb is not free from degeneration, but it consists mainly of fine fibres. There is an abundant fine degeneration in the lateral nucleus of the thalamus, as in the other cases examined.

In the mesencephalon the main mass of degeneration lies in the inner portion of the lateral half of the crusta (fig. 4). The mesial half is free, but the outer portion of the lateral half contains a few degenerated fibres. The point of greatest concentration lies somewhat on the mesial side of the junction of the outer and middle thirds, and from this the degeneration shades off on either side. On the posterior aspect of the degeneration in its outer portion there are a few small detached bundles of degenerated fibres, and a considerable number of single fibres pass backwards through the substantia nigra (in which some of them end) into the tegmentum, but they cannot be traced very far, and there is no fine degeneration in the anterior colliculus.

Throughout the pons the area covered by the degeneration is practically the same as that already described after extirpation of the entire motor region, except that the degenerated fibres are less numerous. The lateral and mesial bundles are not free, but they contain fewer fibres than the middle bundles. Along the anterior margin of the mesial fillet a few scattered fibres are to be seen; these may represent the accessory fillet of von Bechterew described by Hoche, Barnes, and others in the human subject.

Below the pons, in the *medulla oblongata and spinal cord* the degeneration is evenly scattered over the entire area of the anterior pyramid and crossed pyramidal tract. A slightly greater proportion of fibres remain

on the homolateral side than in the case of total extirpation. The only noteworthy point is the fact that although the cortical lesion was limited to the leg area, a few fibres can be seen to enter the grey matter of the cord in the cervical enlargement, and the motor tract suffers some loss as it traverses this region. In the thoracic region the diminution in the number of fibres is very slight, but in the lumbar region they disappear very rapidly, and can be seen to enter the grey matter and to terminate in a copious fine degeneration at the base of the posterior horn. The crossed tract can be traced to the last sacral segment, and the homolateral fibres end one or two segments higher up.

Degeneration following Extirpation of the Arm Area.

The lesion does not include the whole arm area, that portion of the cortex next to the body area having escaped injury (fig. 1). Association and commissural fibres are found radiating from the region extirpated; these consist mostly of fine fibres, the great majority of the coarse fibres mixed with fine passing into the corona radiata, and thence to the internal capsule. In the upper regions of the capsule they extend farther forward than the fibres from the leg area, intermingling with the latter posteriorly, but in the lower levels the fibres from the two areas overlap to a great extent. A large proportion of the fine fibres end in the optic thalamus.

In transverse sections of the mid-brain through the anterior corpora quadrigemina (fig. 4) the degeneration occupies about two-fifths of the crusta, the mesial fifth and lateral two-fifths containing only a few scattered fibres, but these are more numerous in the former than in the latter. The area of densest degeneration lies distinctly on the mesial side of the middle point of the crusta, and from this it fades away on either side. Some fine degeneration can be seen in the substantia nigra behind the crusta, but no fibres can be traced into the tegmentum.

Throughout the pons and medulla oblongata the degeneration is very similar to that already described for the leg area, differing from it only in a few minor details. The mesial (fronto-pontine) and lateral (temporo-pontine) bundles in the pons contain only a few scattered fibres, and some isolated fibres are to be seen along the anterior margin of the mesial fillet, but they are not intermixed with the fillet fibres. In the medulla oblongata the degeneration is scattered uniformly over the entire area of the pyramid, and after the decussation the proportion of crossed to un-

crossed fibres is about the same as that following extirpation of the entire motor area.

In the spinal cord most, but not all, of the fibres of both the homolateral and the crossed pyramidal tracts terminate in the grey matter of the cervical enlargement; a few can be traced to the lower end of the sacral region. This is a point worthy of note, because the lesion in this case certainly did not encroach upon the leg area.

Degeneration following Extirpation of the Face Area.

In this case the lesion is not confined to the face area, but extends into the adjacent arm area to some extent. It includes the cortical centres for the opening of the mouth and for the movement of the larynx, upper lip, eyelids, neck and digits (see fig. 1). The association and commissural fibres were not followed. The degeneration in the *internal capsule* covers an area somewhat more extensive than that following extirpation of the arm centre alone, and its anterior extremity lies farther forwards; beyond this there is nothing worthy of special notice.

In the mesencephalon it occupies about three-fifths of the crusta (fig. 4), its inner limit approaching nearer to the middle line than in the case of the arm lesion. Scattered fibres are found on the mesial and lateral aspects of the main mass of degeneration, but the outer extremity of the latter (temporo-pontine) segment is quite free. Fibres pass into and terminate in the substantia nigra, but in this case none can be followed into the tegmentum.

In the pons the only point calling for special mention is the relatively large amount of fine degeneration which is scattered amongst the cells of the grey matter surrounding the pontine pyramidal bundles. This is more abundant on their mesial aspect than on their lateral, but it ends abruptly at the median raphe. A few scattered fibres are found in the inner portion of the mesial fillet. In the lowermost levels of the pons a few isolated fibres leave the posterior aspect of the pyramidal bundles and can be traced backwards through the fine degeneration to the fillet, where they disappear.

In the medulla oblongata the degeneration is scattered over the whole area of the transverse section of the pyramid, but it is slightly denser in the mesial portion than in the lateral. The ratio of crossed to uncrossed fibres in the first cervical segment just below the decussation is about 100 : 1, and it remains the same throughout the spinal cord.

Spinal Cord.—In the lower part of the cervical enlargement numerous fibres are seen to pass obliquely in from the crossed pyramidal tract and to enter the grey matter at the base of the posterior horn, where they

terminate in a copious fine degeneration. One or two similar fibres are visible on the homolateral side. In longitudinal sections of this segment they are seen to come off at right angles from the main tract and to enter the grey matter. These fibres are more numerous and more easily followed in this specimen than in any that we have examined.

The degeneration does not cease at the lower end of the cervical enlargement, but is continued down through the thoracic and lumbar segments as a few sparsely scattered fibres both on the crossed and on the uncrossed side. A few isolated fibres on the crossed side are still present in the last sacral segment.

Degeneration following Limited Lesions.

In addition to the above, *limited lesions were made within each of the larger motor areas,* and the degenerations traced from these. Within the leg area the following centres were located by stimulation, and small portions of the cortex, including them, were cauterised—centre for extension of hallux and toes (in two animals), for flexion of knee and for flexion of hip (one each). Within the arm area four lesions were made, involving the centres for rotation of arm at shoulder, for flexion of wrist, and for flexion of fingers (two cases); and within the face area the centre for the opening of the mouth was destroyed.

We shall not attempt to describe the degeneration in detail in each case. It is sufficient to say that below the level of the mid-brain there is no evidence of any localisation of the fibres within the area of the pyramidal tract in transverse sections. In the pons, medulla oblongata, and spinal cord they are practically evenly distributed over this area, the only difference between the effects of an extensive and a limited lesion being in the density of the degeneration, *i.e.* in the number of fibres degenerated. Following all lesions of the arm area some fibres can be traced to the lumbar region of the spinal cord, and in all lesions of the leg area some appear to terminate in the cervical enlargement. In the case of the limited lesion within the face area some fibres pass into the pyramidal tract of both sides of the cord, crossed and uncrossed, and can be followed to the 2nd thoracic segment, but not beyond it.

In the lower levels of the internal capsule and in the crusta the fibres from the hip, knee and digits centres are scattered fairly evenly over those segments of the pyramidal tract which degenerate after extirpation of the entire leg area; that is to say, no localisation of fibres *within the leg area* can be made out with certainty in these regions. The same applies to the fibres from the circumscribed lesions within the arm and face areas.

In some of these cases, however, the degeneration could not be satisfactorily followed in the parts of the brain above the mesencephalon on account of the imperfect penetration of the staining fluid.

Summary and Conclusions.

In endeavouring to trace the course of the individual portions of the pyramidal tract we find that our results agree, for the most part, with those obtained by the majority of former observers who have employed the degeneration method with Marchi staining. The fibres arising from the giant pyramidal cells of the motor cortex pass into the corona radiata, at first in fairly distinct groups, but they soon begin to intermingle, and in the highest levels of the internal capsule there is already considerable overlapping. Those from the lateral portion of the Rolandic area (face) enter the capsule most anteriorly, those from the mesial portion (leg area) most posteriorly, while between the two, but intermixing with them, come the fibres from the intermediate parts of the cortex, viz. the arm and trunk areas. In passing down through the internal capsule into the pes pedunculi the tract becomes compressed within narrower limits, and this intermingling of the fibres from the different areas becomes more marked. In the lower levels of the capsule the fields of degeneration resulting from ablation of the face, trunk and limb areas overlap, to a very large extent, and this is more marked still in the crusta (figs. 3 and 4). Below this, in the pontine pyramidal bundles, the anterior pyramids, and the direct and crossed pyramidal tracts in the spinal cord, there is no localisation of fibres at all, the degeneration following the most limited lesion being uniformly scattered over the entire area of the pyramidal tract in transverse section.

With regard to the longitudinal localisation of the fibres in the tract, our results partly support those of Sherrington.* He found that after lesions in the leg area, encroaching little, if at all, on the arm area, the degeneration in the cord stopped short in great part in the cervical enlargement; and after lesion in the arm area, encroaching little, if at all, on the leg area, the degeneration in the cord extended down through the thoracic into the lumbar, and even throughout the sacral region. He concluded from this that the pyramidal tract is a path of cortical visceral (splanchnic) as well as of cortical somatic fibres,—hence fibres descending from the arm area of the cortex into the lumbar region of the cord. We have not found that after pure leg area lesions a *great part* of the degeneration ends in the grey matter of the cervical region, or that a *great number* of fibres can be followed down to the lumbar region after lesions con-

* Sherrington, *Jour. of Physiol.*, vol. x. (1889), p. 429.

fined to the arm area, but in the one case *some* fibres terminate in the cervical enlargement, and in the other *a few* certainly do pass to the lower end of the cord. Mellus denied Sherrington's statement; he could trace no fibres beyond the upper dorsal region following his localised extirpations in the arm area, but his lesions were made in the post-central convolution behind the fissure of Rolando, and so outside the true motor area.

Two possible explanations suggest themselves for the presence of these fibres in the spinal cord, and for those stray fibres which invade the fronto-pontine and temporo-pontine bundles in the internal capsule, crusta, and pons. They may be due to unavoidable injury to the cortex of the brain beyond the motor area during the operation, through interference with the blood supply, or mechanically by pressure, by which a few of the cells are damaged to such an extent as to lead to degeneration of the fibres which take origin in them. Thus, after extirpation of a portion of the arm area, some degenerated fibres may come from damaged cells in the leg and face areas, and even from the cortex outside the true Rolandic area.

On the other hand, the areas for leg, arm, and face may not be so strictly limited as stimulation with minimal currents by the unipolar method would indicate, and a few fibres destined for the innervation of the leg muscles may take origin from cells situated within the arm area, and *vice versa*.

With regard to the degeneration following total extirpation of the motor cortex, we corroborate the results stated in a former paper * by one of us, and are in agreement on most points with others who have used this method. In all our larger lesions the fine degeneration in the nuclei pontis is very evident, indicating an important connection between the pyramidal fibres and the grey matter in this region.

We have not been able to make out the accessory fillet of von Bechterew, described by him, and by Hoche,† Schlesinger,‡ Barnes § and others, in the human subject. This is a bundle of fibres which leaves the pyramidal tract in the mesencephalon, and running outwards and backwards through the substantia nigra, passes into the outer part of the mesial fillet in the upper levels of the pons. A similar bundle of descending fibres (Spitzka's bundle) in the fillet, nearer the middle line, has been named the mesial accessory fillet. These two bundles were believed by Hoche to end in certain of the cranial motor nuclei. In many of our cases numerous fibres leave the pyramidal tract in the crusta, and do pass in the direction described by

* Simpson, *Internat. Monatschr. f. Anat. u. Physiol.*, Bd. xix. (1902), p. 1.

† Hoche, *Arch. f. Psychiat. u. Nervenkr.*, Bd. xxx. (1898), p. 103.

‡ Schlesinger, *Neur. Cent.*, 1896, p. 146. § Barnes, *loc. cit.*

these authors in the human subject, but we are inclined rather to believe that they terminate in the substantia nigra and in the tegmentum behind it, as we cannot pick them up again in the fillet in sections of the pons made at a lower level.* Such fibres ending in the grey matter of the anterior corpora quadrigemina have been described in the cat and dog, and by Horsley and Beevor † in the monkey.

We have never been able to trace fibres from the pyramidal tract to any of the cranial motor nuclei, and have found no evidence of fine or terminal degeneration in the neighbourhood of these nuclei.

In the monkey we have never seen any trace of a ventro-lateral pyramidal tract, such as is described by Barnes ‡ in the human subject. This tract is said to be given off frequently in man: it may arise in the pons, medulla oblongata, or first cervical segment, and it passes down in the region of Helweg's tract on the ventro-lateral aspect of the inferior olivary nucleus. Barnes claims that it is distinct from Helweg's tract; it is best marked in the upper two cervical segments, but occasionally he has succeeded in tracing it to the lumbo-sacral region. We find nothing to represent it in the monkey, neither are there any fibres corresponding to Pick's bundle, another aberrant strand which is sometimes found in the pyramidal system in man.

In the spinal cord the passage of collateral fibres from the crossed pyramidal tract, first described by Schäfer § (and in some cases from the homolateral), is very evident both in the cervical and lumbar enlargements, and to some extent in the thoracic region. They are seen in transverse and in longitudinal sections of the cord, and terminate in the grey matter at the base of the posterior horn. The fine degeneration in this region of the grey matter is very abundant: the corresponding region of the opposite side is free, and the contrast between the two sides in this respect, in the same section, is very marked.

Fibres representing a direct anterior pyramidal tract are found in the first cervical segment only; the direct lateral can be traced to the lower end of the sacral region, and the crossed pyramidal to the first coccygeal segment.

The expenses incurred in this research were defrayed by grants from the Moray Fund and from the Carnegie Trust.

* In some of our sections through the pons we did find a few scattered fibres along the anterior margin of the fillet.

† Horsley and Beevor, *Brain*, vol. xxv. (1902), p. 436. ‡ Barnes, *loc. cit.*

§ Schäfer, *Jour. of Physiol.*, vol. xxiv., p. xxii—Proceedings of the Physiol. Soc.

(*Issued separately October* 3, 1907.)

XXX.—Hydrachnidæ collected by the Lake Survey. By Mr Wm. Williamson. *Communicated by* Sir John Murray, K.C.B.

(MS. received July 2, 1907. Read July 15, 1907.)

The systematic study of the Hydrachnidæ, or water-mites, of Scotland appears to have been entirely neglected until within the last few years, and, so far as I am aware, the only communications dealing with Hydrachnids found in Scotland are two noted in the list of literature cited.

Of the life-history of Hydrachnids we have only a limited knowledge, and that may in time be modified as further information is acquired. At present the life-cycle appears to be in four definite stages—the egg, the larva, the nymph, and the imago. When an egg is hatched, a six-legged larva emerges, and apparently its main function is to find a suitable host to which it may attach itself. I have hatched out at various times the larvæ of different species, but never succeeded in keeping them alive more than a few days, doubtless because a host was not present in the hatching tube. Species of *Dytiscus, Notonecta, Corixa, Ranatra*, and even dragonflies have been found with these larvæ firmly attached. During the period of attachment the larva undergoes a change, and finally emerges as an eight-legged nymph. Although I have been able to keep nymphs alive for several months, I have not yet succeeded in keeping them alive during the whole of their existence as nymphs, and have not therefore been able to determine the duration of the nymphal period. At the end of this period the creature passes into another stage in which it undergoes further development, and this is followed by the emergence of the imago, of which I have been able to keep specimens alive for eleven months.

While the Lake Survey investigations were being carried on at Loch Rannoch in May 1902 a trout was caught, in the stomach of which, on examination by Dr T. N. Johnston, a great number of living Hydrachnid larvæ were found. A short account of these was given by Mr Soar in the *Quekett Journal*, with figures of the larvæ, which were most probably swallowed by the trout just before it was caught.

For the nomenclature followed herein, reference is made to "Hydrachnidæ und Halacaridæ," Piersig und Lohman (*Das Tierreich*, Lief. 13). As

that work contains a very full synonymy, the genera and species now recorded are sufficiently defined for reference.

1. Genus HYDRACHNA, Müll.

Hydrachna globosa, de Geer.—A nymphal form was found by Mr Scourfield at a pond between Mallaig and Morar, June 1902. This species has also been found at Oban.

2. Genus THYAS, C. L. Koch.

Thyas venusta, C. L. Koch.—Found by Mr Murray at Whitebridge Pond, April 1904. This species has also been found in the vicinity of Edinburgh.

3. Genus ARRHENURUS, Ant. Dug.

Arrhenurus, sp.—A female specimen, not yet identified, was found by Mr Murray in St Mary's Loch (at a depth of 100 feet), January 1906.

4. Genus LEBERTIA, Neuman.

Lebertia tau-insignita, Lebert (?)—Specimens collected by Mr Scourfield at Morar, June 1902, and at Loch Tarff, August 1903, were identified by Mr Soar as *Lebertia tau-insignita*, Lebert. I also identified as the same species a specimen taken by Mr Murray from St Mary's Loch at a depth of 100 feet.

The genus Lebertia is undergoing revision by Dr Sig Thor. From his "Lebertia-studien" it would appear that some confusion in the naming of species belonging to this genus has existed for a number of years, and that this has arisen in some measure through the name *tau-insignita* having been erroneously separated from the specific description to which it belonged.

Lebertia (*Pilolebertia*) *porosa*, Sig Thor, var.—This specimen was found by Mr Murray at Loch Ness, and identified by Dr Sig Thor. This is a new Scottish record.

A specimen taken by Mr Scourfield from a loch near Fort Augustus has not yet been identified.

5. Genus OXUS, P. Kram.

Oxus ovalis, Müll.—Found by Mr Scourfield at Loch Rannoch, June 1902. I have not seen any other record of the occurrence of this species in Scotland.

6. Genus HYGROBATES, C. L. Koch.

Hygrobates longipalpis, Herm.—Found at Rannoch and Morar, June 1902; Ghriama, Loch na Claise Fhearna, and Suardalain, September 1902; Loch Tarff, August 1903; Dores Pond, February 1904; Hundland (Orkney), September 1906.

Hygrobates reticulatus, P. Kram.—Found at Morar, June 1902; Loch Ness, August and September 1903.

7. Genus HYDROCHOREUTES, C. L. Koch.

Hydrochoreutes ungulatus, C. L. Koch.—Found by Mr Murray at Loch Lydoch and Dubh Lochan, June 1902.

Hydrochoreutes krameri, Pier.—Found by Mr Murray at Loch na Claise Fhearna, September 1902.

8. Genus PIONACERCUS, Pier.

Pionacercus leuckarti, Pier.—Found by Mr Scourfield near Fort Augustus, August 1903.

A specimen found by Mr Scourfield at Loch Rannoch, June 1902, has not been identified.

9. Genus ATAX, (Fabr.) Bruz.

Atax crassipes, Müll.—Found at Rannoch and at a pond near Mallaig, June 1902; Loch of the Lows (Perth), 1903; St Mary's Loch (taken from a depth of 100 feet), January 1906.

In this species the first pair of limbs are much stouter than the other three pairs, are armed with movable spines, and appear to be used, not for locomotion, but mainly to seize the entomostracans on which the creature feeds. When stalking its prey, it now and then halts for a few seconds, the body sinks to one side, is raised up again to sink to the other side, and in turn to be raised again; or it may be that the posterior end of the body is lowered to rest on the papillæ,—all as if the long slender limbs were too weak to support the weight of the body during its brief rest. If, however, a daphnia or cyclops comes within range, the limbs which hitherto appeared to be rather in the way are suddenly bent round the entomostracan, which is passed under the palpi, and when it is held securely so that the mouth-organs may do their work, the first pair are again unbent and held up in front as before. Although the palpi have now the weight of

the entomostracan to support, the hydrachnid does not appear to have its movements hampered, as it moves about as freely as before, sometimes throwing itself on its back, with its long slender limbs waving above, and then, with a sudden jerk, bounding off to another place. When sufficient nutriment has been absorbed, the palpi are relaxed and the body of the victim released.

The second and third pairs of limbs are used mainly for locomotion, the first pair being held up in front, and the fourth pair apparently trailed behind. Occasionally this order is changed, so that the third and fourth pairs of limbs are used for progression, the second pair being waved about in the water, and the first pair still remaining raised up in front.

Instead of swimming with an easy, regular motion, this hydrachnid rather jerks itself through the water, sometimes forwards, sometimes backwards, and it appears to swim as much with the ventral surface upwards as with the dorsal.

10. Genus NEUMANIA, Lebert.

Neumania vernalis, Müll.—Found by Mr Scourfield near Fort Augustus, August 1903.

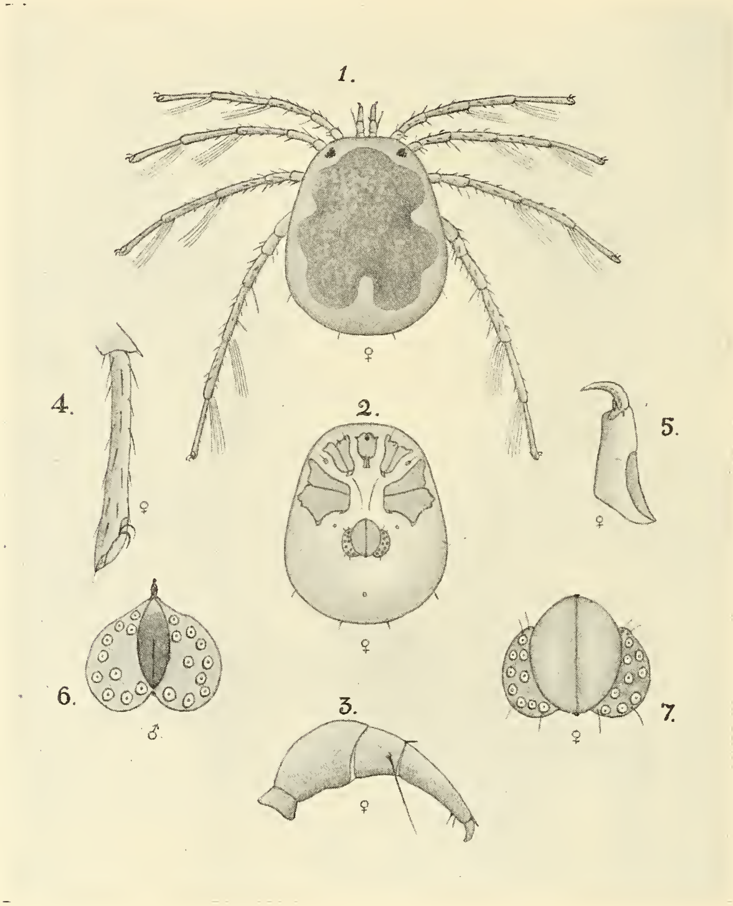
11. Genus HUITFELDTIA, Sig Thor.

Huitfeldtia rectipes, Sig Thor.—Specimens were found by Mr Murray at Hundland, Boardhouse, Housby, and Swanney, in the Orkney Islands. As there was no record of this hydrachnid having been found before in Britain, I sent specimens of the male and female to Dr Thor for verification. He confirmed my identification, and stated that he had not known of the occurrence of this species except in Norway.

Mr Soar has drawn the figures which show the details of this species. The following are the measurements given by him, as well as those given by Dr Sig Thor:—

	C. D. Soar.	Sig Thor.
Female :—		
Length	1·40 mm.	1·20 mm.
Breadth	1·12 „	1·00 „
Length of Mandible	·25 „	...
Length of Palpus	·40 „	·27 „
Length of bristle on Palpus	·16 „	...
Legs, 1st pair	...	1·15 „
„ 2nd „	...	1·27 „
„ 3rd „	...	1·30 „
„ 4th „	...	1·40 „
Distance apart of double eyes	...	·26 „
Male :—		
Length	1·24 „	...
Breadth	·98 „	...
Length of Genital Area	·19 „	...

The eyes, epimera, and legs are very much the same as in the genus *Piona* (= *Curvipes*). In the female, the genital plates are sickle-shaped, with from eight to ten acetabula each; in the male, which is smaller than the female, the plates completely surround the genital opening.



Explanation of Figures—
Huitfeldtia rectipes, Sig Thor.

Fig. 1.—Dorsal view of female.
Fig. 2.—Ventral view of female.
Fig. 3.—Palpus of female, showing characteristic bristle on third segment.
Fig. 4.—Terminal segment of fourth leg of female.
Fig. 5.—Mandible of female.
Fig. 6.—Genital plates of male.
Fig. 7.—Genital plates of female.

The palpus possesses features which are important in the determination of the genus, viz. a long stiff hair or bristle on the outer edge of the third segment of each palpus, and on the distal inner edge of the fourth segment an elongated peg or tooth.

The colours of the living hydrachnid cannot be stated, as the specimens examined had been in formalin for some time.

12. Genus PIONA, C. L. Koch.

The genus now under consideration was known at one time as *Nesæa*, afterwards as *Piona*, and latterly as *Curvipes*. In "Hydrachnidæ und Halacaridæ," Piersig und Lohman, Dr Piersig restored the name *Piona*. This has not met with general acceptance, and consequently the name *Curvipes* is frequently used.

Piona obturbans, Pier.—Found at Rannoch, 1902. A nymphal form found at Dubh Loch, 1902.

Piona paucipora, Sig Thor.—Found at Rannoch and Lydoch, 1902. Very few specimens found in Scotland.

Piona fuscata, Herm.—Found at Dubh Loch, 1902.

Piona rufa, C. L. Koch.—Found at Loch Chlair and Ghriama, 1902.

Piona carnea, C. L. Koch.—Found at Loch of the Lows (Perth) and Loch na Claise.

Piona aduncopalpis, Pier.—Found at Uanagan, 1903.

The identification of the twelve genera and eighteen species recorded above was made partly by Mr Chas. D. Soar and partly by myself. The genus *Huitfeldtia* has not been recorded before, except for Norway, and therefore *Huitfeldtia rectipes*, Sig Thor, is a new British record. *Lebertia porosa*, Sig Thor, and *Oxus ovalis*, Müll, are additions to the previous lists of Scottish Hydrachnids.

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(*Issued separately October* 3, 1907.)

XXXI.—Precipitated Sulphur. By Alexander Smith and R. H. Brownlee.

(Read July 8, 1907. MS. received July 15, 1907.)

(*Abstract.*)

In papers previously read before the Society, the behaviour of sulphur when heated has been the subject of investigation. It has been shown that the transition from a pale-yellow mobile liquid to a deep-brown viscous one, which occurs as the temperature rises in the neighbourhood of 160°, is due to the production from the mobile sulphur (S_λ) of another distinct variety (S_μ). The proportion of the viscous variety (S_μ) is about 4 per cent. at the melting point (114·5°). At 160° it has become 11 per cent., at 170° 19 per cent., and at the boiling point 34 per cent. The complete succession of changes is as follows :—

$$S_{rh} \rightarrow S_{monocl} \rightarrow S_\lambda(S_\mu <) \rightarrow S_{vapor}.$$

When the boiling sulphur is allowed to cool very slowly all these changes take place in the reverse order, and only crystalline, rhombic sulphur is obtained. When, however, the boiling sulphur is chilled suddenly, there is no time for the transformation of the S_μ into S_λ, which alone crystallises, and hence the cooled mass is plastic. The amorphous sulphur which the plastic mass contains is the supercooled S_μ. Certain catalytic agents affect the speed of the change from S_μ to S_λ. Thus, if a few bubbles of ammonia are passed into the boiling sulphur, the reversion to S_λ is greatly accelerated, and no S_μ (amorphous sulphur) is found in the chilled product. The mass after standing for a short time is wholly crystalline. On the other hand, sulphur dioxide, which sulphur acquires by exposure to the air, is a retarding catalyser, and its presence permits the supercooling of the S_μ when ordinary sulphur is employed. Other substances, such as hydrogen chloride and the halogens, have an effect similar to that of sulphur dioxide. Since sulphur precipitated at the ordinary temperature contains in many cases a large proportion of amorphous sulphur, this investigation was undertaken in order to ascertain the exact circumstances under which a variety of sulphur which is stable only at high temperatures can thus be formed at low temperatures.

In this work four actions by which sulphur is precipitated were used, and two of them were investigated in detail. The amorphous sulphur was estimated, after the precipitate had hardened, by extraction of the soluble sulphur with carbon disulphide.

1. In the product of *the interaction of hydrogen sulphide and sulphurous acid* in aqueous solution the proportion of insoluble sulphur was very variable. In one instance it reached 82·6 per cent.

2. *The addition of acids to solutions of calcium or sodium polysulphides* gave sulphur which was almost wholly soluble. Thus, with excess of acetic acid all the sulphur was soluble; with excess of half-concentrated hydrochloric acid (6 N) less than 1 per cent. was insoluble; with excess of concentrated hydrochloric acid (12 N) 2·4 per cent. only was insoluble.

The precipitates, viewed through a polarising microscope, were found to be composed at first of fluid droplets. The droplets could be seen to crystallise, each forming a spherical group of radiating crystals. Later, the cluster suddenly resolved itself into a single crystal, without altering its spherical shape. This observation of the production of liquid sulphur at the ordinary temperature, and its transformation into monoclinic sulphur, and finally into spheres of rhombic sulphur (the only stable form), was most instructive.

3. *The actions of bromine and iodine upon solutions of sodium sulphide and polysulphide* were studied in some detail. When equivalent and excessive amounts of the halogens were used, the proportions of insoluble sulphur varied from 8 per cent. to 56 per cent. Bromine gave the larger proportions.

4. The most detailed investigation was given to *the action of acids upon sodium thiosulphate.* The following were the results of some typical preliminary experiments. In each experiment 100 c.c. of thiosulphate solution was used. The concentration of this solution is given in the first column. In the second column appears the amount and concentration of the acid. In experiment 3 the thiosulphate was added to the acid, in the others the acid was added to the thiosulphate. The last column shows the per cent. of insoluble sulphur:—

	Concen. $Na_2S_2O_3$ (100 c.c.).	Acid.	Per cent. Am S.
1	4 N	50 c.c., 12 N. HCl	34·0
2	4 N	50 c.c., 6 N. HCl	6·5
3	8 N	500 c.c., 12 N. HCl	96·0
4	4 N	25 c.c., 16 N. HNO_3	56·5
5	4 N	300 c.c., glac. $H\bar{A}$	0·0

Some of the conclusions from these results were:—

(1) That, whereas Berthelot held that "positive" sulphur (as in S_2Cl_2), when liberated, gave insoluble sulphur, and "negative" sulphur (as in

polysulphides) gave soluble sulphur, in point of fact the very same compound can yield sulphur wholly soluble (exp. 5), or almost wholly insoluble (exp. 3).

(2) That excess of the acid is not required for the production of insoluble sulphur in this action (exps. 2 and 4), although such excess greatly increases the proportion obtained (exps. 1 and 3).

(3) That free sulphurous acid does not determine the formation of insoluble sulphur: this acid was liberated in large amount in experiment 5, but all the sulphur was soluble.

(4) That the order of mixing makes a great difference in the effect (*e.g.* exp. 3), evidently because when the salt was added to the acid, the latter was present in greatest excess from the very beginning of the precipitation.

(5) That sulphuric acid (experiments not given above) gave smaller proportions of insoluble sulphur than did hydrochloric acid.

(6) That, other things being equal, the higher the temperature of the liquids, the smaller was the proportion of amorphous sulphur. For example, equal volumes of 2 N. thiosulphate and 10 N. hydrochloric acid, maintained before and during mixing at the following temperatures, gave proportions of insoluble sulphur appended: 0° (47·1 per cent.), 25° (34·0 per cent.), 40° (30·8 per cent.).

These preliminary results showed the conditions required to be fulfilled in the quantitative final experiments with the same interaction. It was necessary to maintain the same concentration of the ingredients throughout the process of mixing. This was achieved by arranging to use equal volumes in each experiment, and by allowing the liquids to run with equal speed into a mixing vessel provided with a mechanical stirring apparatus. It was also essential that the liquids should be brought to a constant temperature, and that this temperature should be maintained in the mixing vessel in spite of thermal effects. Three different temperatures were used, namely, 0°, 25°, and 40°, and constancy (± ·5°) was preserved by suitable contrivances.

The following part of a set of observations at 0° will serve as a sample of the results. The first column gives the concentration of the hydrochloric acid used, the second gives the concentration of the acid immediately after mixing with an equal volume of thiosulphate solution. The three following columns show the percentages of amorphous sulphur from 2 N., 4 N., and 6 N. thiosulphate solution, respectively. The last column describes the appearance of the sulphur immediately after precipitation when 2 N. thiosulphate was used:—

Concen. of HCl used	After mixing	Concen. $Na_2S_2O_3$ and Per cent. Am S.			State of Sulphur from 2 N. $Na_2S_2O_3$.
		2 N	4 N	6 N	
2 N	N	6·1	...	...	Yellow-brown viscous oil.
3 N	$1\frac{1}{2}$ N	12·4	14·7	8·3	" "
5 N	$2\frac{1}{2}$ N	21·5	18·9	29·0	More viscous oil.
10 N	5 N	47·0	45·9	57·7	Very viscous oil.
12 N	6 N	68·8	72·3	75·9	Non-viscous powder.

A study of the foregoing, and of the other series not here given, showed—

(1) That the proportion of insoluble sulphur is not dependent upon an excess of acid, for much amorphous sulphur was formed when the salt was in excess.

(2) That the proportion of insoluble sulphur is proportional to the concentration of the acid immediately after mixing.

(3) That the proportions were smaller the higher the temperature of precipitation.

The question which finally presented itself was whether the excess of acid, which is so potent in increasing the yield of amorphous sulphur, operates in this way during the liberation of the sulphur, or exercises this influence by contact with the sulphur after liberation. Actual experiment showed that subsequent mixing of the liberated sulphur with more concentrated acid was as effective as using the larger excess during the precipitation.

The *mechanism of the action* is therefore as follows:—The sulphur from the thiosulphate, when first liberated, is wholly S_μ. With little acid present this changes largely to S_λ, and the S_λ finally solidifies to soluble sulphur. A more concentrated acid retards the change to S_λ, and permits the hardening and preservation of a larger proportion of the S_μ. Acid substances such as retard the transformation $S_\mu \rightarrow S_\lambda$ in melted sulphur operate, therefore, in the same way with suspended liquid sulphur. It is the fact that the sulphur is first liberated as S_μ that alone renders the production of amorphous sulphur possible. This liberation first in the least stable form is in accordance with the law of transformation by steps (Ostwald), which successfully explains the behaviour of so many substances.

University of Chicago,
April 1907.

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XXXII.—A Specimen of Helix pomatia with Paired Male Organs.

By J. H. Ashworth, D.Sc., *Lecturer in Invertebrate Zoology in the University of Edinburgh. Communicated by* Professor J. C. Ewart, M.D., F.R.S. (With Plate, and Two Figures in the Text.)

(MS. received May 10, 1907. Read June 3, 1907.)

The specimen of *Helix pomatia* which forms the subject of this communication was found among the class material * in the Zoological Department of the University of Edinburgh. It presents so unique and interesting an abnormality that it is worthy of description in some detail.

The animal was approximately full-grown and possessed a normal dextral shell. On dissection it was observed that, in addition to the usual set of reproductive organs present on the right side, there were, on the left side, certain other structures undoubtedly homologous with the normal accessory male organs. The roof of the mantle chamber was removed,† the body wall cut through by means of an incision in the mid-dorsal line, the flaps pinned out, the alimentary canal and the reproductive organs carefully uncoiled, and the latter laid out to the right side of the animal. The alimentary canal presented no unusual features, and it has been entirely removed. The remaining structures are shown in fig. 2.

The normal reproductive apparatus may be first briefly described. The ovotestis (O.T.) or hermaphrodite gland,‡ which was embedded in the liver, has been dissected out so as to show its ducts which lead into the sinuous hermaphrodite duct (H.D.). This opens into the common duct, but just before doing so it bears two small closely apposed blind diverticula, about 3 mm. long—the vesiculæ seminales—only one of which (V.S.) is shown in the figure; the other, which is slightly smaller, lies hidden beneath. At the point of junction of the hermaphrodite and common ducts there is a large, somewhat tongue-shaped, albumen gland (Alb.G.).§

The common duct is imperfectly divided by two internal longitudinal

* Collected in the neighbourhood of Würzburg.

† The heart, the pulmonary vessels, and the nephridium were quite normal.

‡ The ovotestis was removed and sectioned, and though the histological details were defective it was obviously quite normal in structure. Various stages in development of spermatozoa and ova were seen, including ripe spermatozoa and a considerable number of large oocytes ·11 mm. in diameter.

§ This is shown in the figure, as it was pinned out, in an extended form ; in its natural condition, as it lies in position in the animal, it is much more curved.

folds into a larger female portion (C.D.F.), the walls of which are strongly plicated, and a smaller male portion (C.D.M.) which is covered by glandular prostatic tissue slightly folded or lobulated. After a course of about 50 mm. the two portions of the common duct become completely separated as the oviduct and the vas deferens.

The oviduct (Ovd.) runs forward for about 10 or 12 mm. and then merges into the wider vagina (V.), which leads into the common genital atrium (Atr.). The vagina bears a large clavate dart sac (D.S.) and two tufts of digitiform mucous glands (Muc.G.). In the figure these glands are to a large extent hidden by the dart sac; they are not quite as large as in many other snails of the same collection examined, but, with this exception, the reproductive organs are fully grown. Opening into the female genital canal near the point of junction of oviduct and vagina is the spermotheca or receptaculum seminis (R.S.), which bears a bulbous dilatation (R.S.B.) at its inner end. In the natural position of the parts the tubular portion of the spermotheca lies alongside the common duct and is bound to it by connective tissue.

The vas deferens (V.D.) has at first a somewhat sinuous course; it then loops round the retracted posterior tentacle (Tent.P.R.), runs parallel to and practically along the whole length of the penis (P.), and bends round so as to enter the base of the latter, opening into it upon a knob-like muscular papilla which acts as a protrusible copulatory organ. This organ is enclosed in a thick muscular sheath, the inner surface of which, in the proximal part of the penis, has a rugose appearance. About the middle of the length of the penis, the sheath bears a muscular introversible tube. The part of the sheath distal to this is slightly thinner than the proximal part, but is also traversed by longitudinal folds. This inner sheath is surrounded by a much thinner outer one. The fibres of the thin, but strong, retractor penis muscle (M.Retr.P.) are chiefly inserted into the outer sheath close to the point of entry of the vas deferens into the penis. This muscle serves to retract the copulatory organ and also the penis as a whole. It arises from the posterior region of the mantle slightly to the left of the middle line. In the figure the greater part of the muscle is seen attached to the penis; the remaining short portion is shown at M.Retr.P′. Just before entering the penis the vas deferens bears a long slender diverticulum, the flagellum (Fl.), which secretes the elongate spermatophore.

The genital atrium (Atr.) is a short wide tube opening to the exterior by an aperture which, in non-excited specimens, is slit-like and about 2·5 mm. long. This common genital pore is situated just behind and ventral to the posterior or ocular tentacle.

Commencing at a point slightly median or dorsal to the pulmonary aperture, there is a distinct but shallow groove in the skin which leads obliquely forwards and downwards, ending just below the ventral lip of the genital pore.

On looking at the left side of the specimen just behind the posterior tentacle it is at once obvious that certain parts of the genital apparatus are repeated; indeed, there is no difficulty in recognising a penis and its retractor muscle, a vas deferens and a flagellum. These may be first described as they are seen in the general dissection, shown in fig. 2.

The additional penis (P.S.) is as well developed as the normal one, and opens to the exterior by means of a slit-like aperture as large as the normal genital pore on the right side. The retractor muscle (M.Retr.P.S.) is inserted in the usual position at the inner end of the penis; only a short portion is shown at that point in the figure, the remainder (M.Retr.P.S′.) is seen attached to the body wall, its origin being close to that of the retractor of the right penis. Enclosing the bases of these retractors there is a thin connective tissue sheath (Sh.), folded at the middle so as to divide the enclosed area into two compartments, one for each retractor. When the flaps of the body wall are replaced in their normal position the origins of the retractor muscles are seen to be a little to the left of the mid-dorsal line, and so placed that the muscles would gradually diverge from one another to their respective penes. The two penes are symmetrically placed with regard to the median plane of the head, and, as already stated, are equally developed.

The extra vas deferens (V.D.S.) is seen in connection with the inner end of the penis, and may be followed along the whole length of that organ until it disappears beneath one of the large retractor muscles of the anterior end; this muscle (M.Retr.) is inserted into the body wall by means of two diverging portions which enclose the distal part of the penis and the proximal part of the vas deferens.

Arising from the vas deferens near the middle of its length is a well-developed flagellum (Fl.S.), the terminal portion of which was accidentally severed and lost during the initial stages of the dissection in class, before the abnormality had been observed. At the point of origin of the flagellum, which is situated somewhat further from the penis than in the case of the normal flagellum, there is a sudden increase in the diameter of the vas deferens, its distal part being about twice as wide as the proximal part. The ocular tentacle (Tent.P.L.) and its retractor muscle pass to the outer side of the penis and neighbouring portion of the vas deferens, as is the case on the right side.

The penial nerve on both sides leaves the cerebral ganglion (C.G.) laterally, anterior to the root of the ocular nerve, and passes into the interval between the penis and vas deferens, being supported by a connective tissue sheet which partially binds these two structures together. The nerve divides into three or four branches, which pass into the penis near the middle of its length. The nerve to the right penis really originates in the pedal ganglion, and traverses the cerebro-pedal connective and a portion of the cerebral ganglion before becoming free as the penial nerve (N.P.), seen in the figure. The actual origin of the left penial nerve (N.P.S.) could not be determined by dissection, so the nerve, the cerebral ganglion, and the cerebro-pedal and cerebro-pleural connectives were excised and sectioned. The preservation of the tissues enclosed within the sheath of the ganglion was, however, so defective that the complete course of the penial nerve fibres could not be followed. The left penial and ocular nerves, on being traced back into the cerebral ganglion, are seen to penetrate the sheath of the ganglion close together and then unite, but soon after this the tissue is so badly preserved that the two bundles of fibres are no longer distinguishable. Whether the penial nerve fibres arise from the ocular nerve, as is *apparently* the case, or whether they actually arise from the pedal ganglion, is unfortunately impossible to decide. In the majority of normal specimens there is, on the left side, a nerve given off from the cerebral ganglion just anterior to the ocular nerve, and reaching the body wall immediately behind the ocular tentacle—that is, in the region of the supplementary penis in the abnormal specimen. It is possible that in the abnormal specimen this nerve has been elevated along with the up-growth of the supernumerary organs.

Dissection of the tissues around the base of the extra penis and vas deferens permits their further course to be traced (see fig. 3). The penis passes through the body wall and opens to the exterior by a slit-like pore (G.O.S.), but probably the portion of the tube immediately internal to the aperture is to be regarded as atrium (see below, pp. 316, 317). The vas deferens (V.D.S.), followed from the inner end of the penis, courses alongside the penis until it reaches the body wall, along which it runs obliquely dorsally for a short distance; it then comes into close association with the skin, so close, indeed, that it was at first believed that the tube actually perforated the skin and opened externally by a minute pore. Further examination, checked by sections of this region, proves that there is no actual opening to the exterior; the vas deferens ends blindly in the sub-epidermal tissue and is bound to the epidermis by a short cord of connective tissue (Con.T.).

On reference to fig. 1 it is seen that the left genital aperture (G.O.S.) is

in the same relative position as the normal one—that is, ventral and slightly posterior to the ocular tentacle (Tent.P.L.). The blind end of the vas deferens is situated immediately below the papilliform area of the skin indicated by the + in fig. 1—that is, about 2·5 mm. posterior and slightly dorsal to the external aperture of the penis. There is also on the left side a groove (Ep.Gr.) * in the skin, the course of which corresponds to that of the groove of the right side, already described (p. 314). The blind end of the vas deferens lies very close to the dorsal margin of this groove, just before the latter bends ventrally on approaching the genital orifice.

Examination of the internal structure of these extra organs shows that they are quite normally and perfectly developed. The flagellum shows, in section, the usual longitudinal curved or scroll-like internal fold, exactly as in the normal organ, where its function is presumably to aid in moulding the spermatophore. The wall of the flagellum consists of an internal granular epithelium surrounded by a thick layer of longitudinal and circular muscle fibres. The vas deferens is lined by a folded epithelium, outside which is a thin sheath of muscular and connective tissue.

The penis has the usual structure (see p. 313); it possesses a thin outer sheath (fig. 3, P.Sh.O.), which surrounds the much stouter inner sheath (P.Sh.I.), the latter having about the middle of its length a muscular tubular introvert (Intr.). Enclosed by the proximal part of this inner sheath, the inner surface of which presents the usual rugose appearance, is a muscular copulatory organ (C.O.), on which the vas deferens opens. This organ is fully as large (3 mm. long) as that of the right side of the same specimen and of other specimens examined. In the distal part of the inner sheath there are also longitudinal folds exactly as in the normal organ (see fig. 4). One of these folds ends in a somewhat tongue-like depressed papilla (Pa.), about a millimetre wide, the distal margin of which is raised well above the general surface of the sheath. In all the normal specimens which I have examined, a papilla similar in form and size is also present, and lies close to the point at which the penis enters the atrium. Its position and structure suggest that it forms a rudimentary valvular arrangement serving to partially or completely close the penial aperture, especially when the lips of the latter are in the contracted condition. But whatever its function may be, this papilla apparently marks the distal end of the penis; beyond lies the atrium. In the supernumerary penis this papilla † is about 2·5 mm.

* This groove is present in normal specimens on both right and left sides.

† It is interesting to note that the papilla is situated upon the internal wall, *i.e.* the wall adjacent to the middle line, in both the normal and supernumerary penes, another instance of their symmetrical relations.

from the external aperture, and probably this last portion of the tube ought therefore to be regarded as atrium. This view is supported by the character of the external opening, which agrees in every respect with that of the pore of the normal genital atrium. This left atrium, the inner surface of which is traversed by shallow transverse folds, is rather shorter than the right (normal) atrium, and its walls are slightly thinner.

There is no trace of connection between these supernumerary organs and the ovotestis or the genital ducts of the right side.

The duplication of genital organs in this specimen of *Helix* presents features of interest from three aspects, (1) on account of its rarity, (2) because the form of the supernumerary organs has a significance in relation to the phylogeny of the genital ducts in *Helix*, and (3) in regard to certain points in the ontogeny of the male organs of *Helix*.

This abnormality is not in the same category as those moderately numerous instances in which organs, normally situated in the middle line, have been found in a state of more or less complete division—for example, the penis of mammals. The penis of *Helix* is never median, and there is no trace of any connection between the two penes of this abnormal specimen to suggest that they have been produced by division of a single penis rudiment. In the duplication of the penis and its associated structures this snail presents an example of lateral homœosis which is extremely rare in the Mollusca; in fact the only case,* known to me, at all comparable to this one is Appellöf's (1893, p. 14) record of a specimen of *Eledone cirrhosa* (=*Moschites cirrosa*), in which not only was the third right arm hectocotylised, as usual, but the third left one also. There were on the right arm 57 suckers and on the left 66; both arms presented the normal plan of hectocotylisation and were practically equally developed. There was no corresponding duplication of the internal sexual organs. This specimen therefore presents interesting parallels to the abnormal *Helix* in that (1) the duplicated sperm-transferring organ and the normal one are symmetrically placed with regard to the median plane and are practically equally developed, and (2) there is no modification of the normal sexual organs.

Repetition of the penis on the right side has been twice recorded in *Helix pomatia*, but reference to these cases shows that they are essentially different to the one under consideration. Pegot (1900) described a specimen with three penes, one in its usual position and communicating with the genital atrium by a large canal, the two others, similar in structure but

* A Pteropod with paired penes has recently been described by Meisenheimer; see Addendum, p. 327.

smaller, situated in the vagina. The vas deferens was single in its proximal part but bifurcated distally, each of the two portions passing to one of the supernumerary penes. The vas deferens of the normal penis was represented by a small bud. Each of the penes had a flagellum and a retractor muscle. Paravicini (1898) had previously described a somewhat similar case in which three penes were present and the vas deferens was branched, but only one of the penes was functional. Both these are examples of meristic variation, possibly produced by division of the penis rudiment at an early period of its development, and they differ essentially from the case of the *Helix* with paired penes.

Before passing to the consideration of the significance of the abnormality in regard to the origin and relations of the genital ducts in Pulmonata, it will be of advantage to briefly review the comparative anatomy of the genital ducts of a few Gastropods.

In the Pectinibranchiata (Monotocardia) the genital duct of the female opens into the pallial chamber near the anus. In many of the Taenioglossa and some of the Rachiglossa and Heteropoda the aperture of the genital duct in the male is also situated in the pallial chamber, and in many of the less specialised forms of these groups there is, in those males which possess a penis, a ciliated seminal groove which extends from the genital pore forwards along the right side of the body to the base of the penis, which organ is usually non-introversible and is situated on the head or "neck." The spermatozoa are conducted by this groove from the genital pore to the base of the penis, where they are led into a deep groove or tube which traverses the penis to its tip.

A comparable condition is found in many Tectibranchs, in which the aperture of the hermaphrodite duct is on the right side, within but near the opening of the mantle cavity. From this aperture the fertilised ova escape directly to the exterior, but the spermatozoa pass into a ciliated groove which runs along the right side of the body and head to the penis, which is introversible (except in *Actaeon*).

Within the Pulmonata there are forms which exhibit stages of specialisation of the genital ducts connecting the condition just described with that seen in *Helix*. The most interesting of these stages in relation to the subject of the present paper is that presented by the primitive Auriculid, *Pythia scarabeus*, L., described by Plate (1897) (see text-figure A). In this animal the hermaphrodite- or common-duct opens at the genital pore (G.O.) situated just outside the mantle chamber but near the pulmonary aperture; and from this point a ciliated groove (CIL.GR.) runs forwards on the right side of the body to the aperture of the vas deferens. Here the

spermatozoa are received and conducted by means of the vas deferens (V.D.) to the inner end of the muscular eversible tubular penis (P.), whose external aperture is only a short distance anterior to that of the vas deferens, and just behind the right lip. In the retracted condition the penis, which is

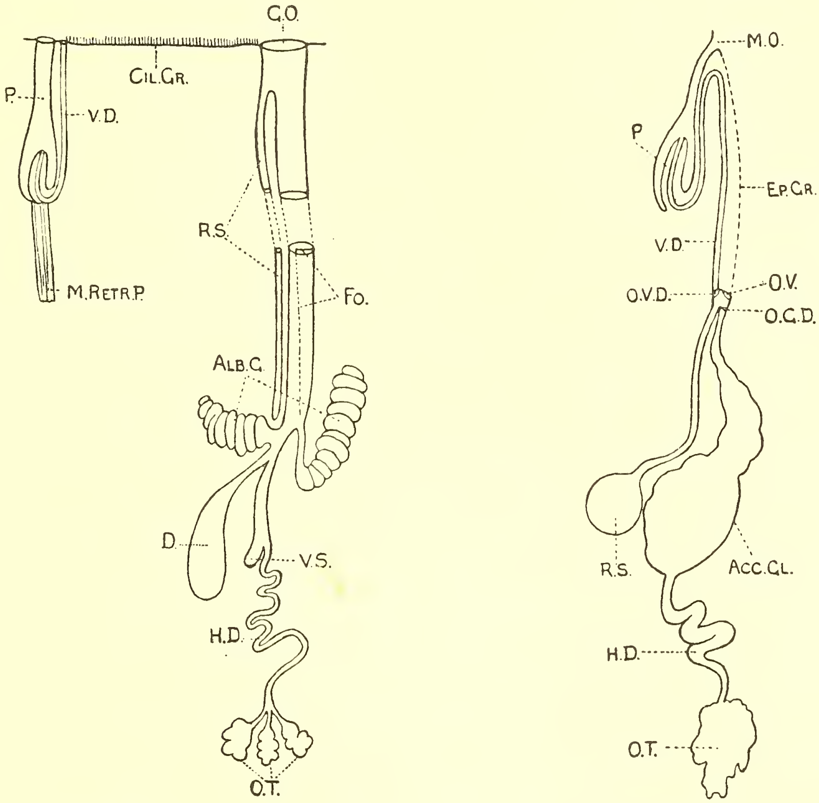

FIG. A.   FIG. B.

FIG. A.—Genital organs of *Pythia scarabeus*, L. ×5. After Plate (1897, p. 134, fig. 1).

Fig. B.—Genital organs of *Auricula myosotis*, Draparnaud. ×16. After Pelseneer (1893. Plate xxii fig. 199).

ACC.GL., accessory glands on common duct; ALB.G., albumen glands; CIL.GR., ciliated grove on the surface of the right side of the body; D., diverticulum, probably a mucous gland; EP.GR., edge of lateral groove in the skin; Fo., fold partially dividing common duct into male and female portions; G.O., opening of genital (common) duct to the exterior; H.D., hermaphrodite duct; M.O., male orifice; M.RETR.P., retractor muscle of penis; O.T., ovotestis; O.V., opening of vagina to exterior; O.V.D., point of origin of vas deferens; P. penis; R.S., receptaculum seminis; V.D., vas deferens; V.S., vesicula seminalis.

provided with a retractor muscle, and vas deferens together form a U-shaped loop open at both ends to the exterior.

A further stage is represented by another member of the family

Auriculidae, namely, *Auricula myosotis*, Draparnaud (see text-figure B). The primitive (hermaphrodite) genital opening (O.V.) is just outside the mantle chamber and serves only for the exit of the ova. From this pore there is a lateral groove (Ep.Gr.) running forward to the penis; but lying in the sub-epidermal tissues, just below the groove, there is a ciliated tubular vas deferens (V.D.) which receives the spermatozoa (at O.V.D.) as they issue from the hermaphrodite duct, and conducts them to the large muscular eversible tubular penis (P.). Thus, by conversion of the ciliated seminal groove into a closed tube, the hermaphrodite aperture has been converted into a functionally female aperture, and the male opening (M.O.) has been secondarily moved forwards from its primitive position so that male and female pores are some distance apart.

In the rest of the Pulmonata the hermaphrodite duct divides into male and female ducts. In the Basommatophora the female aperture retains a position similar to that of the primitive genital opening of *Auricula*, so that the male and female pores are separate.* In the Stylommatophora the female pore has undergone secondary forward movement and is closely associated with the male pore, the two openings being situated in a common genital atrium.†

The penis and vas deferens of the abnormal specimen of *Helix* are interesting as showing, as it were, two different phylogenetic stages—namely, the *Pythia*-stage on the left and the normal condition for *Helix* on the right. The supplementary vas deferens and the retracted penis form a U-shaped loop, both ends of which are in contact with the epidermis, as in *Pythia*; in fact, they so closely resemble the corresponding structures figured by Plate in *Pythia* that these organs may be described as having identical relations in the two forms, except that in the supplementary organs of *Helix* the vas deferens has no pore at its epidermal end, and that it bears a blind outgrowth—the flagellum—which is not present in *Pythia*. While it is impossible to explain the cause of the development of the extra organs in this specimen of *Helix*, it would seem that, assuming that like causes produce like effects, whatever cause (in the ontogeny) has brought about their formation must have been closely similar to, if not identical with, that which has been responsible for the development of the corre-

* Except in the genus *Amphibola* and the family Siphonariidae where the male and female ducts open into a common genital atrium.

† Except in the families Oncidiidae and Vaginulidae and the genus *Atopos*, where a separation of the genital pores has been brought about by detorsion, which has caused the anus and the female opening to move backwards, so that the latter now lies, in *Vaginula* about the middle of the right side and in *Oncidiella* and *Atopos* posteriorly near the anus. These are to be regarded as specialised rather than as primitive forms (Pelseneer, 1901, p. 27).

sponding normal organs, since the results are so similar in the two cases. But on the left side the penis and its annexes have been free to develop without the disturbing influences of the vagina, oviduct, and their accessory structures, which, on the right side, have been secondarily moved forwards, in phylogeny, into the sphere of the male copulatory organ. Under these circumstances the supernumerary organs would be more likely to assume a condition more closely resembling that which they would present in the ancestral form in which the male and female genital apertures were some distance apart, and in which, consequently, the male organs developed free from the disturbing elements introduced in later forms by the shifting forwards of the female aperture and its associated structures.

The form of these extra organs (the vas deferens having a close association at one end with the epidermis) which it is suggested, by the above argument, might also be assumed by the corresponding normal organs if they were likewise free to develop independently of the female structures, supports the already well-established view that the present condition of the genital ducts in *Helix* and other Stylommatophora has been derived from a condition existing in the ancestral form in which the vas deferens and penis were connected with the primitive genital aperture by means of a lateral groove, such as is still found in *Pythia.*

The supernumerary genital organs have also a significance in relation to the ontogeny of the male genital apparatus. It is very difficult to reach a definite conclusion concerning the mode of development of the male organs of the Pulmonata from a study of the published accounts; evidently the conditions are complicated and vary considerably in different genera or even species.

Eisig (1869), working on *Limnaea auricularis,* concluded that the genital apparatus is formed in three separate sections which afterwards become connected together: (1) the ovotestis and the hermaphrodite duct; (2) the oviduct, its glands and the prostatic part of the vas deferens; and (3) the penis and the lower (or distal) part of the vas deferens. The penis arises as a solid ingrowth from the epidermis, and the vas deferens is formed as a diverticulum from the free inner pole of the penis. According to Rouzaud (1885) the whole genital apparatus of Pulmonates is the product of a single cellular bud on the inner side of the wall of the neck region at a point which later becomes the site of the common genital orifice in the Helicidae and of the female pore in the Limnaeidae. The penis arises as a secondary bud, which in *Helix* is situated upon the primitive bud, but in *Limnaea* upon the wall of the neck region a short distance from the primitive bud. Rouzaud attributes the separation of the penis rudiment in

Limnaea to its passive migration, but Klotz (1889) denies that any such migration takes place.

Von Jhering and Brock assign to the penis a quite different mode of origin. The former (1875) states that the parts of the genital apparatus in *Helix pomatia* and *H. nemoralis* are not separately developed (differing from Eisig's results on *Limnaea*), but arise as differentiations of a mesodermal rudiment. Brock (1886) describes the first appearance of the penis in *Agriolimax agrestis* (L.), not as a separate structure but as a swelling, afterwards a blind sac, on the purely mesodermal primitive genital duct, which latter is not at first in connection either with the ovotestis or the exterior. He also found that the vas deferens develops as an outgrowth from the inner end of the penis, and its free end unites with and opens into the common duct. The genital duct establishes communication with the exterior by means of the genital atrium opening outwards; there is, according to Brock, no trace of invagination of the skin to form either the penis or the atrium. For several reasons it is unfortunate that Brock's careful work was carried out on *Agriolimax*, which, however convenient it may be from the point of view of microtome technique, is so modified in various directions that it could scarcely be expected that the ontogeny of the genital organs would remain unaffected.* Especially from the point of view of the development of the penis, the particular species (*agrestis*) investigated was very ill adapted for the purpose, as Simroth (1887, pp. 646–647) remarked, because of the indefinite form and structure of that organ in the adult.

Subsequent to the publication of Brock's results Klotz (1889) investigated the development of the sexual organs in *Limnaea ovata* and affirms that the penis arises independently at the hinder margin of the right tentacle, as an ectodermic invagination, thus closely agreeing with Eisig (who, however, described the ingrowth as solid).

A consideration of the above accounts seems to indicate that in *Limnaea* the penis is formed directly from the epidermis either as a solid or a hollow ingrowth, but in *Helix* and *Agriolimax* its development is bound up with that of the genital atrium and the neighbouring parts of the female duct, so that the penis appears to arise from the atrium or from the oviduct, more probably from the former which is relatively long in the earlier stages. The vas deferens, according to Eisig and Brock, arises as an outgrowth from the inner end of the penis, but according to Rouzaud, Simroth and

* See also the observations of Simroth and Babor below (p. 324), which show that the penis in some Limacidae, *e.g.* *Agriolimax laevis* and *Limax maximus*, is frequently delayed in development.

Klotz, it seems to be produced by a process of splitting from the penis rudiment.

The condition of the supernumerary penis and vas deferens in the abnormal *Helix* indicates that the penis arises from the epidermis as a solid or hollow ingrowth which has been secondarily carried inwards by the invaginating atrium. Its development would be difficult to explain if the penis were differentiated from, and dependent for its origin upon, a primitive mesodermal sexual duct, as held by Brock, for there is no trace whatever of any such duct on the left side of this *Helix*. The supernumerary vas deferens probably arose in the normal way as a diverticulum from the inner end of the penis.

The origin of the penis from the epidermis is in agreement with its phylogenetic history, which points to its appearance as an elevated epidermal organ, at first non-introversible and grooved, but later introversible and tubular, and also better accords with other observations upon abnormalities in the Pulmonata than would the origin of the penis from any part of the female or common sexual ducts. Bietrix (1886) records a specimen of *Helix pomatia* in which the genital apparatus is divided into three quite separate portions: (1) the ovotestis and albumen gland, (2) the dart sac and mucous glands, and (3) the penis and its annexes. The penis and flagellum are quite normal,* but the vas deferens is represented by a short cæcum 4 mm. in length. The atrium appears to be a single swollen prolongation of the penis sheath. Mangenot (1883) describes a specimen of *Helix pomatia* in which the external genital aperture is closed, in consequence of which the atrium and vagina are much dilated by imprisoned fluid contents. The atrium is somewhat oval in shape, and the vagina and penis open into it at opposite ends of its longer axis. The penis has the usual form and size, but the vas deferens is a slender cæcal appendage 6 mm. long, arising from the inner end of the penis. The vas deferens, which bears a flagellum of approximately normal size, has no connection whatever with the common duct, and the only connection between the penis and the vagina is the purely secondary one afforded by the atrium. Both these cases indicate that the penis arises quite independently of the common duct or the vagina, and that its development is, in *Helix*, closely associated with that of the atrium.

That the penis "Anlage" in some cases fails to develop seems to be indicated by the recorded examples of absence of the male sexual organs in the Pulmonata. Such a defect is more readily understood if these organs arise from a separate "Anlage" than if they form an integral part of the

* Except that the retractor muscle is represented by two strands with separate origins, but these unite so that the muscle is single at its point of insertion on the penis.

genital duct from its earliest appearance, for in many cases the rest of the genital apparatus is quite normal. Von Jhering (1885, p. 207) remarks upon the frequent absence of the male apparatus in specimens of *Limax*, and Simroth (1883–1885, p. 223) describes three specimens of *Agriolimax laevis* as being purely female,* though the one figured (Taf. 9, fig. 22H) shows a small papilliform rudiment of the penis but no trace of vas deferens.† Simroth (1890, pp. 21, 22) also figures two examples of *Vitrina lamarcki* which show different phases of reduction of the male apparatus. In one case (Taf. 2, fig. 11) the penis is quite rudimentary in the form of a small nodule on the wall of the atrium; it has no retractor muscle and no connection with the sperm-duct. The figure (Taf. 2, fig. 10) of another example shows a penis only slightly larger, without a retractor muscle, but the vas deferens opens into it. A similar condition to this last is also figured by Simroth (1891, p. 228, and Taf. 9, fig. 13) in a specimen of *Plutonia atlantica*. Collinge records specimens of *Helix aspersa* (1893, p. 237), *Arion intermedius* (1893, p. 238), and *A. hortensis* (1904, p. 15), in which the male organs are wanting; in the two latter the common duct consisted of oviduct only. It is possible that in some of the cases above mentioned the animals concerned were young and proterogynously hermaphrodite, and that although the male organs had not yet made their appearance, they would have done so later. Babor's observations (1894) show that this is probably the case in *Agriolimax laevis*. All the young examples (not more than 2 cm. long) of this slug which he examined (1894, p. 56) possessed only female genitalia, and the gonad contained only ova or ova along with a few spermatogonia. After further growth the penis and its annexes were formed upon the atrium, and so the hermaphrodite organs became complete. In two large specimens, 4 cm. long, the penis was hypertrophied and the gonad was purely male. *Limax maximus* is also proterogynous, but even young animals have a rudimentary penis although the gonad at this time contains ova only (*loc. cit.*, p. 57). When they have reached their definitive size, the animals are hermaphrodite or male, the gonad swarms with spermatozoa, and the penis is large. Babor (p. 58) concludes that in most Limacidae there is a cycle of development, the animal being at first unisexual (in the cases above cited—female), then hermaphrodite, and finally again unisexual (male, in the cases cited).

The examples described by von Jhering, Simroth, and Babor are there-

* The gonad contained ova only.

† Simroth (1892–1893) recorded a specimen of *Limax primitivus* in which a retractor muscle was attached to the wall of the atrium, but there was no penis. Babor (1894, p. 57), however, having examined this example, concludes that a penis is present.

fore probable cases of proterogynous hermaphroditism. The gonad first assumes the female condition, in many Limacidae and possibly in some other Stylommatophora,* in correlation with which the common duct consists of oviduct only, and the penis "Anlage" is, for a time at least, suppressed or remains rudimentary.

It is interesting to note that all the cases (with one exception—Collinge's *Helix aspersa*) of absence or great reduction of male organs † in the Pulmonata have been recorded in genera in which the shell is depressed, reduced, or absent—that is, in the most specialised forms. But in the figures of these specimens of *Limax*, *Agriolimax*, and *Vitrina* given by Babor and Simroth the penis rudiment is invariably found on the wall of the atrium, and there is nothing in the figures to show that the penis is formed upon the primitive genital (which becomes later—the common) duct.

Consideration of the abnormalities described in the preceding pages leads to the conclusion that the penis develops as an epidermal structure which, in the Stylommatophora, is closely associated with, and is secondarily carried inwards by, the invaginating atrium. The intimate connection of the atrium and penis is especially well seen in the supernumerary organs of the specimen of *Helix* described in the earlier part of this paper (pp. 314 to 317) and in the example recorded by Bietrix (see above, p. 323). The origin of the vas deferens as a diverticulum from the inner end of the penis, which only later fuses with and opens into the male part of the common duct, is strongly supported by the conditions seen in the specimens described by Bietrix and Mangenot (p. 323). In proterogynously hermaphrodite forms,

* The absence of penis in other Opisthobranchs seems to be attributable to a similar cause. For instance, Peck (1887–1890, p. 349) describes specimens of *Cymbuliopsis calceola* in which the gonad is in a state of female activity and there is no penis, and Pelseneer (1893, p. 24, footnote 2) gives a similar record in the case of *Clio striata*.

Giard (1888) refers to a specimen of *Paludina* infected with *Distomum militare*, probably a case of parasitic castration, in association with which the penis is reduced ; he attributes to a similar cause the occasional absence of penis in specimens of *Pterotrachea*, otherwise showing male characters.

† Reduction of the female ducts has been rarely observed in the Pulmonata or in any other hermaphrodite mollusc (Pelseneer, 1895, pp. 37–38). Schuberth (1892, p. 50) has described an example of *Helix pomatia* in which the oviduct is almost completely atrophied for a distance of 36 mm. from the albumen gland, so that it is present above the prostate as a mere streak. The anterior portion of the oviduct is, however, fully developed. In one or two of the large male examples recorded by Babor the female ducts have undergone partial reduction, but were probably formerly present in a fully-developed condition.

The list of recorded abnormalities in the genital apparatus of Pulmonates, as far as they have come under my notice, may be completed by reference (1) to an anomalous specimen of *Clausilia martensi* mentioned by Babor (1900), in which the penis and vas deferens are fused so as to form a hoop, and (2) to a specimen of *Urocyclus ehlersi*, described by Simroth (1905), in which occlusion of the opening of the penis into the atrium has taken place, and the distal end of the penis has dilated to form a thin-walled spermatophore-sac.

especially in those in which the shell is depressed, reduced, or absent, the development of the penis rudiment may be postponed until the male elements in the gonad are further developed.

Summary.

1. An account is given of a fully grown specimen of *Helix pomatia*, which, in addition to the normal set of reproductive organs present on the right side, possesses also on the left side a set of accessory male organs—namely, a penis and its retractor muscle, vas deferens, and flagellum. There is no trace of connection between these supernumerary organs and the ovotestis or the genital ducts of the right side.

2. The normal and supernumerary penes are equally developed and are symmetrically placed with regard to the median plane of the head.

3. The extra penis has the usual structure; it possesses a fully-developed copulatory organ and a muscular tubular introvert upon the inner sheath.

4. The extra vas deferens opens at one end into the penis and at the other terminates blindly in the sub-epidermal tissue; the blind end is bound to the epidermis by a short cord of connective tissue.

5. The extra flagellum is normal in size and in its internal structure.

6. The penial nerves on both sides have a corresponding course from the cerebral ganglion to their distribution on the penes. Owing to defective preservation it was impossible to determine whether the fibres of the extra penial nerve arise from the ocular nerve, as is apparently the case, or whether they really arise from the pedal ganglion, as do those of the normal penial nerve.

7. The penis opens into a short atrium which communicates with the exterior by a genital aperture of normal size and shape. This supernumerary genital aperture occupies a position on the left side exactly corresponding to that of the normal one on the right side. About 2·5 mm. posterior and slightly dorsal to it is the blind end of the vas deferens, which is situated close to the dorsal margin of the epidermal groove, which traverses the anterior portion of the animal.

8. In the duplication of the penis and its associated structures this snail presents an example of lateral homœosis. The only comparable cases recorded in the Mollusca are a Pteropod with paired penes (see Addendum, p. 327) and a specimen of *Moschites cirrosa*, in which not only was the third right arm hectocotylised as usual, but the third left one also.

9. The supplementary vas deferens and the retracted penis form a U-shaped loop, both ends of which are in contact with the epidermis, and

so closely resemble the corresponding structures figured by Plate in the primitive Pulmonate, *Pythia scarabeus,* that these organs may be described as having identical relations in the two forms, except that in the supplementary organs of *Helix* the vas deferens has no actual opening at its epidermal end and that it bears a flagellum which is not present in *Pythia.* It is suggested that the supplementary organs of *Helix,* having been able to develop free from the disturbing influence of the vagina, oviduct, and their accessory structures (which on the right side have been secondarily moved forwards in phylogeny), have assumed a condition closely resembling that which they would present in the ancestral form in which male and female apertures were some distance apart. The form of the extra organs (the vas deferens having a close connection at one end with the epidermis), which, it is suggested, might also be assumed by the corresponding normal organs if they were also free to develop independently of the female structures, supports the view that the present condition of the genital ducts in *Helix* and in other Stylommatophora, has been derived from a condition existing in the ancestral form in which the vas deferens and penis were connected with the primitive genital opening by means of a lateral groove, such as is still found in *Pythia.*

10. Consideration of this and of other abnormalities which have been described in the Pulmonata leads to the conclusion that the penis develops as an epidermal structure which in *Helix* and other Stylommatophora is closely associated with, and is secondarily carried inwards by, the invaginating atrium. In proterogynously hermaphrodite forms, especially in those in which the shell is depressed, reduced, or absent, the development of the penis rudiment may be postponed until the male elements in the gonad are further developed.

ADDENDUM. (*August* 3, 1907.)

After this paper was in type a record of another abnormality, presenting a close parallel to the one described on pp. 312 to 317, was brought to my notice by Professor Simroth. Meisenheimer (Pteropoda, in *Wiss. Ergebn. Deutsch Tiefsee-Exped. auf dem Dampfer "Valdivia," 1898-1899,* Bd. ix. pp. 290–291, text-fig. 32: Jena, 1905) describes a single example of the gymnosomatous Pteropod *Halopsyche gaudichaudi,* Souleyet, in which there are two penes symmetrically placed on the right and left sides of the buccal mass. The internal genital organs are quite normal, and the genital duct opens to the exterior on the right side of the body. From this aperture a seminal groove leads forwards to the vas deferens and penis

(*cf.* text-fig. A, p. 319). On the left side of the animal, in a position corresponding to that of the right penis, there is an equally well-developed penis, with a vas deferens, from which a seminal groove extends backwards some distance, and finally disappears on reaching the dorsal surface of the body. There is no connection between the supplementary penis and the normal genital organs.

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DESCRIPTION OF THE PLATE.

Key to the Reference Letters.

Alb.G.	Albumen gland.
Art.	Artery.
Atr.	Genital atrium.
C.D.F.	Common duct, female portion.
C.D.M.	Common duct, male portion, covered with prostatic tissue.
C.G.	Cerebral ganglion in its sheath.
Cn.	Connectives (cerebro-pleural and cerebro-pedal) in their sheath.
C.O.	Copulatory organ.
Con.T.	Connective tissue cord connecting the blind end of the supernumerary vas deferens to the epidermis.
D.S.	Dart sac.
E.	Eye.

Ep.	Epidermis.
Ep.Gr.	Groove in epidermis.
F.	Foot.
Fl.	Flagellum.
Fl.S.	Supernumerary flagellum.
G.O.S.	Supernumerary external genital opening.
H.D.	Hermaphrodite duct.
H.D.S.	Hermaphrodite duct, saccular portion.
Intr.	Muscular introvert on inner sheath of penis.
Mo.	Mouth.
M.Retr.	Retractor muscle of foot, its two portions embracing the epidermal ends of the supernumerary penis and vas deferens.
M.Retr.A.	Retractor muscles of the anterior end.
M.Retr.B.	Retractor muscle of buccal mass.
M.Retr.P. / M.Retr.P′	The two portions of the retractor muscle of the penis.
M.Retr.P.S. / M.Retr.P.S.′	The two portions of the retractor muscle of the supernumerary penis.
M.Retr.Tent.P.L.	Retractor muscle of left posterior tentacle.
Muc.G.	Mucous glands.
N.B.	Buccal nerves (cut short).
N.F.	Pedal nerves.
N.Fac.	Facial nerve.
N.G.	Genital nerve.
N.L.E.	External labial nerve (also supplies anterior tentacle).
N.L.I.	Internal labial nerve.
N.Oc.	Ocular nerve.
N.P.	Nerve to penis (penial nerve).
N.P.L.	Left pallial nerve.
N.P.M.	Median pallial nerve.
N.P.R.	Right pallial nerve.
N.P.S.	Nerve to supernumerary penis.
O.T.	Ovotestis.
Ovd.	Oviduct.
P.	Penis.
Pa.	Papilla.
P.S.	Supernumerary penis.
P.Sh.I.	Inner sheath of penis.
P.Sh.O.	Outer sheath of penis.
R.S.	Receptaculum seminis (or spermotheca).
R.S.B.	Bulb of receptaculum seminis.
Sh.	Connective tissue sheath enclosing the origins of the retractor muscles of the penes.
Tent.A.L.	Left anterior tentacle.
Tent.A.R.	Right anterior tentacle.
Tent.P.L.	Left posterior (or ocular) tentacle.
Tent.P.R.	Right posterior (or ocular) tentacle.
V.	Vagina.

Ashworth: HELIX POMATIA

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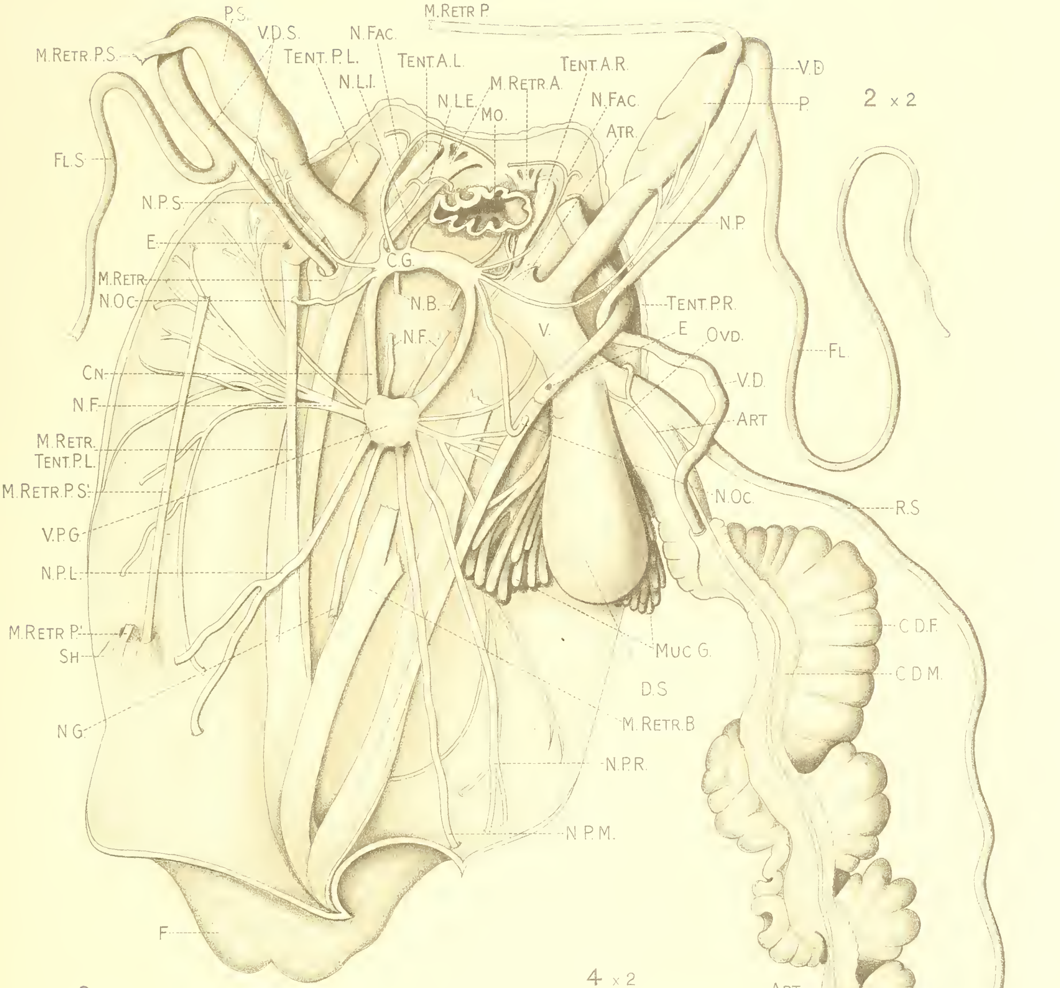
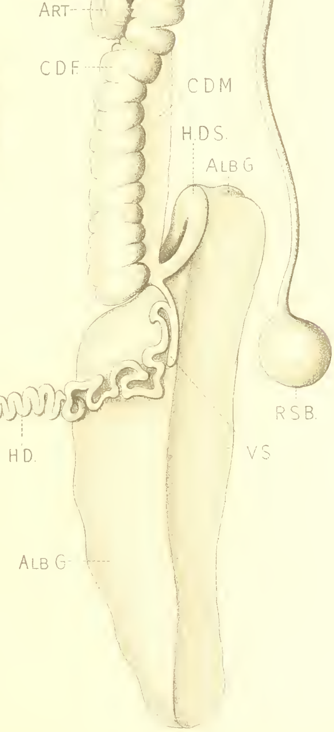


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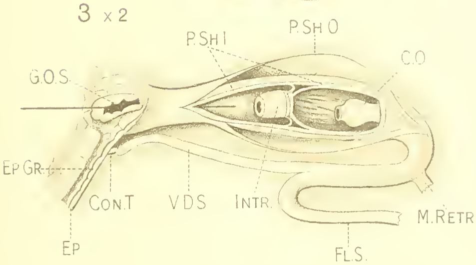

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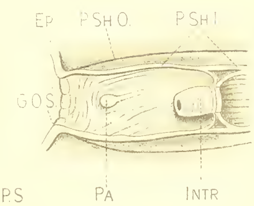

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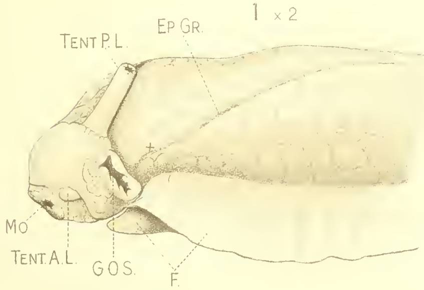


J. H. Ashworth, del.
Lith. Anst. v. Werner & Winter, Frankfurt a/M.

V.D.	Vas deferens.
V.D.S.	Supernumerary vas deferens.
V.P.G.	Viscero-pleural ganglion.
V.S.	Vesicula seminalis.

+ In fig. 1 indicates the position of the epidermal blind end of the supernumerary vas deferens.

Figures* of the Abnormal Specimen of Helix pomatia, L., all twice natural size.

Fig. 1. The anterior end seen from the left side, to show the supernumerary genital orifice (G.O.S.). The + marks the position of the epidermal blind end of the vas deferens. For further description, see pp. 315, 316.

Fig. 2. The dissection seen from the dorsal aspect. The alimentary canal, heart, nephridium, and roof of mantle chamber have been removed and the reproductive organs spread out so as to display the various parts. The supernumerary flagellum (Fl.S.) is incomplete, its terminal portion having been accidentally severed and lost. The cerebral ganglia (C.G.) are shown still enclosed in their sheath (as the latter could not be safely removed), as are also the connectives (Cn.). The sheath has, however, been removed from the viscero-pleural ganglion (V.P.G.), which is seen to be divisible into five portions—a median abdominal ganglion, right and left of which is a visceral ganglion, to the outer side of each of which is a pleural ganglion. The pedal ganglia are more ventrally situated and are not shown in the figure; only a few of the pedal nerves are shown; no attempt has been made to represent those which run posteriorly. On the right side the muscle (M.Retr.) which retracts the anterior end and the foot has been severed; the retractor of the buccal mass (M.Retr.B.) has also been cut through.

For descriptions of the normal and supernumerary reproductive organs, see pp. 312, 313, and 314, 315.

Fig. 3. A dissection of the supernumerary organs from the left or outer aspect. The outer and inner sheaths of the penis have been cut open to show the copulatory organ (C.O.), the introvert (Intr.), and also the rugose folds on the inner surface of the inner sheath. The blind end of the vas deferens (V.D.) is seen close beneath the epidermis, to which it is attached by a short cord of connective tissue (Con.T.). Only a small portion of the flagellum (Fl.S.) is shown. A bristle has been passed through the external genital opening into the lumen of the penis. For further description, see p. 316.

Fig. 4. A further dissection of the distal portion of the penis, from the left or outer aspect. The incision in the sheaths of the penis shown in fig. 3 has been continued to the left (that is, distally), so as to extend to the genital opening. The flaps have been turned back so as to afford a clear view of the inner surface of the inner sheath in its distal portion. Note the longitudinal folds and the papilla (Pa.). Probably the region from this papilla to the external opening (G.O.S.) ought to be regarded as atrium. For further description, see pp. 316, 317.

* The cost of drawing the figures on stone has been defrayed by a grant from the Carnegie Trust for the Universities of Scotland.

(*Issued separately October* 4, 1907.)

XXXIII.—A Sketch of Japanese National Development, more especially with reference to Education. By Baron Kikuchi, M.A., D.Sc., LL.D., Emeritus Professor of Mathematics in the Imperial University of Japan (ex-Minister of Education, and formerly President of the Imperial University).

(An Address delivered at the request of the Council, 17th June 1907.)

Fifty years ago, Japan was almost a mythical land to the Western world; what little was known of it was through the medium of the Dutch, who were allowed to trade in the single port of Nagasaki. What we Japanese knew of the outer world was also derived mostly from them, and from such books as were brought into the country by them. In 1853 came Commodore Perry, and demanded that we should open up our country to the commerce of the world; we were but ill prepared for such a course, but it was not in our power to resist the demand, and unwillingly enough the Shōgun's government was obliged, notwithstanding the strong opposition of the Imperial Court and of the conservative elements in the country, to contract a treaty of commerce with America, which was speedily followed by those with other nations.

Brought thus face to face with the Occidental nations, we saw how immensely superior to us they were in power, and that if we would not be treated as inferiors—nay, if we would preserve our existence as an independent nation—we must acquire the knowledge and learn the arts which gave them this power. We therefore earnestly and resolutely set to work to do this. The results, some of which were manifested during the late war, seem to have astonished those who have not watched us closely. In fact, I think in former days we were not taken seriously by the world. We were regarded as children, merry, carelessly enjoying the life, never earnest in anything; Japan was regarded merely as a land of curios, of ivory netsukés, of flowers, of pretty sceneries and pretty arts, of geishas, of Madame Chrysanthèmes. One of my friends who was in England at the time of the war told me that he was asked how the Japanese had managed to jump out of their skin. Now, I hope to show you to-day, as well as I can in so short a time, that we did nothing of the kind—that all we have done is simply a natural outcome of our historical development.

I think it may be said to be one of the most characteristic traits of the

Japanese people that we are eager to find out what other people have got to teach us; we are apt pupils, and when we think that the adoption of new methods, material or intellectual, is conducive to the progress of our country, we do not hesitate to adopt them, wherever we find them. I know that we have been reproached for this: we have been told that we are nothing but a nation of plagiarists, only capable of imitating what other people have done and not of originating anything. For the present, we are quite content to bear this reproach; we have been so busy with introducing Western civilisation that we have not perhaps done much original. And yet I think, in some respects, we have succeeded, if not in doing anything original, at least in adapting what we have adopted from others to our needs, and in improving upon them so far as our own circumstances are concerned. I would even point out on behalf of my countrymen, if I may be allowed, that we have already made some original investigations in science, the results of which are very important, and some, I have been told, even epoch-making; but it is not now my purpose to enlarge upon this point.

The present is not the only time that we have deliberately introduced an alien and, as we considered, a superior civilisation. In the sixth century we were brought into contact with the Chinese civilisation; at first through Corea, and afterwards directly, it was introduced into Japan. Our ancestors saw, as we see in more recent times, the superiority of the new civilisation, and, perceiving that its adoption would tend to the betterment of their political, social, and moral conditions, deliberately set about to adopt it. We learned many new arts and crafts; we had no letters, so we borrowed their ideographs, we even adopted their literature bodily; astronomy, mathematics, medicine, and other branches of knowledge were imported and taught to our young nobles in the University. And, because our old and somewhat primitive system of government was not fitted to the new and more complex state of society induced by the introduction of the new civilisation, and also because many abuses had crept in, laws and administrative system were reformed after the Chinese model; "the Reformation of Taikwa era" (middle of the seventh century) and the promulgation of "the Taihō code" (701), covering the whole range of government administration, including what would not now come under such, can only be compared to the reforms and changes of the present era of Meiji. We have thus twice within the historic age made almost complete changes in our system of government, in our laws, and in our institutions, entirely after foreign models.

Yet, amidst all these changes, we have kept the essential characteristics of our nationality. That we should have been most deeply affected in our thoughts and ideas by the Chinese literature is only natural, and quite true.

But there was something peculiar to our people, which has persisted through all those changes, and that is what we call "Yamato Damashii," or the Japanese spirit. This spirit, which Motoori, the founder of Neo-Shintoism, has characterised in a famous ode as "like the blossom of the mountain cherry blooming in the morning sun," consists essentially in the intense love of country and reverence for and devotion to the Imperial House. The peculiar reverence that we Japanese have for the Imperial House, our loyalty and devotion, is a heritage of many centuries. The relation between the Emperor and the people is not simply one between the present Emperor and the present generation of the Japanese people, but a relation between his ancestors and our ancestors for many centuries. On this point we are in direct opposition to the teaching of Chinese philosophers, for the Chinese are democratic in this respect; two of the most revered of their Emperors, Yao and Shung, bequeathing their throne not to their sons but to whom they deemed to be the worthiest successors. We have borrowed a great deal from China, but in our exaltation of loyalty and reverence to the Imperial House we have kept throughout all ages our own peculiar ideals as distinguished from the Chinese, notwithstanding the respect in which the Chinese ethics have always been held.

This relation between the Imperial House and the people is the fundamental character of our nationality. It goes back to the days of mythology, when the first ancestor of the Imperial House, the goddess Ama-Terasu-Ō-mi-Kami, or the great goddess of Celestial Light who reigned in the Taka-Ma-ga-Hara or High Heavenly Plain, sent down her grandson to Japan, saying, "This is the land of which my descendants shall be the lords. Do thou proceed thither and govern it. Go! *The prosperity of thy dynasty shall be coeval with Heaven and Earth.*" These last words are continually occurring in our literature, and are regarded as "the charter of the land" by the Japanese. It was, however, the great-grandson of this prince who finally succeeded in establishing himself in the province of Yamato, and founded the Japanese empire. From this first Emperor, known as Jimmu Tennō, there has been *one unbroken line of descent* to the present Emperor. This unique character of our Imperial dynasty, together with the ancestor-worship or reverence for ancestors, has brought about the peculiar relation that I have mentioned as existing between the Imperial House and the people.

The ancestor-worship which I have just mentioned is another of our national characteristics which has persisted from the earliest times to the present day, and it is closely connected with the relation between the Imperial House and the people; for were there no spirit of reverence for

ancestors, this relation would not derive the importance it has from the fact of its persistence for generations of our ancestors.

I have tried to explain briefly the essential or fundamental characteristics of the Japanese people: I would refer you to my inaugural address at London University, published in the *Nineteenth Century* for June, for some further details, and now pass on to a brief account of what happened within more recent times, in order that you may know in what condition we were just before we were brought into contact with the Occidental nations.

From the beginning of the sixth century, and for more than a century after, some of the great noble houses took advantage of the somewhat peculiar condition of the Imperial House at the time to usurp a great deal of the power of government into their hands; and, although it was restored for some time into the Imperial hands, the great house of Fujiwara again succeeded gradually in establishing itself in authority. Empresses were always chosen from daughters of their house, and none but issues of these ascended the throne, and the Fujiwaras became practical rulers of the land; high government offices were almost exclusively conferred on members of their house. The court was given up to refinement and luxury, and governors of provinces stayed in the capital, preferring easy and pleasant life at the court to the rough and hard duties in the provinces. Under such conditions it is not to be wondered at that maladministration and abuses of all kinds were rampant; then it was that the military class first came into existence. At first they were willing to serve in minor and rougher offices about the court, and to undertake military expeditions into outlying provinces, and even to serve as guards to the nobles of the Fujiwara house. Gradually the two houses of Taira and Minamoto, who were both of them descended from the emperors, became leaders of this class, and, as was almost inevitable, they were not content long to occupy subordinate positions. At first the Tairas got hold of the government, but they soon followed in the footsteps of the Fujiwaras, and became arrogant, luxurious, and degenerate. The Minamoto house, which had been almost destroyed by its rival, the Tairas, arose under its able leader, Yoritomo, and in its turn destroyed the Taira house entirely. It is a remarkable fact, as showing the reverence in which the Imperial House has always been held through all its vicissitudes, that in the rising of Yoritomo the Imperial rescript given him to rise and destroy the Tairas and deliver the court from their arrogance played a highly important part. Yoritomo, wise enough to perceive the mistakes of the Tairas, established himself in Kamakura, instead of Kyōto, and, leaving the court to itself, proceeded to frame a

scheme by which all the real powers of government became his in virtue of his offices as the generalissimo of forces to subdue the barbarians (Sei-I-Tai-Shōgun) and as the superintendent of the police for sixty-six provinces (Rokujūroku-Koku-Sō-Tsuihoshi). I cannot enter into the most adroit manner in which all these changes were accomplished; but since this, which we may perhaps date from 1190, the year of the appointment of Yoritomo to the office of the superintendent of the police, until 1868, with a very brief interval in the fourteenth century, the country was under the rule of the military class, the Shōguns or their regents being the practical rulers of the land. Again I repeat that nevertheless all this time the people in general, including the military class and the rulers themselves, held the Imperial House in utmost reverence. I lay great stress on this point, because, as I have stated, I consider this a most important factor in our national development.

And here I must say a few words about a particular sect of Buddhism, viz., the Zen, which has exercised a very great influence on the military class, from the time when it became the ruling class of the country. Its influence is felt in our character even at the present day, and something must be said about it if we would fully understand the spirit of our people. The Zen sect was first introduced into Japan just about the time of the establishment of the feudal system by Yoritomo; it was warmly welcomed by the military chiefs, just as the earlier sects introduced had been by the court; and many Zen temples were built by them in Kamakura and afterwards in Kyōto. There were many reasons why it should be so well received by the military class. It teaches a man how to arrive, by means of a certain form of contemplation, at a habitual state of mind which will enable him to meet with calmness any event and endure with indifference any hardship. Such a power of perfect self-control and undisturbable peace of mind must indeed be invaluable to a man of any profession, but more especially to a Samurai of old days, whose life had to be spent amid continual dangers and hardships. There was also charm in that it did not require on the part of its votaries any profound book-knowledge; it was also in accord with that simplicity and frugality which were the characteristics of the Samurai. I may mention that at the present time there are many earnest young men who perform this contemplation at the temples of the Zen sect.

Coming down now to the fifteenth century, we find that, owing to the weakness of the central government, military chiefs arose in all parts of the country and were continually fighting against each other. Among these, there appeared three great men. The first was Nobunaga, who from

being a petty lord of Owari brought the whole of central Japan under his power. After he was attacked and killed by one of his generals, another of his generals, Hideyoshi, better known by his subsequent title of the Taikō, succeeded him in power and established his authority over the whole country. After his death in 1598, Tokugawa Iyeyasu, who had been gradually strengthening himself, biding his time and patiently waiting for his opportunity under Nobunaga and Hideyoshi, became the Sei-I-Tai-Shōgun in 1603, and established his government in Yedo, where his successors ruled for fifteen generations until 1868.

By the defeat and death of Hideyori, Hideyoshi's son, in 1615, Iyeyasu's authority became undisputed, and the country enjoyed profound peace for two centuries and a half, during which the feudal system of government received its most perfect development, and the Bushidō or "the Way of Samurai" was fully elaborated.

The first three Shōguns of the House of Tokugawa, all three men of great capacity, assisted by many followers of devoted loyalty and highest ability, succeeded in thoroughly consolidating their authority and in completing the organisation of government. As an example of the method by which the Shogunate kept the daimyōs completely under their power, I may take the case of Maeda, the daimyō of Kaga. As this was a powerful house, whose territories extended over the three provinces of Kaga, Noto, and part of Echizen, and whose annual revenue exceeded one million *koku* of rice, a cadet of the Tokugawa house was made the Lord of Echizen, to block the route of Maeda to Kyōto, and several faithful retainers of Tokugawa were made to be daimyōs in Echigo, to the north; thus completely shutting in Maeda. A similar policy was pursued towards all daimyōs who were likely to be a cause of anxiety to the Shogunate. Another device was that of requiring the sojourn in Yedo of every daimyō at fixed intervals and for a fixed period; while their wives and children were obliged to reside in Yedo, thus serving as hostages. Thus, although daimyōs enjoyed almost complete autonomy within their territories in almost everything, financial, military, judicial, educational, and industrial, the Shogunate seldom interfering except in extreme cases, yet this device effectually obviated too dangerous independence.

So well was this policy towards the daimyōs planned, that the shutting up of the empire from all foreign intercourse, which, as you are aware, was deliberately ordered by the third Shōgun, and consequent absence of all external competition and stimulus, was compensated for by the balance of power, as it were, between the different daimyōs, and did not give rise to such stagnation and decline as were likely to happen to a nation situated in such

circumstances. On the contrary, the daimyōs and their counsellors were ever strenuous in their endeavours to uphold and increase the power and honour of their clans, on the one hand by promoting the education and training of their retainers and stimulating their spirit, and on the other by encouraging the industrial arts and crafts, tending to the increase of wealth within their territories. Indeed, the propagation of education and the development of industry, fostered by peaceful rivalry, were the two great products of two centuries and a half of the Tokugawa Shogunate, which not only preserved the country from stagnation and decline during that period, but enabled the nation, when it was brought face to face with the Occidental nations at the close of that period, to assimilate their knowledge, and gave it flexibility enough to adopt their methods.

Up to the beginning of the Tokugawa Shogunate, Buddhist temples were almost the only places where people could obtain any learning; even the sons of military chiefs were mostly taught at these temples, and they continued to be schools for the common people long after. It is also to be remarked that from the thirteenth to the sixteenth centuries most of the great generals and lords were assisted by the counsels of priests, mostly of the Zen sect. There were in those days, indeed, no learned scholars who were not Buddhist priests. Towards the close of the sixteenth century, a man named Fujiwara Seikwa, who had been a Buddhist priest but had renounced Buddhism, was the first to teach Confucian moral philosophy as interpreted by Shu-shi, a philosopher of the Sung dynasty, and distinct from Buddhist teaching. Iyeyasu was a great admirer of Seikwa, and the teachings of the Shu-shi school of the Confucian philosophy formed the basis of the teaching of Chinese classics, that is to say, of the education of the upper classes, thus causing the emancipation of our moral teaching from all religious influences. Other schools of Confucian philosophy arose, but none of them was so influential as the Shu-shi school.

Iyeyasu and his immediate successors were, however, too busy with the work of consolidation to do very much for the advancement of learning and education; but Tsunayoshi (1680–1709), the fifth Shōgun, who was a great Chinese scholar, gave a great impulse to the study of Chinese literature. He himself delivered courses of expository lectures on Chinese classics, which were attended by daimyōs and his own immediate retainers. An Academy was opened for the first time in Yedo, where learned scholars of Chinese literature gave expository lectures of classics and made commentaries on them. This Academy continued till the beginning of the present era of Meiji to be a sort of University for the Chinese learning. Daimyōs established schools in their territories for their retainers, not only

for the study of Chinese learning but also for training in military arts, such as archery, fencing, use of spears, riding, Jujutsu, swimming, etc. The education of Samurais in those days consisted almost entirely of the study of Chinese literature and training in military arts. The former was chiefly cultivated not so much for literary purposes as for the humanities. Its system of moral philosophy was studied, that they might thereby be better fitted for the task of regulating their own individual conduct; of properly managing the affairs of their house, that is to say, their family relations; of taking share in the conduct of government of their lord's territories in a fit and proper manner; and, if need be, of helping their lord in the wider sphere of national administration and government. History also was read, not for historical facts but for the lessons it gave of how states rose, declined, and fell; how great men of the olden days dealt with different problems of government, and so on.

Pedagogically speaking, there was great waste of time. Boys not older than five or six were made to learn books of Confucius. At first they were taught merely to read without understanding the meaning, but as they grew older they were taught to understand it, partly by lectures, which often consisted as much in moral sermons hung on to these texts as in their exposition, partly by their own thinking. Time, however, was no object in those old days; and the mental culture and moral training they obtained in this way were very valuable indeed. Those who would not or could not acquire mental culture still obtained a certain amount of moral training; and for the rest, they went in for military exercises, which also were largely encouraged, and in which they got a great deal of discipline and moral training. The masters demanded and obtained implicit obedience from their pupils, and considered themselves not mere teachers in the practice of these arts, but mental and moral mentors. Whether in literature or in military arts, if anyone showed special proficiency, he was allowed or even sometimes ordered to proceed to study with noted masters at schools belonging to the Shōgun's government, or to some other daimyō, or at private schools. A daimyō who was fortunate enough to have a great master among his retainers was proud of the fact, and not averse to those of other daimyōs coming to receive instruction from him. Then, again, there were some masters who were unwilling or unable to serve a daimyō, and opened schools which would often be very largely attended for the sake of their teaching. It may also be stated that going in their lord's retinue to reside in Yedo was a very good education, giving them opportunities of obtaining views of a larger world and coming into contact with a variety of people.

Other studies besides Chinese were often taken up by different individuals, either according to their own taste or by the order of their lords. Thus, those who would be doctors had to study the Chinese system of medicine with other doctors, for there was no school at which it was taught. Mathematics, originally introduced from China, were cultivated by some few. Seki, a contemporary of Newton, was a great mathematician. He invented a system of algebra, which was entirely original and gave him immense power in his further investigations. The results obtained by Seki and his disciples by their original investigations in metrical geometry, algebra, theory of series, etc., were of a very high order. In natural sciences, also, some progress was made, and Siebold, when he came to Japan to study our flora and fauna, found some very eager and enthusiastic pupils. There were many other branches of learning which were pursued by various people. Fine arts, industrial arts, and even games, such as Go and Chess, were encouraged and pursued with good results. Foreign learning was not wholly unknown towards the latter part of the Shogunate. I cannot forbear from telling you of one doctor named Maeno Rankwa (1723–1803), who, after succeeding in getting a knowledge of the Dutch alphabet and some two or three hundred words with great difficulty, got hold of a Dutch book of anatomy, and, comparing the figures in it with the body of an executed criminal which he secretly dissected, and being greatly struck by their correctness as contrasted with the old Chinese idea, determined with a few friends of like spirit to translate the book. It was almost like deciphering hieroglyphics, but after four years of hard work he at last succeeded in the task. Such was the spirit which actuated some, at least, of the old Samurai in their search for knowledge.

This introduction of foreign knowledge may be considered a part of a remarkable intellectual revolutionary movement towards the end of the eighteenth century, somewhat similar to the Illuminism in Europe at the same period. It appeared in almost every branch of intellectual activity, in literature, in arts, in religion, and in politics. The rise of Neo-Shintoism and study of the history of Japan prepared the mind of more earnest thinkers to comprehend the real nature of the essential constitution of our empire. The usurpation of the powers of government by the Shōguns was beginning to be resented as an act of disloyalty and unrighteousness. Moreover, occasional collisions with the Russians in the north were enough to cause a great deal of anxiety in thoughtful minds. Thus the country was ripe for a great change when Commodore Perry first appeared on our coast, in 1853, and by introducing the complications of foreign relations hastened the downfall of the Tokugawa Shogunate. This took place in 1868, only

fifteen years after the coming of Commodore Perry. The cry of the opponents of the Shogunate was "Restoration of the Imperial Government," "Great Righteousness," "Loyalty to the Emperor," and with at least a large section of it, "The Expulsion of Barbarians." Yet scarcely was the last of the Shōguns overthrown than one of the first acts of the new government was to grant the foreign representatives an audience with the Emperor, an act without precedent in our history. This so incensed the Conservatives, who had really believed in the cry of the expulsion of the foreigners under the Imperial banner, that some of them attacked the British representative, Sir Harry Parkes, as he was proceeding to the palace; fortunately, they were prevented from doing harm by the bravery of two Samurai, who had been sent as guards. One of them, Goto, was afterwards a leader of the progressive party, who made earnest representations for a national assembly as early as 1874.

I must turn back and say a few words about the education of the common people under the Tokugawa Shogunate. There were no provisions made for their education either by the Shōgun's government or those of the daimyōs or by their own communities in cities, towns, and villages, where the people were allowed to enjoy a great deal of self-government in local matters, under their headsmen or mayors. But almost throughout the whole country there were private schools where elementary teaching was given in reading, writing, and arithmetic. Writing formed a most important part of the curriculum, and it was mostly through lessons in writing of Chinese ideographs that they learned to know them, and therefore reading was an adjunct to writing. The text-books from which they learned were, after they had mastered the Japanese alphabet, sentences in which Chinese ideographs were mixed with the Japanese letters; they were mostly moral maxims, so that here again we have moral teaching forming an essential part of the education. There were other books, by means of which knowledge of Chinese ideographs necessary in various arts and crafts, in geography, and in history was given. The fees for this tuition were very low, and facilities of education were thus within the means of even the poor people. The custom of going on pilgrimages to various shrines, scattered throughout the country, was very useful in imparting a practical knowledge of geography and even of history. There was, besides, a species of entertainment, very popular in some parts of the country, notably in Tōkyō, called story-telling, in which professional story-tellers gave accounts of historical events in a most graphic if not always very accurate manner. It would not be an exaggeration to say that these and the drama served to show people in general the standard of morals, especially of loyalty in men

and chastity in women, of self-sacrifice in performance of duty, that were demanded of the Samurai and even of the common people.

In April of 1868 the Emperor swore the following memorable oath, known as the Imperial Oath of Five Articles :—

1. Deliberative assemblies shall be established, and all measures of government shall be decided by public opinion.
2. All classes, high and low, shall unite in vigorously carrying out the plan of the government.
3. Officials, civil and military, and all the common people shall, as far as possible, be allowed to fulfil their just desires, so that there may not be any discontent among them.
4. Uncivilised customs of former times shall be broken through, and everything shall be based upon the just and equitable principles of nature.
5. Knowledge shall be sought for throughout the whole world, so that the welfare of the empire may be promoted.

Thus, as I said at the beginning of my address, we set out deliberately to introduce Western civilisation; and I have tried to show that we were not wholly unequipped mentally and morally for the task, and that, through it all, we have tried to keep the Japanese spirit, the fundamental character of our nationality, intact.

(*Issued separately October* 5, 1907.)

XXXIV.—The Variation of Young's Modulus under an Electric Current. By Henry Walker, M.A., B.Sc. *Communicated by* Professor J. G. MacGregor, F.R.S.

(MS. received May 20, 1907. Read June 24, 1907.)

Object of the Investigation.

This investigation was carried out for the purpose of extending to other substances an inquiry into the effect of an electric current on Young's Modulus, which was carried out by Miss Noyes on a steel wire at Cornell University, and described in the *Physical Review*, No. 4, vol. ii. In the investigation of Miss Noyes, the wire under examination was heated by a coil, the current being supplied by a storage battery. To vary the method of heating, the current was also sent through the wire with the expectation that the same effects would be obtained as when the wire was heated by the coil. This, however, was found not to be the case, and Professor Nichols, under whose superintendence the investigation was carried out, in his *Laboratory Manual of Physics*, vol. ii. p. 293, states that an extension of the inquiry to other materials than iron would be of interest.

In the present series of experiments the behaviour of steel, soft iron, copper, and platinum was examined when a current was passed through the wires.

General Plan of the Experiments.

The wire to be tested was mounted horizontally on a solid block of wood and carefully adjusted so as to be parallel to lines ruled on it. Care was also taken to see that the wire was horizontal. It was placed in a glass tube of about 4 cms. diameter for protection against air-currents. This tube was 130 cms. long, and two holes were drilled in it, whose centres were 98 cms. apart, through which the positions of two marks on the wire were observed by microscopes. To prevent currents of air, the ends of the tube were stopped with cotton-wool. The temperature of the wire was determined by its electrical resistance.

The microscopes employed had micrometers in the eye-pieces which were intended to be ·01 mm. between each division; but on carefully measuring them it was found that in the right-hand microscope each

division was ·00950 mm., and in the left-hand, ·00944 mm. It was possible to estimate a tenth of a division.

The temperature coefficient of the electrical resistance of the wires was determined by coiling them on a glass tube covered with silk, and immersing them in a vessel of oil. The resistance was determined for temperatures ranging from near zero to 150°. Great care was taken to obtain accuracy in measuring the various temperatures at which the resistance was determined. The temperature was taken by a platinum resistance thermometer. It had been calibrated by finding its resistance at 0° and 100°, and drawing the graph on the assumption that its resistance was zero at $-240°$, according to the results obtained by Professor Callendar. This graph was then tested by finding the resistance in the vapour of boiling ethyl-alcohol and of boiling amyl-alcohol, and the differences were within the limits of experimental error.

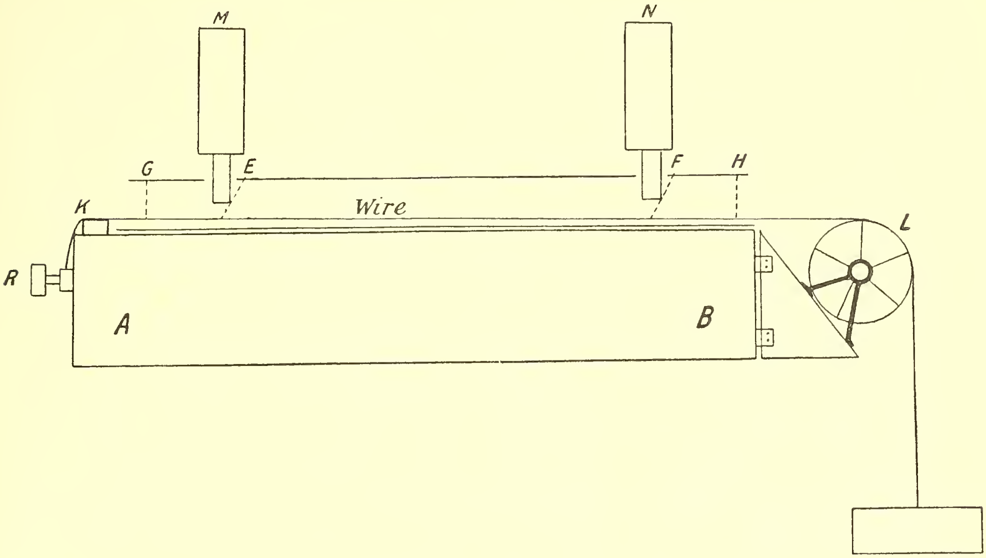
The method adopted for finding the temperature coefficient of resistance was the following:—The platinum thermometer was immersed in the oil close to the wire whose resistance was being determined. Two ordinary mercury thermometers were also placed in the vessel, one near the top, the other near the bottom, their use being to ensure that the temperature did not vary while the resistances of the wire and of the platinum thermometer were being determined. The oil was kept well stirred; when it reached a temperature at which a reading was desired the flame was carefully regulated; and when the mercury thermometers had remained steady for not less than five minutes, the resistances of the thermometer and the wire were taken in the usual way by a metre-bridge. The resistance of the copper side-pieces of the bridge had been determined, the wire calibrated, and, to provide against any alteration in these through their temperature being raised by heat from the oil bath, the connecting wires were several metres long. The resistance of the connecting wires was allowed for by measuring that of similar wires lying parallel to them.

The diameter of each wire was determined by weighing a measured length in air and in water, and calculating the diameter from the density. The diameter at different points was also measured by the micrometer gauge, and the average of the two methods was that used in the calculations.

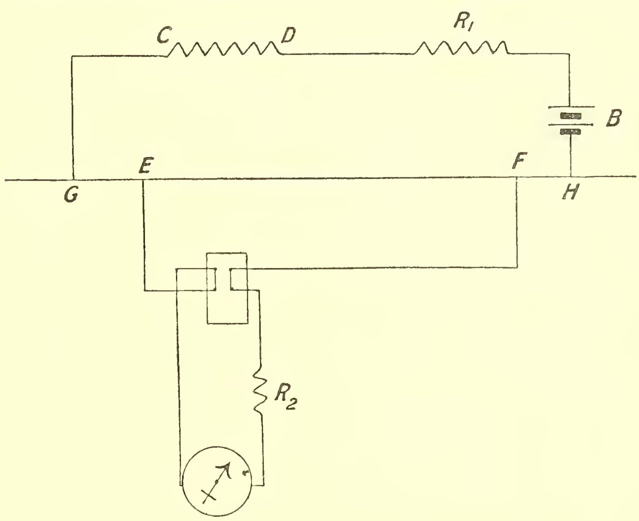
Arrangement of the Apparatus.

AB is the wooden block, GH the glass tube resting on wooden blocks, and having openings at E and F, through which the readings were taken by the microscopes M and N. The wire was fastened at R, passed over a

grooved piece of wood at K, through the tube and over the wheel L to a pan in which the weights were placed. The wheel was mounted on anti-



friction rollers. A pasteboard tube protected the portion of the wire between the glass tube and the wheel from currents of air. The wires



for the current and for measuring the resistance were attached near G and H.

The plan of connections for measuring the resistance is given in fig. 2, the fall of potential method being used. The portion EF of the wire

to be tested was in circuit with the battery B, the rheostat R_1 being adjusted so as to give the desired temperature. EF was connected in multiple arc with the galvanometer G. The resistance R_2 in the galvanometer circuit was one of 5000 ohms, its purpose being to reduce the deflection, and also to make the resistance so large that the current in EF would not be appreciably diminished by closing the galvanometer circuit. CD was a resistance in the circuit EF which was varied with the different wires; it never differed much from the resistance of the wire which was under examination. By placing it in multiple arc with the galvanometer instead of EF, and comparing the deflections, the resistance of EF could be determined.

With each wire preliminary experiments had to be carried out for the purpose of finding suitable weights to remain permanently in the pan to stretch the wire, and also what weights should be used to put on and take off so as to get a suitable elongation. If the stretching weight was too small the wire was not horizontal, and when the additional weight was put on the mark was drawn out of focus, and the reading could not be accurately made.

For most of the determinations four sets of readings were taken, each set consisting of the changes in length produced by adding the elongation weight and then removing it, thus making the result the average of eight determinations. When the wire was heated, care was taken to see that it returned very closely to the original length when the weights were removed; if there was much variation, it was concluded that the temperature had altered, and the set was discarded.

For each measurement, the microscope was set twice, and the average reading taken; if the readings differed from one another by more than one-tenth of a division, more observations were made. The weights were placed in position a minute or two before the readings were taken, the aim being to make the conditions of determining the modulus as nearly identical in all cases as possible, so that the results at different temperatures might be strictly comparable. In order to bring it to the state of ease, each wire was stretched for about twenty-four hours by a weight somewhat greater than the maximum weight to be used in determining the modulus, before any observations were made.

The procedure in each case was first to determine the modulus at the temperature of the room before any current had been passed, then to pass a weak current by which the temperature was raised through two or three degrees and again to determine the modulus. After this, the current was gradually increased and readings taken at various temperatures. When it

was found that the modulus was approaching a turning value, the readings were taken at shorter intervals, so that the behaviour could be accurately represented in its neighbourhood.

When the highest temperature to which it was thought necessary to carry the experiment had been reached, the current was gradually diminished and readings taken with a decreasing current. In all the graphs the readings with the increasing current are denoted by circles, and those with the decreasing current by crosses.

The formula used in computing the modulus was—

$$M = \frac{P.L}{a.l},$$

where

$P =$ stress in dynes.
$L =$ length of wire.
$a =$ area of cross-section.
$l =$ elongation.

The results are given in the C.G.S. system of units, the value of g at Stranraer being 981·4.

In order to see if the friction of the pulley introduced an error, pieces of each wire from the same coil were suspended vertically. To make the conditions of this set of experiments as nearly similar as possible as when the wires were horizontal, the tube was clamped in a vertical position with the wire inside it. The ends of the tube were stopped with cotton-wool, and the ends of the wire outside the tube were protected from air-currents by pasteboard cylinders. The same weights were used as when the wire was horizontal, and the elongations were measured by the microscopes. It was fitted up in a corner of the room and protected from air-currents. The temperature was also taken by the platinum thermometer, three mercury thermometers being placed at different parts of the tube to make sure that the temperature throughout it had become uniform.

Iron Wire.

The modulus was first determined without any current, the wire being at the temperature of the room. It was then heated by a weak current by which the temperature was raised about 3°·5, and it was found that the modulus had fallen in value. The strength of the current was next gradually increased, the effect of this increase being to cause a steady rise in the modulus until the temperature had risen to about 53°. Beyond this there was a regular fall in the value, the rate of decrease being less than

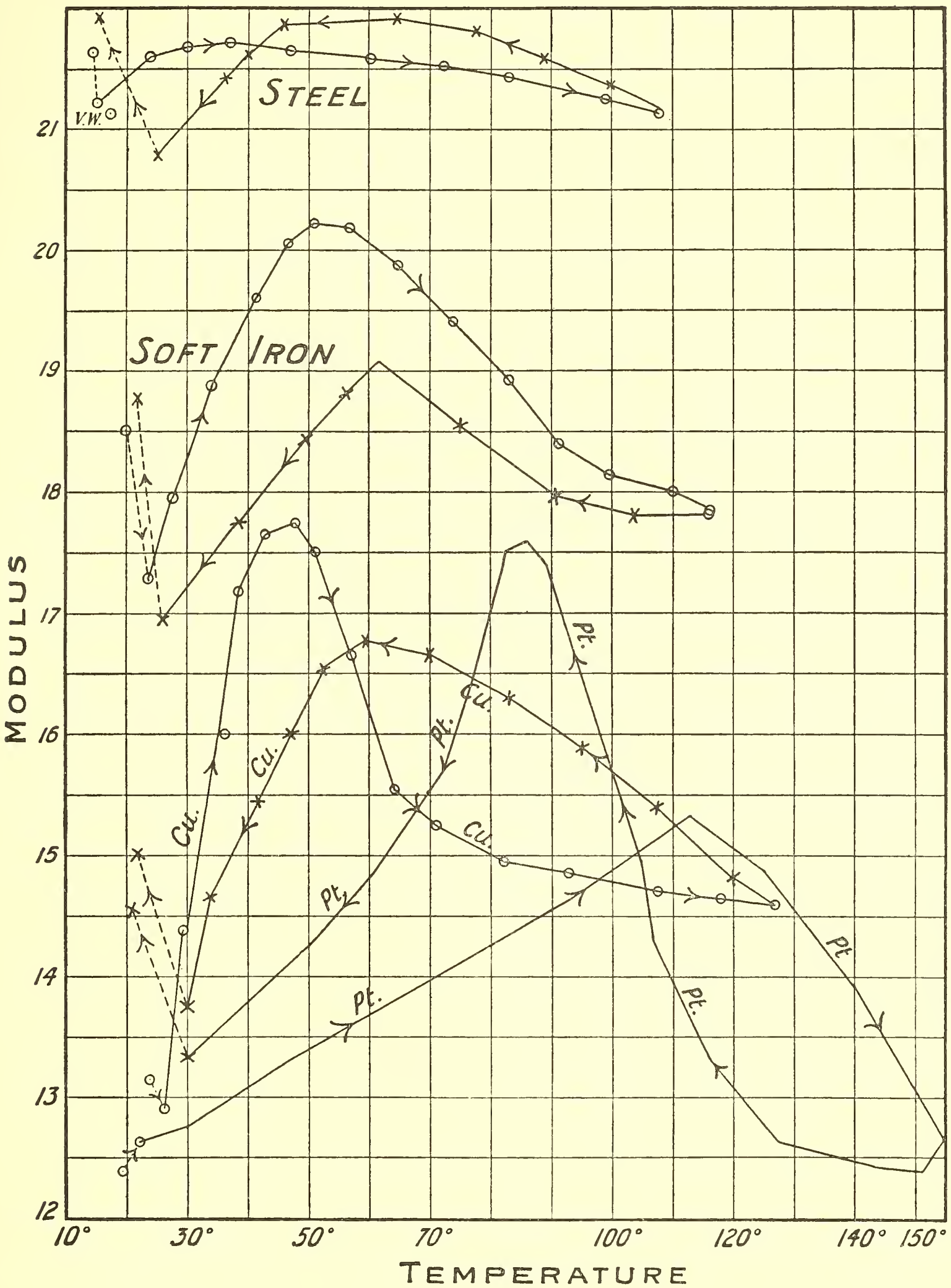
the rate of increase when the modulus was rising in value. This diminution continued up to the highest temperature to which the experiment was carried, but the rate was not uniform, for at about 105° the rate of decrease fell. Moreover, the modulus did not fall to so low a value as it had when the current was first started. This part of the curve is denoted by the circles.

Iron Wire.

Length = 97·9 cms.
Area of cross-section = ·0005474 sq. cms.
Stretching weight = 500 grams.

No.	Temp.	Elongation for 500 grams.	No. of Observations.	M.
1	20°·4 C.	·04741 cm.	8	$18{\cdot}51 \times 10^{11}$
2	23 ·9	·05073	8	17·30
3	27 ·5	·04880	8	17·98
4	34 ·2	·04658	8	18·84
5	40 ·8	·04482	10	19·58
6	47 ·1	·04387	8	20·04
7	50 ·6	·04347	8	20·19
8	57 ·0	·04357	6	20·14
9	64 ·9	·04421	7	19·85
10	73 ·9	·04526	8	19·39
11	82 ·8	·04633	8	18·94
12	90 ·7	·04764	8	18·42
13	100 ·2	·04833	8	18·16
14	108 ·0	·04880	10	17·98
15	116 ·1	·04916	6	17·85
16	104 ·3	·04935	8	17·78
17	90 ·5	·04880	8	17·98
18	75 ·0	·04736	8	18·53
19	67 ·2	·04623	9	18·98
20	61 ·8	·04588	8	19·13
21	60 ·6	·04626	5	18·97
22	55 ·6	·04663	7	18·82
23	49 ·5	·04771	8	18·38
24	38 ·7	·04958	8	17·70
25	25 ·3	·05174	8	16·96
26	21 ·3	·04686	8	18·73
Vertical Wire.				
27	16 ·1	·04852	8	18·09

The current was then gradually diminished in strength, and readings taken at various temperatures. It was found that the value was not so high as when the current was increasing, and about 104° it reached a minimum. From this point on to 62° the rate of increase was nearly the same as the rate of decrease with the increasing current. The reading at 60°·6 is probably incorrect, as it does not lie on any smooth curve that


STEEL
V.W.
SOFT IRON
Cu.
Pt.
MODULUS
21
20
19
18
17
16
15
14
13
12
10°
30°
50°
70°
100°
120°
140°
150°
TEMPERATURE

would pass through the values on opposite sides of it. After this, there was again a steady fall, and at 25°·3, the last reading taken with a current, the value was the lowest obtained.

The last reading of all was at the temperature of the room, and the value was higher than that got before any current was passed through the wire. The current had thus produced a permanent increase in the modulus.

Steel Wire.

Length = 97·60 cms.
Area of cross-section = ·0004714 sq. cms.
Stretching weight = 500 grams.

No.	Temp.	Elongation for 500 grams.	No. of Observations.	M.
1	13°·7 C.	·04697 cm.	8	$21·63 \times 10^{11}$
2	15 ·0	·04791	8	21·21
3	19 ·0	·04744	4	21·42
4	24 ·2	·04702	6	21·58
5	30 ·0	·04686	6	21·68
6	34 ·0	·04682	8	21·70
7	37 ·0	·04678	8	21·72
8	41 ·0	·04682	10	21·70
9	46 ·4	·04697	7	21·63
10	59 ·2	·04706	8	21·59
11	72 ·6	·04717	8	21·54
12	83 ·2	·04735	8	21·46
13	98 ·6	·04779	8	21·26
14	108 ·0	·04814	9	21·15
15	100 ·3	·04763	6	21·33
16	88 ·8	·04702	8	21·58
17	85 ·8	·04697	10	21·63
18	76 ·9	·04662	12	21·79
19	64 ·2	·04637	9	21·91
20	57 ·0	·04631	8	21·94
21	45 ·0	·04659	8	21·82
22	40 ·3	·04691	8	21·66
23	34 ·5	·04750	8	21·39
24	25 ·0	·04891	8	20·77
25	15 ·6	·04622	8	21·98
Vertical Wire.				
26	17 ·3	·04815	6	21·10

Steel Wire.

As stated in the general description, the same series of experiments was followed with all the wires; therefore the first reading was taken at the temperature of the room before any current had been passed. A weak current was then used, and it produced a fall in the modulus. The current

was next increased, with the result that the value rose until a maximum was reached about 34°, after which there was a regular decrease until the temperature was 108°, this being the highest temperature reached. Throughout the latter part of this curve the rate of decrease in the modulus was less than that of increase when the value was rising.

The current was then diminished, with the result that the modulus rose until the temperature had fallen to 57°, the maximum at this point being higher than that got with the increasing current. Throughout this stage the rate of increase was higher than the rate of fall with the increasing current. Below this temperature the modulus diminished along with the temperature, the rate of fall being greater than that at the same temperature with the increasing current, so that the graph cuts at about 41°·5. The fall was then quite regular to 25°, this having been the last reading taken with a current. The value was finally determined at the temperature of the room without a current, and found to be higher than what it was before any current had been passed. There was therefore a permanent increase in the value of the modulus.

Copper Wire.

The modulus was determined at the temperature of the room, and when a weak current was passed it produced a fall in its value. As the current was increased, there was a rapid increase in the modulus, which continued until the temperature had risen to about 45°, when a maximum was reached. Above this temperature there was a diminution which was fairly rapid at first, but the rate was not so great as that of the increase before the maximum. The rate of fall began to alter about 60°, and after 80° it was fairly uniform up to 127°, beyond which point the readings were not continued.

The current was then diminished, with the result that the modulus increased in value till the temperature had fallen to about 60°, and throughout this stage the value was higher than at the same temperatures with the increasing current. Until the temperature had fallen to 80°, the rate of increase in the modulus was greater than that of decrease for the increasing current; but at lower temperatures the rate of increase became less than that of decrease, and the two curves cut at about 57°. Further, the maximum with the diminishing current was not so high as that obtained when the current was increasing. The modulus still kept on falling, the rate also being less than that with the increasing current, until the

Copper Wire.

Length = 97·50 cms.
Area of cross-section = ·0006026 sq. cms.
Stretching weight = 300 grams.

No.	Temp.	Elongation for 300 grams.	No. of Observations.	M.
1	23°·8 C.	·03630 cm.	8	$13·15 \times 10^{11}$
2	25 ·3	·03693	8	12·90
3	29 ·7	·03308	10	14·40
4	35 ·5	·02977	10	16·00
5	39 ·1	·02765	9	17·23
6	42 ·6	·02703	8	17·62
7	47 ·4	·02693	7	17·69
8	50 ·1	·02727	12	17·47
9	56 ·8	·02861	8	16·66
10	59 ·0	·02938	8	16·14
11	63 ·7	·03065	8	15·54
12	70 ·5	·03126	6	15·24
13	81 ·3	·03186	8	14·95
14	93 ·2	·03205	8	14·86
15	108 ·0	·03236	10	14·72
16	118 ·5	·03249	10	14·66
17	127 ·0	·03267	11	14·58
18	119 ·9	·03201	9	14·88
19	107 ·0	·03085	8	15·44
20	95 ·5	·02999	8	15·88
21	82 ·8	·02919	8	16·32
22	70 ·0	·02852	5	16·70
23	60 ·0	·02835	7	16·80
24	52 ·9	·02878	8	16·55
25	47 ·3	·02979	6	16·02
26	41 ·2	·03087	8	15·43
27	34 ·5	·03254	8	14·64
28	30 ·0	·03452	8	13·80
29	21 ·2	·03175	8	15·02
Vertical Wire.				
30	16 ·3	·03736	6	12·75

temperature reached 30°. On allowing the wire to cool to the temperature of the room, there was a distinct increase in the modulus, so that a permanent change had been produced.

Platinum Wire.

The behaviour of platinum was different from that of the other three metals. With them the effect of a weak current was to produce a fall in the modulus, whereas in the case of platinum there was a rise with a current which produced a change of less than 3°. As the current was gradually strengthened, there was a steady increase in the modulus; in

this case, the rate of increase was slower than with the others, and the maximum not reached until the temperature had risen to 110°. Beyond this the modulus fell till the temperature rose to 155°, the rate of fall being greater than that of rise.

Platinum Wire.

Length = 62·10 cms.
Area of cross-section = ·0007548 sq. cms.
Stretching weight = 200 grams.

No.	Temp.	Elongation for 200 grams.	No. of Observations.	M.
1	20°·0 C.	·01302 cm.	8	$12·40 \times 10^{11}$
2	22 ·9	·01280	8	12·62
3	29 ·7	·01266	10	12·75
4	46 ·9	·01215	9	13·30
5	61 ·5	·01189	10	13·58
6	66 ·5	·01157	8	13·95
7	81 ·6	·01136	12	14·21
8	93 ·1	·01101	8	14·67
9	101 ·8	·01066	8	15·15
10	107 ·6	·01049	8	15·40
11	113 ·3	·01055	10	15·31
12	124 ·9	·01087	6	14·85
13	139 ·7	·01159	8	13·93
14	155 ·0	·01276	8	12·65
15	150 ·3	·01298	9	12·44
16	143 ·2	·01295	10	12·47
17	128 ·0	·01273	11	12·68
18	116 ·1	·01213	10	13·31
19	110 ·4	·01149	12	14·06
20	107 ·0	·01131	8	14·28
21	104 ·5	·01078	8	14·97
22	100 ·6	·01037	8	15·57
23	96 ·5	·009539	6	16·45
24	89 ·4	·009308	7	17·35
25	85 ·3	·009144	8	17·60
26	82 ·5	·009228	8	17·50
27	77 ·0	·009800	8	16·48
28	71 ·8	·01030	7	15·68
29	60 ·7	·01087	8	14·85
30	50 ·5	·01132	10	14·27
31	40 ·1	·01167	8	13·83
32	29 ·4	·01193	12	13·54
33	20 ·6	·01111	8	14·53
Vertical Wire.				
34	20 ·2	·01314	7	12·29

The current was then diminished, and this caused a further decrease in the modulus, the minimum being about 148°. Below this, there was a rapid rise in the modulus, and the two curves cut one another at 103°. It

continued to increase, the maximum being reached near 86°; it again fell rapidly till about 77°, when the rate of decrease diminished, but the modulus was always higher than at the same temperature with the increasing current. On allowing the wire to cool to the temperature of the room, there was again, as with the other metals, a permanent increase in the value of the modulus.

A Comparison of the Results.

On reviewing the effects on the metals, we find there are likenesses and differences. With weak currents the initial effect on iron, steel, and copper is a decrease in the modulus; while, on the other hand, there is an increase in the platinum with even the weakest current that was used

In all cases, an increase in the strength of the current produced a rise in the modulus until a maximum value was attained, the maximum being at different temperatures for the different metals. A further increase in the current then produced a fall in the modulus, which, in each case, was continued up to the highest temperature to which the experiment was carried.

With a diminishing current the effects were more varied. In two cases—viz. the iron and the platinum—there was at first a decrease in the modulus, but a marked difference between them soon made itself manifest. The modulus of the iron was always less than that at the same temperature with the increasing current, while, on the contrary, the modulus of the platinum rose rapidly, and attained a higher value than it had at any temperature with an increasing current.

On the other hand, when a diminished current was passed through steel and copper, the modulus rose in value, and with both a maximum was reached which, in the steel, was higher than the maximum with the increasing current, whereas, in the copper, it was less than the maximum with the increasing current.

In the iron and the steel, the final values with the current were less than the initial values, although the temperatures of these final readings were higher than the initial temperatures with a current; so that the effect of a weak diminishing current is, while it is still passing, to produce a diminution in the modulus. The result, so far as the experiment was carried, was the same with the copper; the value was less than at the same temperature with the increasing current, but, at the lowest temperature at which a reading was taken with the diminishing current, the modulus was higher than the first reading with a current. The platinum, however,

differed from the others, inasmuch as the final readings with the diminishing current gave a higher value for the modulus than it had with the initial increasing current.

It is interesting to examine in more detail the results for soft iron and steel. Here we see the effects on the same chemical substance, and therefore, on the assumption that a change in the modulus is caused by a regrouping of the molecules, are able to follow the changes in the groups of molecules, to see when they break up, and how the regrouping has affected the modulus. When the rate of change is small, we may assume that the groups are more stable than when the rate is great.

There is a general similarity in the behaviour of soft iron and steel, but there are also marked differences. With an increasing current the results are, qualitatively speaking, the same. The effect of a weak current is to produce a fall in the modulus; next, as the current increases the modulus also increases, and rises to a higher value than it had before any current was passed; then a maximum is reached, after which a steady decline sets in. There is a great difference, however, in the quantitative results; for, while the increase from the lowest value to the maximum is 14·3 per cent. in soft iron, it is only 2·3 per cent. in steel.

With a diminishing current, there is a marked difference between them. The value of the modulus is smaller in the soft iron than at the same temperatures with the increasing current; whereas, in the steel, it has at first a higher value than it had at the same temperatures with an increasing current. They are, however, alike in this respect, that the modulus in both attains a maximum and then diminishes, the final value being less than with the increasing current at the same temperatures.

After cooling to the temperature of the room, the modulus in each case undergoes a distinct increase. In the soft iron this value is not so great as the greatest value reached with the current, but in the steel it is the highest of all the readings.

It is to be observed that the variation of the modulus in steel is not so great as in iron, and from this we may infer that the groups of molecules are more stable than they are in iron. This result might be expected, for the molecules of steel will be affected to a smaller extent by the circular magnetic field produced by the current than those of soft iron. This expectation, however, is not borne out at all points, for, when one would have looked for the coercive force of the steel to manifest itself, we find a distinct change in the modulus after the current has been stopped.

In all cases, after the wire had been allowed to cool to the temperature

of the room, and the modulus determined without any current, it was found there was an increase in its value compared with what it was before any current had been passed through the wire. The final effect, then, in each case was to produce a permanent increase in its value.

It was also intended that brass wire should be put through the same course of experiments, but the temperature coefficient of resistance was so small that it was found impossible to determine the temperature with a suitable degree of accuracy. Probably it would be advisable to determine the temperature of brass and other alloys from the expansion of the wire.

(*Issued separately October* 7, 1907.)

XXXV.—On the Periods of the Elliptic Functions of Weierstrass.

By R. T. A. Innes, Director of the Government Observatory, Johannesburg, F.R.A.S.

(MS. received June 19, 1907. Read July 15, 1907.)

I.

The elliptic functions introduced by Professor K. Weierstrass of Berlin are best known through the admirable pages of Professor H. A. Schwarz's *Formeln und Lehrsätze zum Gebrauche der elliptischen Functionen*, Göttingen, 1885. These pages have been translated literally into French by Dr Henri Padé, Paris, 1894. Halphen's well-known three-volumed treatise on elliptic functions is devoted almost entirely to the Weierstrassian forms. In the English language, the following references may be useful:—Whittaker, *Modern Analysis*; Greenhill, *Elliptic Functions*; Harkness and Morley, *Theory of Functions*; Forsyth, *Theory of Functions*.

II.

The Weierstrassian forms are intimately connected with a cubic equation which may be written

$$4(s-e_1)(s-e_2)(s-e_3)\,,$$

or

$$4s^3-g_2s-g_3\,,$$

yielding the well-known relations between the roots and the invariants

$$e_1+e_2+e_3=0\,,\qquad e_1e_2+e_1e_3+e_2e_3=-\tfrac{1}{4}g_2\,,\qquad e_1e_2e_3=\tfrac{1}{4}g_3\,,$$

with the discriminant

$$G=\tfrac{1}{16}(g_2^3-27g_3^2).$$

As the object of this paper is mainly a numerical one, we limit our attention to real values of the roots; e_1 is the largest positive root, e_3 the largest negative root, and e_2 is intermediate in value.

III.

The problem usually presented in physical applications is:—Given the roots or the invariants of the cubic, to find the periods of the elliptic functions.

If the roots are given, the formulæ required have been given with all necessary fulness by Schwarz and Halphen. If, instead of the roots, the invariants are given, it would seem, at first glance, that the cubic must be solved before the periods can be found, but it has been shown that this is

not so. In his paper, *Ueber der Perioden der elliptischen Integrale*, Dorpat, 1875, H. Bruns gave several formulæ which can be used; but the unfortunate choice of an absolute invariant leads to unnecessary complication. It will be seen later that by the choice of another absolute invariant the formulæ take simple and symmetrical forms.

The formulæ given by Schwarz and later mathematicians have been developed from the purely mathematical point of view, and suffer from a surfeit of π and imaginaries. Had the mathematician skilled in the theory of functions been responsible for our trigonometrical tables and formulæ, it is possible that, instead of, to give one example, the values of $\sin\theta$ being tabulated, it might have been $\left(\frac{\pi}{2}\right)^2\frac{1}{\sin^2\left(\frac{\mu\pi}{2}\right)}$, so that the simplest trigonometrical operations would have involved a great deal of unnecessary work and writing. These remarks apply in a lesser degree to Legendre's great tables of his elliptic integrals. After computing $\frac{2}{\pi}\int_0^{\frac{\pi}{2}} f\theta d\theta$, he carefully multiplied the results by $\frac{\pi}{2}$, so that in nearly every case of practical application the computer has to divide by $\frac{\pi}{2}$. For such reasons, the symbols ordinarily used have in this paper been replaced by others, as follows:—

$\frac{2}{\pi}\omega_1$	by	w_1
$\frac{2}{\pi}\eta_1$	,,	n_1
$\frac{2}{\pi}\frac{1}{i}\omega_3$	,,	w_3
$\frac{2}{\pi}i\eta_3$	,,	n_3
$\frac{\tau}{i}=\frac{\omega_3}{i\omega_1}=\frac{1}{\pi}\log_e\frac{1}{q}$	,,	v_1.

With these changes, the fundamental formulæ become:—

$$n_1w_3+n_3w_1=\frac{2}{\pi}$$

$$w_3=w_1v_1$$

$$\sqrt{w_1}=\frac{2}{\sqrt[4]{e_1-e_3}+\sqrt[4]{e_1-e_2}}(1+2q_1^4+2q_1^{16}+\ .\ .\ .\ )$$

$$n_1w_1=\frac{1}{3}\ \frac{1-3^3q_1^2+5^3q_1^6-\ .\ .\ .}{1-3\ q_1^2+5\ q_1^6-\ .\ .\ .}$$

$$v_1v_3=1\,.$$

The relation of n_1 and w_1 to q_1 is the same as that of n_3 and w_3 to v_3. If the roots e_1, e_2, and e_3 are given, q_1 has to be computed by known formulæ, and then w_1 and n_1 by the above relations.

IV.

When the roots are not given explicitly, it is either necessary to find them by solving the cubic or to use the invariants. To make use of the invariants, put

$$\cos \iota_1 = \sqrt{27} g_3 / g_2^{\frac{3}{2}}, \qquad 0° < \iota_1 < 180°$$

(*N.B.*—All radicals are to take the positive sign; thus $\cos \iota$ is of the same sign as g_3),

and to abbreviate put

$$\lambda_1 = \tfrac{3}{4} g_2 ;$$

then

$$w_1 = \frac{1}{12} \frac{1}{\lambda_1^{\frac{1}{4}}} \frac{1}{\cos^2 \frac{\iota_1}{2}} \left\{ 7F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2 \frac{\iota_1}{2} \right) + 5 \cos \iota_1 F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2 \frac{\iota_1}{2} \right) \right\}.$$

$$n_1 = \frac{1}{36} \lambda_1^{\frac{1}{4}} \frac{1}{\cos^2 \frac{\iota_1}{2}} \left\{ 7 \cos \iota_1 F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2 \frac{\iota_1}{2} \right) + 5F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2 \frac{\iota_1}{2} \right) \right\}.$$

$$w_3 = \frac{1}{12} \frac{1}{\lambda_1^{\frac{1}{4}}} \frac{1}{\sin^2 \frac{\iota_1}{2}} \left\{ 7F\left(-\frac{1}{6}, \frac{7}{6}, 2, \cos^2 \frac{\iota_1}{2} \right) - 5 \cos \iota_1 F\left(\frac{1}{6}, \frac{5}{6}, 2, \cos^2 \frac{\iota_1}{2} \right) \right\}.$$

$$n_3 = \frac{1}{36} \lambda_1^{\frac{1}{4}} \frac{1}{\sin^2 \frac{\iota_1}{2}} \left\{ -7 \cos \iota_1 F\left(-\frac{1}{6}, \frac{7}{6}, 2, \cos^2 \frac{\iota_1}{2} \right) + 5F\left(\frac{1}{6}, \frac{5}{6}, 2, \cos^2 \frac{\iota_1}{2} \right) \right\}.$$

As the hypergeometrical series concerned are always convergent, these four formulæ constitute a formal solution of the problem.

V.

Although the two series converge for all values of the argument, the convergence is slow when the variable $\sin^2 \frac{\iota_1}{2}$ or $\cos^2 \frac{\iota_1}{2}$ exceeds $\frac{1}{2}$. It is therefore convenient to compute w_1 and n_1 when ι_1 is under 90°, and w_3 and n_3 when ι_1 exceeds 90°. If we suppose ι_1 less than 90° and we require w_3 and n_3, we compute w_1 and n_1 and also v_1. The latter quantity is found as follows:—

Put

$$\frac{1}{\sqrt{3}}\tan\frac{\iota_1}{3} = \cos 2\gamma_1 ,$$

and

$$\sqrt{\tan\gamma_1} = \cos 2\gamma_1' ;$$

then

$$l_1 = \tan^2\gamma_1' ,$$

giving

$$q_1 = \tfrac{1}{2}l_1 + 2(\tfrac{1}{2}l_1)^5 + \text{etc.}$$

and

$$v_1 = \frac{1}{\pi}\log_e\frac{1}{q_1} .$$

Then

$$w_3 = w_1 v_1$$

and

$$n_3 = \frac{\dfrac{2}{\pi} - n_1 w_3}{w_1} .$$

If we put $\iota_3 = 180^\circ - \iota_1$, we can interchange the suffixes $_1$ and $_3$ so that all the processes are available in either case.

VI.

The two hypergeometrical series of § 4 are not sensitive; and their tabulation by Mr Frank Robbins is given in the Appendix. The periods are sensitive, but in the present method of calculation almost the whole of the variation is thrown on the simple factors $1/\cos^2\frac{\iota_1}{2}$ and $1/\sin^2\frac{\iota_1}{2}$.

If we remember that $\cos\frac{\iota_3}{2} = \sin\frac{\iota_1}{2}$, the following argument, here limited to $\sin\frac{\iota_1}{2}$, can be extended. When ι_1 is under 90°, and an accuracy of two units in the seventh place of the decimals of the logarithms of the hypergeometrical series will suffice, the use of series can be avoided by introducing the following approximate formulæ:—

$$\log F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota_1}{2}\right) = \log\left\{1 - \frac{\frac{7}{72}\sin^2\frac{\iota_1}{2}}{\left(\sqrt{1 - \frac{2}{3}\sin^2\frac{\iota_1}{2}}\right)^{1-\frac{7}{72}}}\right\} - [6\cdot1011]\left(\frac{\iota_1^\circ}{100}\right)^{\frac{100}{12}}$$

and

$$\log F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_1}{2}\right) = \log\left\{1 + \frac{\frac{5}{72}\sin^2\frac{\iota_1}{2}}{\left(\sqrt{1 - \frac{2}{3}\sin^2\frac{\iota_1}{2}}\right)^{1+\frac{5}{72}}}\right\} + [6\cdot0049]\left(\frac{\iota_1^\circ}{100}\right)^{\frac{100}{12}}$$

The march of these series, computed both accurately and by the approximate formulæ, is shown in Table I.

TABLE I.

ι_1	$\log F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota_1}{2}\right)$.	By Approx. Formula.	$\log F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_1}{2}\right)$.	By Approx. Formula.
0°	0·0000000	00	0·0000000	00
10	9·9996784	84	0·0002297	97
20	9·9987132	32	0·0009184	84
30	9·9971030	30	0·0020653	53
40	9·9948459	59	0·0036690	90
50	9·9919398	98	0·0057269	69
60	9·9883830	30	0·0082349	48
70	9·9841755	54	0·0111868	67
80	9·9793202	00	0·0145732	32
90	9·9738259	59	0·0183800	00
180	9·9130246		0·0591526	

The following relations hold:—

$$7F_1\left(-\frac{1}{6}, \frac{7}{6}, 2\right) = 5F_1\left(\frac{1}{6}, \frac{5}{6}, 2\right) = 9 \cdot \frac{2}{\pi}.$$

$$F\left(-\frac{1}{6}, \frac{7}{6}, 2, \frac{1}{2}\right)F\left(\frac{1}{6}, \frac{5}{6}, 2, \frac{1}{2}\right) = \frac{108}{70} \cdot \frac{2}{\pi}.$$

$$F\left(-\frac{1}{6}, \frac{7}{6}, 2, \cos^2\frac{\iota_1}{2}\right)F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_1}{2}\right) + F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota_1}{2}\right)F\left(\frac{1}{6}, \frac{5}{6}, 2, \cos^2\frac{\iota_1}{2}\right) = \frac{108}{35} \cdot \frac{2}{\pi}.$$

$$v_1 = \cot^2\frac{\iota_1}{2}\left\{\frac{7F\left(-\frac{1}{6}, \frac{7}{6}, 2, \cos^2\frac{\iota_1}{2}\right) + 5F\left(\frac{1}{6}, \frac{5}{6}, 2, \cos^2\frac{\iota_1}{2}\right) - 10\cos^2\frac{\iota_1}{2}F\left(\frac{1}{6}, \frac{5}{6}, 2, \cos^2\frac{\iota_1}{2}\right)}{7F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota_1}{2}\right) + 5F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_1}{2}\right) - 10\sin^2\frac{\iota_1}{2}F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_1}{2}\right)}\right\}.$$

Also

$$\frac{5}{\cos^2\frac{\iota_1}{2}}F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_1}{2}\right) =$$

$$\frac{2}{\pi}\,\frac{108}{\sin^2\frac{\iota_3}{2}}\,\frac{1}{\left[7F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota_3}{2}\right) + 5\cos\iota_3 F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_3}{2}\right)\right]} - v_1\frac{5}{\cos^2\frac{\iota_3}{2}}F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_3}{2}\right),$$

and

$$\frac{7}{\cos^2\frac{\iota_1}{2}}F\left(-\frac{1}{6},\frac{7}{6},2,\sin^2\frac{\iota_1}{2}\right)=$$

$$\frac{2}{\pi}\frac{108}{\sin^2\frac{\iota_3}{2}}\frac{1}{\left[7F\left(-\frac{1}{6},\frac{7}{6},2,\sin^2\frac{\iota_3}{2}\right)+5\cos\iota_3F\left(\frac{1}{6},\frac{5}{6},2,\sin^2\frac{\iota_3}{2}\right)\right]}$$

$$+v_1\frac{7}{\cos^2\frac{\iota_3}{2}}F\left(-\frac{1}{6},\frac{7}{6},2\,.\,\sin^2\frac{\iota_3}{2}\right).$$

VII.

As there is an infinite number of cubic equations with real roots, it is impossible to tabulate the periods of all possible elliptic functions with either the roots or the invariants as arguments. The formulæ of § 5 show that the periods depend on the absolute invariant ι_1 and the quantity λ_1, a simple function of the invariant g_2.

For the purposes of tabulation we may assume that we can find a factor which will change e_1 into unity or a factor which will similarly reduce λ_1 to unity. Let us assume that we have found a factor m^2 which will reduce e_1 to unity. Whatever m may be, the following relations hold:—

$$m^2e_1+m^2e_2+m^2e_3=0$$

$$m^2e_1m^2e_2+m^2e_1m^2e_3+m^2e_2m^2e_3=-\tfrac{1}{4}m^4g_2$$

etc.

$$\frac{1}{m^2}\wp\left(\frac{u}{m}\,\middle|\,\frac{w_1}{m},\frac{w_3}{m}\right)=\wp(u\,/\,w_1,w_3)$$

$$\frac{1}{m^2}\wp\left(\frac{u}{m}\,\middle|\,m^4g_2,\,m^6g_3\right)=\wp(ug\,/\,_2,g_3).$$

This assumption will enable us to form a table which will include within its limits every possible case of three real roots. Such a table follows. As there is a close relationship between the forms used by Legendre and Jacobi and those of Weierstrass, I have included in the table the values of Legendre's modular angle θ and k and k_1, and Jacobi's nome q_1.

The relation between Legendre's modular angle and the absolute invariant ι is:—

$$\sin^2\theta=\frac{2\tan\frac{\iota_1}{3}}{\sqrt{3}+\tan\frac{\iota_1}{3}}.$$

The period n_1 vanishes for $\log q_1=9{\cdot}2857$.

TABLE II.

m^2e_1	m^2e_2	m^2e_3	k	k_1	m^4g_2	m^6g_3	$16m^{12}G$	$\log g$	θ	ι_1	$\log q_1$	$\log q_3$	$\log \frac{w_1}{m}$	$\log mn_1$	$\log \frac{w_3}{m}$	$\log mn_3$	$\log v_1$
1	−0·5	−0·5	0·0000	1·0000	3·00	1·00	0·0000	0·0000	00° 00′	00° 00′	−∞	0·0000	9·9120	9·6109	∞	−∞	+∞
1	−0·4	−0·6	0·3536	0·9354	3·04	0·96	3·2113	0·0527	20 42	19 46	7·9214	9·1044	9·9123	9·6098	0·0952	9·2004	0·1828
1	−0·3	−0·7	0·4851	0·8745	3·16	0·84	12·5033	0·2191	29 01	39 01	8·2243	8·9517	9·9134	9·6065	0·0278	9·3999	0·1144
1	−0·2	−0·8	0·5774	0·8165	3·36	0·64	26·8739	0·5353	35 16	57 19	8·4035	8·8340	9·9153	9·6009	9·9835	9·4871	0·0682
1	−0·1	−0·9	0·6489	0·7608	3·64	0·36	44·7293	1·1393	40 27	74 22	8·5328	8·7312	9·9179	9·5926	9·9495	9·5418	0·0315
1	0·0	−1·0	0·7071	0·7071	4·00	0·00	64·0000	∞	45 00	90 00	8·6356	8·6356	9·9215	9·5814	9·9215	9·5814	0·0000
1	0·1	−1·1	0·7559	0·6546	4·44	−0·44	82·3012	1·2239	49 06	104 09	8·7223	8·5431	9·9260	9·5666	9·8975	9·6124	9·9715
1	0·2	−1·2	0·7977	0·6030	4·96	−0·96	97·1407	0·6905	52 55	116 51	8·7986	8·4505	9·9317	9·5473	9·8765	9·6380	9·9448
1	0·3	−1·3	0·8341	0·5516	5·56	−1·56	106·1724	0·4176	56 31	128 11	8·8681	8·3554	9·9388	9·5222	9·8576	9·6598	9·9189
1	0·4	−1·4	0·8660	0·5000	6·24	−2·24	107·4954	0·2537	60 00	138 18	8·9335	8·2546	9·9475	9·4888	9·8406	9·6789	9·8930
1	0·5	−1·5	0·8944	0·4472	7·00	−3·00	100·0000	0·1497	63 26	147 19	8·9968	8·1444	9·9585	9·4431	9·8249	9·6959	9·8664
1	0·6	−1·6	0·9199	0·3921	7·84	−3·84	83·7591	0·0829	66 54	155 22	9·0605	8·0187	9·9725	9·3766	9·8105	9·7113	9·8380
1	0·7	−1·7	0·9428	0·3333	8·76	−4·76	60·4662	0·0409	70 32	162 33	9·1273	7·8669	9·9911	9·2681	9·7970	9·7253	9·8059
1	0·8	−1·8	0·9636	0·2673	9·76	−5·76	33·9190	0·0161	74 30	168 59	9·2025	7·6657	0·0176	9·0392	9·7844	9·7382	9·7668
1	0·9	−1·9	0·9826	0·1857	10·84	−6·84	10·5495	0·0036	79 18	174 47	9·2999	7·3441	0·0624	8·1110n	9·7726	9·7502	9·7102
1	1·0	−2·0	1·0000	0·0000	12·00	−8·00	0·0000	0·0000	90 00	180 00	0·0000	−∞	∞	−∞n	9·7614	9·7614	−∞

APPENDIX.

The hypergeometric series $F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota}{2}\right)$ and $F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota}{2}\right)$ introduced in Section IV. of the foregoing paper have been computed for each degree of the quadrant, at the suggestion of the author, by Mr Frank Robbins. Their seven-place logarithmic values, with first and second differences, were published as an appendix to the author's paper on "The Computation of Secular Perturbations" in the *Monthly Notices* of the Royal Astronomical Society, vol. lxvii., May 1907, pp. 444–447.

On the recommendation of the referees, these tables, with the descriptive preliminary paragraphs, are here reproduced (with Mr Robbins's permission), so as to facilitate the application of the method described in the present paper:—

In the hypergeometric series $F(\alpha\,\beta\,\gamma\,x)$ in the first case

$$\alpha \text{ has the value } -\frac{1}{6} \qquad \beta=\frac{7}{6} \qquad \gamma=2 \qquad x=\sin^2\frac{\iota}{2}\,.$$

and in the second case

$$\alpha=\frac{1}{6} \qquad \beta=\frac{5}{6} \qquad \gamma=2 \qquad x=\sin^2\frac{\iota}{2}\,.$$

For convenience of designation the tables are headed *Minus* F and *Plus* F according to the sign of α.

Vega's (1794) ten-figure logarithms, corrected by collation with the copy in use at H.M. *Nautical Almanac* Office, were used, and the natural values of the individual terms were taken out to ten places of decimals. These were obtained in duplicate for each end of the quadrant, and the whole were examined by differencing to the sixth order. Lastly, the seven-figure logarithms of the functions were taken from the eight-figure table of the Service Géographique de l'Armée (Paris, 1891), reference being made to Vega where the eighth figure was approximately five.

The log *Minus* F has been increased by 10 as customary, to avoid the inconvenience of printing negative characteristics.

The whole has been examined by Mr J. Abner Sprigge, of H.M. *Nautical Almanac* Office, so as to make it possible to use the tables with confidence in their accuracy to the seventh place.

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
1°	0·0000023			9·9999968		
		+ 69			− 97	
2	0092		+46	9871		−63
		115			160	
3	0207		45	9711		65
		160			225	
4	0367		47	9486		65
		207			290	
5	0574		46	9196		64
		253			354	
6	0827		46	8842		64
		299			418	
7	1126		45	8424		64
		344			482	
8	1470		46	7942		65
		390			547	
9	1860		47	7395		64
		437			611	
10	2297		45	6784		64
		482			675	
11	2779		46	6109		65
		528			740	
12	3307		46	5369		64
		574			804	
13	3881		46	4565		65
		620			869	
14	4501		46	3696		64
		666			933	
15	5167		45	2763		64
		711			997	
16	5878		47	1766		65
		758			1062	
17	6636		45	9·9990704		64
		803			1126	
18	7439		46	9·9989578		65
		849			1191	
19	8288		47	88387		64
		896			1255	
20	0·0009184		45	87132		65
		941			1320	
21	0·0010125		46	85812		64
		987			1384	
22	11112		45	84428		65
		1032			1449	
23	12144		47	82979		64
		1079			1513	
24	13223		45	81466		65
		1124			1578	
25	14347		46	79888		64
		1170			1642	
26	15517		45	78246		65
		1215			1707	
27	16732		46	76539		64
		1261			1771	
28	0·0017993		+46	9·9974768		−66
		+1307			−1837	

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
29°	0·0019300		+46	9·9972931		−64
		+1353			−1901	
30	20653		46	71030		64
		1399			1965	
31	22052		45	69065		66
		1444			2031	
32	23496		46	67034		64
		1490			2095	
33	24986		45	64939		65
		1535			2160	
34	26521		46	62779		64
		1581			2224	
35	28102		46	60555		66
		1627			2290	
36	29729		45	58265		64
		1672			2354	
37	31401		46	55911		65
		1718			2419	
38	33119		45	53492		65
		1763			2484	
39	34882		45	51008		65
		1808			2549	
40	36690		46	48459		64
		1854			2613	
41	38544		46	45846		66
		1900			2679	
42	40444		45	43167		65
		1945			2744	
43	42389		45	40423		64
		1990			2808	
44	44379		45	37615		66
		2035			2874	
45	46414		46	34741		64
		2081			2938	
46	48495		44	31803		66
		2125			3004	
47	50620		46	28799		64
		2171			3068	
48	52791		46	25731		66
		2217			3134	
49	55008		44	22597		65
		2261			3199	
50	57269		45	19398		65
		2306			3264	
51	59575		45	16134		64
		2351			3328	
52	61926		45	12806		66
		2396			3394	
53	64322		45	09412		65
		2441			3459	
54	66763		45	05953		66
		2486			3525	
55	69249		45	9·9902428		64
		2531			3589	
56	0·0071780		+44	9·9898839		−66
		+2575			−3655	

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
57°	0·0074355		+45	9·9895184		−64
		+2620			−3719	
58	76975		45	91465		66
		2665			3785	
59	79640		44	87680		65
		2709			3850	
60	82349		44	83830		65
		2753			3915	
61	85102		45	79915		65
		2798			3980	
62	87900		44	75935		65
		2842			4045	
63	90742		44	71890		65
		2886			4110	
64	93628		44	67780		65
		2930			4175	
65	96558		44	63605		65
		2974			4240	
66	0·0099532		45	59365		65
		3019			4305	
67	0·0102551		43	55060		65
		3062			4370	
68	105613		43	50690		65
		3105			4435	
69	108718		45	46255		65
		3150			4500	
70	111868		42	41755		65
		3192			4565	
71	115060		44	37190		64
		3236			4629	
72	118296		43	32561		65
		3279			4694	
73	121575		44	27867		65
		3323			4759	
74	124898		43	23108		64
		3366			4823	
75	128264		42	18285		66
		3408			4889	
76	131672		43	13396		63
		3451			4952	
77	135123		43	08444		64
		3494			5016	
78	138617		42	9·9803428		65
		3536			5081	
79	142153		42	9·9798347		64
		3578			5145	
80	145731		43	93202		64
		3621			5209	
81	149352		41	87993		64
		3662			5273	
82	153014		42	82720		63
		3704			5336	
83	156718		42	77384		64
		3746			5400	
84	0·0160464		+41	9·9771984		−64
		+3787			−5464	

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
85°	0·0164251		+42	9·9766520		−62
		+3829			−5526	
86	168080		40	60994		64
		3869			5590	
87	171949		40	55404		62
		3909			5652	
88	175858		42	49752		64
		3951			5716	
89	179809		+40	44036		−62
		+3991			−5778	
90	0·0183800			9·9738258		

(*Issued separately October* 7, 1907.)

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7. Index Slips.—In order to facilitate the compilation of Subject Indices, and to secure that due attention to the important points in a paper shall be given in General Catalogues of Scientific Literature and in Abstracts by Periodicals, every author is requested to return to the Secretary, along with his final proof, a brief index (on the model given below), of the points in it which he considers new or important. These indices will be edited by the Secretary, and incorporated in Separate Index Slips, to be issued with each part of the Proceedings and Transactions.

MODEL INDEX.

Schäfer, E. A.—On the Existence within the Liver Cells of Channels which can be directly injected from the Blood-vessels. Proc. Roy. Soc. Edin., vol. . 1902, pp. .

Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Liver,—Injection within Cells of.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

SESSION 1906-7.

No. V.] VOL. XXVII. [Pp. 369-432.

CONTENTS.

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MDCCCCVII.

Price Three Shillings.

REGULATIONS REGARDING THE PUBLICATION OF PAPERS IN THE PROCEEDINGS AND TRANSACTIONS OF THE SOCIETY.

The Council beg to direct the attention of authors of communications to the Society to the following Regulations, which have been drawn up in order to accelerate the publication of the Proceedings and Transactions, and to utilise as widely and as fairly as possible the funds which the Society devotes to the publication of Scientific and Literary Researches.

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3. Proofs.—In general, a first proof and a revise of each paper will be sent to the author, whose address should be indicated on the MS. If further proofs are required, owing to corrections or alterations for which

[*Continued on page* iii *of Cover.*

INDEX SLIP.

XXXVI.—Preliminary Note on the action of Sodium Ethylate on Trichloromethyl-Sulphonic Chloride. By Professor A. Crum Brown and Thomas F. Cowie, B.Sc.

(Read July 8, 1907. MS. received July 19, 1907.)

Trichloromethyl-sulphonic chloride in ethereal solution acts readily on sodium ethylate, and the same reaction takes place when the solutions of the two substances in absolute alcohol are mixed.

The final products are ether, sodium chloride, sodium carbonate, and sodium sulphite, but small quantities of ethyl carbonate, ethyl orthocarbonate, and sodium ethyl sulphite were obtained.

Experiments are now in progress by which it is hoped that these and other intermediate products may be obtained in larger quantities.

The ethereal liquid left after the ether was distilled off from the solution obtained by the action of ethereal solution of Cl_3C-SO_2-Cl on dry sodium ethylate was fractionated, and thus separated into two portions, one passing over between 125° and 127°, when the thermometer rose rapidly to 158°, and another portion passed over between 158° and 161°, leaving scarcely any residue.

These fractions were analysed with the following results.

First fraction 125° to 127°:—

I. 0·0888 grm. gave 0·1642 CO_2 and 0·0703 H_2O
$C=50·4\%$; $H=8·9\%$

II. 0·1087 grm. gave 0·2012 CO_2 and 0·0853 H_2O
$C=50·5\%$; $H=8·8\%$

III. 0·0798 gave 0·1484 CO_2 and 0·0614 H_2O
$C=50·7\%$; $H=8·6\%$

The formula $(C_2H_5)_2CO_3$ requires $C=50·80\%$ and $H=8·54\%$. The boiling point of ethyl carbonate is 126°.

Second fraction 158° to 161°:—

I. 0·0813 gave 0·1659 CO_2 and 0·0766 H_2O
$C=55·7\%$; $H=10·5\%$

II. 0·0954 gave 0·1950 CO_2 and 0·0880 H_2O
$C=55·7\%$; $H=10·3\%$

The formula $(C_2H_5)_4CO_4$ requires $C=56·33\%$ and $H=10·49\%$. The boiling point of ethyl orthocarbonate is 158°.

The mixture of sodium salts was extracted with absolute alcohol, and gave a small quantity of a salt which was purified by recrystallisation from absolute alcohol. This salt agreed in general character and reactions with the ethyl potassium sulphite described by Warlitz (*Liebig's Annalen*, 143, p. 78), and gave the following results on analysis:—

I. 0·1991 grm. gave 0·1313 CO_2 and 0·0673 H_2O
$C = 18{\cdot}0\%$; $H = 3{\cdot}8\%$

II. 0·1567 grm. gave 0·1027 CO_2 and 0·0521 H_2O
$C = 17{\cdot}9\%$; $H = 3{\cdot}7\%$

III. 0·1200 grm. gave 0·0651 Na_2SO_4
$Na = 17{\cdot}6\%$

IV. 0·2143 grm. gave 0·1176 Na_2SO_4
$Na = 17{\cdot}8\%$

The formula $NaSO_2{\cdot}OC_2H_5$ requires $C = 18{\cdot}17\%$, $H = 3{\cdot}80\%$, $Na = 17{\cdot}43\%$. There was not a sufficient quantity for a sulphur determination, but there can be little doubt that the substance was ethyl sodium sulphite. It gave alcohol and sodium sulphite when treated with caustic soda; charred when heated, giving off vapour containing sulphur, and gave SO_2 when *warmed* with hydrochloric acid. Heated in a tube with solid KCN, it gave a distinct odour of isonitrile.

The most probable explanation of the reaction of sodium ethylate on trichloromethyl-sulphonic chloride is that the ester $(C_2H_5O)_3C{\cdot}SO_2{\cdot}OC_2H_5$ is formed, and acted on by sodium ethylate thus,—

$$(C_2H_5O)_3C{\cdot}SO_2{\cdot}OC_2H_5 + C_2H_5ONa =$$
$$(C_2H_5O)_4C + NaSO_2{\cdot}OC_2H_5 \quad \text{and}$$
$$(C_2H_5O)_3C{\cdot}SO_2{\cdot}OC_2H_5 + C_2H_5ONa =$$
$$(C_2H_5O)_2CO + (C_2H_5)_2O + NaSO_2{\cdot}OC_2H_5,$$

—and that the carbonic esters are acted on by sodium ethylate giving ether and sodium carbonate,* while the ethyl sodium sulphite with sodium ethylate gives ether and sodium sulphite.

* Bassett, *Liebig's Annalen*, 132, p. 57.

(*Issued separately November* 8, 1907.)

XXXVII.—Note on Quaternion Integral Theorems. By Heinrich Hermann, D.Sc. (Tübingen). *Communicated by* Professor C. G. Knott.

(Read July 15, 1907. MS. received September 21, 1907.)

The transformations of volume integrals into surface integrals, and of surface integrals into line integrals, are usually established first for a small parallelepiped and parallelogram.* These elements are not, however, very convenient, or indeed altogether satisfactory, for the extension of the theorems over finite domains. If arranged in straight rows and plates, the elements cannot be fitted to a general boundary; and if they are arranged in curved rows and shells like the stones of a vault, there will be, in general, between various elements of the first order, spaces of the second not represented. This inconvenience disappears if tetrahedra and triangles are taken as elements. In Quaternions, the theorems are easily proved for such elements.

Let $\alpha\,\beta\,\gamma$ be three small vectors drawn from the point O and forming a tetrahedron, the vectors being arranged in the positive order of circuition. Let q be the value (generally quaternionic) at O of a function of space.

There exists the identity—

$$S\alpha\beta\gamma\,.\,\nabla q = \sum_{\alpha,\,\beta,\,\gamma} V\beta\gamma S\alpha\nabla\,.\,q$$

which may be written—

$$-\frac{1}{6}S\alpha\beta\gamma\,.\,\nabla q = \left\{\sum_{\alpha\beta\gamma}\frac{V\beta\gamma}{2}\right\}\left\{q - S\frac{\alpha+\beta+\gamma}{3}\nabla\,.\,q\right\}$$
$$+\sum_{\alpha\beta\gamma}\left\{\frac{V\gamma\beta}{2}\left(q - S\frac{\beta+\gamma}{3}\nabla\,.\,q\right)\right\}$$

Now $-\frac{1}{6}S\alpha\beta\gamma$ is the volume of the tetrahedron; the first term on the right side is the vector area of the triangle α, β, γ multiplied by the value of q at the centre of gravity of the area; and the second term contains the corresponding quantities on the other faces of the tetrahedron. Hence we may write the relation in the form—

$$dv\,\nabla q = \Sigma(d\nu q)$$

where dv is the volume and $d\nu$ the vector area measured outwards.

* See M'Aulay, *Utility of Quaternions in Physics*, p. 19; and Joly, *Manual of Quaternions*, pp. 71, 215.

By the usual extension this gives—

$$\iiint dv \nabla q = \iint d\, q$$

Similarly, the identity—

$$V(V\alpha\beta)\nabla \,.\, q = (\alpha S\beta\nabla - \beta S\alpha\nabla)q$$

may be written—

$$V\frac{V\alpha\beta}{2}\nabla \,.\, q = \alpha\left(q - S\frac{\alpha}{2}\nabla \,.\, q\right) + \left(\beta - \alpha\right)\left(q - S\frac{\alpha+\beta}{2}\nabla \,.\, q\right) - \beta\left(q - S\frac{\beta}{2}\nabla \,.\, q\right)$$

Each term on the right represents one of the vector sides multiplied by the value of q at the centre of the corresponding side. The relation may therefore be written—

$$Vdv\nabla \,.\, q = \sum d\rho q$$

for a small triangle.

By the usual extension this gives—

$$\iint Vdv\nabla \,.\, q = \int d\rho q$$

[Tait's original proof of the second theorem (*Quat.*, 3rd edition, § 498) is *fundamentally* the same as the one here given, but it is probably more difficult to follow at a first reading.—C. G. K.]

(*Issued separately November* 8, 1907.)

Meetings of the Society—Session 1906–1907.

THE 124TH SESSION.

Monday, 22nd October 1907.

GENERAL STATUTORY MEETING. Election of Office-Bearers. p. 1.

FIRST ORDINARY MEETING.

Monday, 5th November 1906.

Professor Crum Brown, LL.D., F.R.S., Vice-President, in the Chair.

Dr THOMAS DEWAR and Mr FRANK A. NEWINGTON were admitted Fellows of the Society.

The following Communications were read:—

1. Obituary Notice of the Rev. George Matheson, D.D., LL.D. By the Rev. JAMES LINDSAY, D.D. *P.*, xxvi. pp. 550–551.
2. On the Partition of Energy in certain Systems. By WM. PEDDIE, D.Sc. pp. 181–194.
3. The Hessians of certain Invariants of Binary Quantics. By THOMAS MUIR, LL.D. *P.*, xxvi. pp. 529–532.
4. The Sum of the r-line Minors of the Square of a Determinant. By THOMAS MUIR, LL.D. *P.*, xxvi. pp. 533–539.
5. Some New Symmetric Function Tables. By Professor W. H. METZLER.

SECOND ORDINARY MEETING.

Monday, 19th November 1906.

Dr R. H. Traquair, F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. On a New Siphonogorgid Genus, *Cactogorgia*, with Descriptions of three new Species. By JAMES J. SIMPSON, M.A., B.Sc., Carnegie Research Scholar, University of Aberdeen. Communicated by Professor J. ARTHUR THOMSON, M.A. *Trans.*, vol. xlv. pp. 829–836.
2. Craniometric Observations on the Skull of *Equus Prejvalskii* and other Horses. By Professor O. CHARNOCK BRADLEY, D.Sc. pp. 46–50.
3. On Skulls of Horses from the Roman Fort at Newstead, near Melrose, with Observations on the Origin of Domestic Horses. By Professor J. C. EWART, M.D. F.R.S. *Trans.*, vol. xlv. pp. 555–587.

4. The Inversion of Cane Sugar by Optically-Active Acids. By THEODORE RETTIE, B.Sc., and W. W. TAYLOR, M.A., D.Sc. Preliminary Note. Communicated by Professor CRUM BROWN.

A Paper by Professor J. Y. SIMPSON was postponed to the next meeting.

The following Candidates for Fellowship were balloted for, and duly elected Fellows of the Society:—Mr CHARLES E. S. PHILLIPS, Mr EDWARD OSWALD FERGUS, and Mr EDWIN BRAMWELL, M.B., F.R.C.P.E., M.R.C.P. (Lond.).

THIRD ORDINARY MEETING.

Monday, 3rd December 1906.

Professor Crum Brown, F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. On the Sporulation of *Amœba proteus*. By Professor J. Y. SIMPSON, D.Sc.

2. Results of Removal and Transplanation of Ovaries. By F. H. A. MARSHALL, D.Sc., and W. A. JOLLY, M.B. (*With Lantern Illustrations.*) *Trans.*, vol. xlv. pp. 589–599.

3. The Influence of an Excessive Meat Diet on the Osseous System. By CHALMERS WATSON, M.D. Communicated by Professor SCHÄFER, F.R.S. pp. 2–5.

4. The Effect of a Meat Diet on Fertility and Lactation. By B. P. WATSON, M.D., F.R.C.S.E. Communicated by Professor SCHÄFER, F.R.S. pp. 6–10.

5. The Effects of Diet on Fertility and Structure of the Uterus. By MALCOLM CAMPBELL, M.D., and CHALMERS WATSON, M.D. Communicated by Professor SCHÄFER, F.R.S. pp. 11–13.

6. The Minors of a Product-Determinant. By THOMAS MUIR, LL.D. pp. 79–87.

FOURTH ORDINARY MEETING.

Monday, 17th December 1906.

Professor A. Gray, LL.D., F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. The Hæmorenal Salt Index as a Test of the Functional Efficiency of the Kidney. By DAWSON TURNER, M.D., F.R.C.P.E. (*With Lantern Illustrations.*)

2. Magnetisation and Resistance of Nickel at High Temperatures. Part II. By C. G. KNOTT, D.Sc. *Trans.*, vol. xlv. pp. 547–554.

3. The Relation between Normal Take-up or Contraction and Yarn Number for the same Degree of Twist in Twisted Threads. By THOMAS OLIVER, B.Sc. Communicated by Dr C. G. KNOTT. pp. 93–106.

4. The Relation between Normal Take-up or Contraction and Degree of Twist in Twisted Threads when the Singles are of Unequal Sizes. By THOMAS OLIVER, B.Sc. Communicated by Dr C. G. KNOTT. pp. 107–116.

5. The Superposition of Mechanical Vibrations (Electric Oscillations) upon Magnetisation, and Conversely, in Iron, Steel, and Nickel. Part II. By James Russell, Esq.

The following Candidates for Fellowship were balloted for, and declared duly elected Fellows of the Society:—Messrs Donald Alexander MacAlister, A.R.S.M., F.G.S., and William Ramsay Smith, D.Sc., M.B., C.M.

FIFTH ORDINARY MEETING.

Monday, 7th January 1907.

Professor Crum Brown, LL.D., F.R.S., Vice-President, in the Chair.

Dr David Ellis signed the Roll, and was admitted a Fellow of the Society.

The following Communications were read:—

1. Notes on Aboriginals of the Northern Territory of South Australia. By W. Ramsay Smith, D.Sc., M.B., C.M., Permanent Head of the Health Department, South Australia. Communicated by Professor D. J. Cunningham, F.R.S. pp. 51–63.
2. Exhibition of Skeletons of Monkeys showing Effects produced by Improper Feeding. Professor D. J. Cunningham, F.R.S.
3. On the Partition of Heat Energy in the Molecules of Gases. By Dr P. Ehrenfest. Communicated by Dr Wm. Peddie. pp. 195–202.
4. On Vibrating Systems which are not subject to the Maxwell-Boltzmann Law. Second Paper. By Dr Wm. Peddie. pp. 181–194.
5. Note on Cases of Contour Zones of Molecular Arrangement resulting from Surface Disturbance. By Dr James Hunter.

SIXTH ORDINARY MEETING.

Monday, 21st January 1907.

Professor Crum Brown, LL.D., F.R.S., Vice-President, in the Chair.

Mr Chas. E. S. Phillips and Dr Duncan Scott Macnair signed the Roll, and were admitted Fellows of the Society.

The following Communications were read:—

1. On Homer Lane's Problem of a Spherical Gaseous Nebula. By the Rt. Hon. Lord Kelvin, President.
2. On the Means of Testing experimentally the Motion of the Earth relatively to Ether. By the Same.

3. The Theory of Axisymmetric Determinants in the Historical Order of Development up to 1860. By Thomas Muir, LL.D. pp. 135–166.

The following Candidates for Fellowship were balloted for, and duly elected Fellows of the Society:—Mr Peter Macnair, Mr James M. P. Muirhead, Professor James Musgrove, M.D., Mr E. Wynston-Waters, and Mr Ernest Maclagan Wedderburn, M.A., LL.B.

SEVENTH ORDINARY MEETING.

Monday, 4th February 1907.

Dr R. H. Traquair, LL.D., F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. On the Fossil Osmundaceæ. By Robert Kidston, F.R.S., F.G.S., and D. T. Gwynne-Vaughan, M.A., Lecturer on Botany at Queen Margaret's College, University of Glasgow. *Trans.*, vol. xlv. pp. 759–780.

2. The Development of the Anterior Mesoderm, and Paired Fins, with their Nerves, in Lepidosiren and Protopterus. By W. E. Agar, B.A., Junior Demonstrator in Zoology, University of Glasgow. Communicated by Professor J. Graham Kerr. *Trans.*, vol. xlv. pp. 611–639.

3. Scottish Tardigrada, collected by the Lake Survey. By James Murray, Esq. Communicated by Sir John Murray, K.C.B. *Trans.*, vol. xlv. pp. 641–668.

4. Arctic Tardigrada, collected by W. S. Bruce. By James Murray, Esq. Communicated by W. S. Bruce, Esq. *Trans.*, vol. xlv. pp. 669–681.

5. *Prymnothonus Hookeri*, Poisson pélagique de l'"Erebus" et de la "Terror" retrouvé par l'Expédition Antarctique Nationale Écossaise. Note préliminaire, par M. Louis Dollo, Conservateur au Musée royal d'Histoire naturelle, à Bruxelles. Présentée par M. le Dr. R. H. Traquair, F.R.S. pp. 35–45.

EIGHTH ORDINARY MEETING.

Monday 18th February 1907.

Dr Robert Munro, Vice-President, in the Chair.

The following Communications were read:—

1. The Coat Colour in Horses. By Professor J. C. Ewart, F.R.S.

2. The Geology of Ardrossan. By J. D. Falconer, M.A., D.Sc., F.G.S. Communicated by Professor James Geikie, F.R.S. *Trans.*, vol. xlv. pp. 601–609.

The following Candidates were balloted for, and declared duly elected Fellows of the Society:—Dr William Cramer, Dr John Downie Falconer, Mr Gilbert Henry Gulliver, Mr John Strong, B.A., and Mr Andrew Watt, M.A.

NINTH ORDINARY MEETING.

Monday, 4th March 1907.

Professor Crum Brown, LL.D., F.R.S., Vice-President, in the Chair.

Mr Andrew Watt signed the Roll, and was duly admitted a Fellow of the Society.

The following Communications were read:—

1. Algebra after Hamilton, or Multenions. By Alexander M'Aulay, M.A., Professor of Mathematics in the University of Tasmania. Communicated by Dr C. G. Knott.

2. Note on the Change produced in the Conductivity and Density of Lead Wires by Permanent Stretching. By James A. Donaldson and Robert Wilson, Natural Philosophy Department, Edinburgh University. Communicated by Professor J. G. MacGregor. pp. 16–20.

3. On the Dynamical Theory of Seismometers. By C. G. Knott, D.Sc.

4. Temperature Observations in the North Sea. By Professor D'Arcy W. Thompson, C.B. (*With Lantern Illustrations.*)

5. The Boiling and Freezing Points of Concentrated Aqueous Solutions, and the Question of the Hydration of the Solute. By Rev. S. M. Johnston, B.A., D.Sc., F.R.S.E., Carnegie Research Fellow, etc. *Trans.*, vol. xlv. pp. 855–884.

TENTH ORDINARY MEETING.

Monday, 18th March 1907.

Dr R. H. Traquair, F.R.S., Vice-President, in the Chair.

Dr Wm. Cramer signed the Roll, and was duly admitted a Fellow of the Society.

The following Communications were read:—

1. On the Influence of Temperature on the Photo-Electric Discharge from Platinum. By W. Mansergh Varley, D.Sc. (Manchester and Leeds), Ph.D. (Strassburg), B.A. (Cantab.), Assistant Professor of Physics and Electrical Engineering, and Fred. Unwin, M.Sc. (Manchester), Assistant Lecturer in Physics, Heriot-Watt College, Edinburgh. Communicated by Professor F. G. Baily, M.A., M.I.E.E. pp. 117–134.

2. On the Discovery of a new Genus of Thread-Bacteria (*Spirophyllum ferrugineum*, Ellis). By David Ellis, D.Sc., Ph.D., Lecturer in Botany and Bacteriology, Glasgow and West of Scotland Technical College. pp. 21–34.

3. The Functions of the Rolandic Cortex in Monkeys. By Wm. A. Jolly, M.B., and Sutherland Simpson, M.D., D.Sc. (From the Physiological Laboratory, University of Edinburgh.) Communicated by Professor E. A. Schäfer, F.R.S. pp. 64–78.

The following Candidates were balloted for, and declared duly elected Fellows of the Society:—Mr John Anderson Gilruth, M.R.C.V.S., and Mr Wm. Robertson, M.R.C.V.S.

ELEVENTH ORDINARY MEETING.

Monday, 6th May 1907.

Dr Robert Munro, Vice-President, in the Chair.

Lieut. Geo. Johnstone signed the Roll, and was duly admitted a Fellow of the Society.

The following Communications were read:—

1. Inbreeding in the Barbary Sheep (*Ovis tragelaphus*) and in the Common Goat (*Capra hircus*). By J. C. Ewart, M.D., F.R.S., Regius Professor of Natural History, University of Edinburgh. (*With Lantern Illustrations.*)
2. Report on Fossil Fishes collected by the Geological Survey of Scotland in Lower Carboniferous Rocks near Gullane, East Lothian. By R. H. Traquair, M.D., LL.D., F.R.S. (*With Lantern Illustrations.*)
3. Heusler's Magnetic Alloy. By Alexander D. Ross, M.A., B.Sc., Houldsworth Research Scholar, University of Glasgow. Communicated by Professor A. Gray, F.R.S. pp. 88–92.
4. The Physical Properties of Mixed Solutions of Independent Optically-Active Substances. By Clerk Ranken, B.Sc., Carnegie Research Scholar, and W. W. Taylor, M.A., D.Sc. Communicated by Professor Crum Brown, F.R.S. pp. 172–180.

TWELFTH ORDINARY MEETING.

Monday, 20th May 1907.

Dr R. H. Traquair, F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. A New Method of determining the Degree of Twist in Single Threads. By Thomas Oliver, B.Sc., Carnegie Research Scholar. Communicated by Dr C. G. Knott. pp. 264–268.
2. The Influence of Twist on the Strength of a Thread. By Thomas Oliver, B.Sc., Carnegie Research Scholar. Communicated by Dr C. G. Knott. pp. 308–311.
3. Notes on some Oligochaets found on the Scottish Loch Survey. By C. H. Martin, B.A. Communicated by Sir John Murray, K.C.B.
4. Notes on some Turbellaria from Scottish Lochs. By C. H. Martin, B.A. Communicated by Sir John Murray, K.C.B.
5. The Composition of the Red Clay. By F. W. Clarke, D.Sc., LL.D., Chief

Chemist, United States Geological Survey. Communicated by Sir John Murray, K.C.B. pp. 269–270.

6. The Glaciation of East Lothian, south of the Garlton Hills. By P. F. Kendall, M.Sc., Professor of Geology in the University of Leeds, and E. R. Baily, B.A. Communicated by Dr John Horne, F.R.S. *Trans.*, vol. xlvi.

Mr Frederick Alexander Black, Professor Alexander Brown, M.A., B.Sc., Mr John Kemp, M.A., and Mr John Hannay Thompson, M.Inst.C.E., M.Inst.Mech.E., were balloted for, and declared duly elected Fellows of the Society.

THIRTEENTH ORDINARY MEETING.

Monday, 3rd June 1907.

Professor Crum Brown, LL.D., F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. Temperature Changes occurring in Fresh-water Lochs. (*With Experimental Illustrations.*) By E. M. Wedderburn, LL.B.

2. A Specimen of *Helix pomatia*, with Paired Male Organs. By J. H. Ashworth, D.Sc. Communicated by Professor J. C. Ewart. (*With Lantern Illustrations.*) pp. 312–331.

3. Encystment of Tardigrada. By James Murray, Esq. Communicated by Sir John Murray, K.C.B. *Trans.*, vol. xlv. pp. 837–854.

FIRST SPECIAL MEETING.

Monday, 10th June 1907.

Dr Robert Munro, Vice-President, in the Chair.

The following Communications were read:—

1. A Contribution to the Craniology of the Natives of Borneo, the Malays, the Natives of Formosa, and the Tibetans. By Principal Sir Wm. Turner, K.C.B. *Trans.*, vol. xlv. pp. 781–818.

2. On the Histology of the Ephedreæ, with special reference to the value of Histology for Systematic Purposes. (*With Lantern Illustrations.*) By R. J. D. Graham, M.A., B.Sc., Carnegie Research Scholar, Botanical Department, University of St Andrews. Communicated by R. A. Robertson, M.A., B.Sc., F.L.S.

3. The Variation of Young's Modulus under an Electric Current. By Henry Walker, B.Sc. Communicated by Professor J. C. MacGregor, F.R.S. pp. 343–356.

FOURTEENTH ORDINARY MEETING,
held in the Queen's Hall, 5 Queen Street.

Monday, 17th June 1907.

Professor Andrew Gray, LL.D., F.R.S., Vice-President, in the Chair.

At the request of the Council, Baron Kikuchi, M.A., D.Sc., LL.D., Emeritus Professor of Mathematics in the Imperial University of Japan (ex-Minister of Education, and formerly President of the Imperial University), gave an Address:—

"A Sketch of Japanese National Development, more especially with reference to Education." pp. 332–342.

SECOND SPECIAL MEETING.

Monday, 24th June 1907.

Dr R. H. Traquair, F.R.S., Vice-President, in the Chair.

Mr Fred. A. Black signed the Roll, and was duly admitted a Fellow of the Society.

The following Communications were read:—

1. The Evolution of the Eyebrow Region of the Forehead, with special reference to the significance of its Excessive Development in the Neanderthal Race. By Professor D. J. Cunningham, F.R.S.
2. On the Origin of the Amniotic and Allantoic Fluids. By Professor D. Noël Paton, M.D., and B. P. Watson, M.D. *Trans.* vol. xlvi.
3. On the Application of a Differential Densimeter to the Study of some Mediterranean Waters. By John J. Manley, M.A., Daubeny Curator, Magdalen College, Oxford. Communicated by Sir John Murray, K.C.B. pp. 210–232.
4. The Electric Conductivity and Refracting Power of Ninety Samples of Sea-water, and a Comparison of these with the Salinity and Density. By E. G. Hill, B.A., B.Sc., Professor of Chemistry, Muir College, Allahabad, India. Communicated by Sir John Murray, K.C.B. pp. 233–243.

Messrs James Archibald, M.A., William Forster Lanchester, M.A., and Muhammad Badre were balloted for, and declared duly elected Fellows of the Society.

FIFTEENTH ORDINARY MEETING.

Monday, 1st July 1907.

Professor Crum Brown, LL.D., F.R.S., Vice-President, in the Chair.

At the request of the Council, Professor C. Michie Smith, B.Sc., F.R.A.S., Director of the Kodaikánal and Madras Observatories, gave an Address:—

Some Account of the Work at the Solar Observatory, Kodaikánal, S. India. (*With Lantern Illustrations of Solar Prominences, etc.*)

The Chairman intimated to the Society that the following gentlemen had been proposed by the Council for ballot on 4th November 1907:—

As British Honorary Fellows.

Sir Alexander B. W. Kennedy, LL.D., F.R.S., Pres. Inst.C.E.

Edwin Ray Lankester, LL.D., F.R.S., Director of the Natural History Departments, British Museum.

James A. H. Murray, LL.D., D.C.L., Editor of the Oxford English Dictionary.

Charles S. Sherrington, M.A., M.D., LL.D., F.R.S., Holt Professor of Physiology in the University of Liverpool.

As Foreign Honorary Fellows.

Emil Fischer, Professor of Chemistry, University of Berlin.

George William Hill, Ph.D., Sc.D., LL.D., West Nyack, New York.

Friedrich Wilhelm Georg Kohlrausch, Pres. of the Physikalisch-Technische Reichsanstalt, Charlottenburg.

Henry Fairfield Osborn, Professor of Zoology, Columbia University, and Curator of the Department of Vertebrate Palæontology, American Museum of Natural History.

Ivan P. Pavlov, Professor of Physiology, Military Medical Academy, St Petersburg.

Gustaf Retzius, formerly Professor of Anatomy, Stockholm.

Augusto Righi, Professor of Physics in the University of Bologna.

Louis Joseph Troost, Member of the Institute of France, formerly Professor of Chemistry at the Sorbonne, Paris.

THIRD SPECIAL MEETING.

Monday, 8th July 1907.

Dr Robert Munro, Vice-President, in the Chair.

The following Communications were read:—

1. The Plant Remains in the Scottish Peat Mosses. Part III.—The Scottish Highlands and the Shetlands. By Francis J. Lewis, F.L.S., Lecturer in Botany, University of Liverpool. Communicated by Professor Geikie, F.R.S. (*With Lantern Illustrations.*) *Trans.*, vol. xlvi.
2. Note on the Abyssal Temperature of Fresh-water Lakes. By E. M. Wedderburn, M.A., LL.B. (*With Lantern Illustrations.*)
3. On the Action of Sodium Ethylate on Trichlormethyl-sulphonic Chloride. By Professor A. Crum Brown, F.R.S., and T. F. Cowie, B.Sc. pp. 369–370.
4. On a Hybrid between Prejvalsky's Horse (*Equus prejvalskii*) and a Highland Pony. By Professor J. C. Ewart, F.R.S.
5. A Note on Reflected Mirage. By C. G. Knott, D.Sc.
6. The System Sulphur-Iodine. By Professor Alex. Smith and C. M. Carson.
7. Precipitated Sulphur. By Professor Alex. Smith and R. H. Brownlee. pp. 308–311.
8. Preliminary Note on the Optical Rotations (throughout the Spectrum), the Electrical Conductivities, and the Densities, of Mixtures of Sodium-Potassium-Tartarate and Ammonium-Molybdate in Aqueous Solution. By James R. Milne, D.Sc. pp. 271–280.
9. The Composition of Terrigenous Deposits. By F. W. Clarke, D.Sc., LL.D., Chief Chemist U.S. Geological Survey. Communicated by Sir John Murray, K.C.B., F.R.S., etc. pp. 269–270.

SIXTEENTH AND LAST ORDINARY MEETING.

Monday, 15th July 1907.

Dr Robert Munro, Vice-President, in the Chair.

PRIZES.

The Keith Prize for the biennial period 1903–1905 was presented to Thomas H. Bryce, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxa*," published in the *Transactions* of the Society within the period.

The Chairman, in presenting the Prize, read the following statement:—

Dr Bryce's articles deal with the structure and development of the blood corpuscles and the development of the blood-forming organs in

Lepidosiren paradoxa. The papers are a model of careful description, and are profusely illustrated by accurate drawings and microphotographs. The work is of a laborious description, and bears upon the face of it the stamp of accuracy. The part which deals with the origin of the leucocytes is of great interest, both practical and theoretical, and it may be truly said that Dr Bryce's observations have thrown a considerable amount of light upon a very intricate subject. Nor is the account which he has given of the development and structure of the erythrocyte of less importance. The description of the mitotic changes of the dividing erythrocyte has an intimate relationship to similar changes which occur in other somatic cells, and the author has been able to clear up more than one obscure point in connection with this subject, and in connection with cell-structure in general. He has also succeeded in demonstrating in an objective manner, by sections through the erythrocytes, that the fibrillar structure which was originally described by Meves, encircling the Amphibian erythrocyte, is not merely the optical expression of folds in the cell-membrane, but that the appearance is due to the presence of actual fibrils within the border of the corpuscle; and as a result of Dr Bryce's work this interpretation of the appearances is now universally accepted.

The Makdougall-Brisbane Prize for the biennial period 1904–1906, awarded by the Council to JACOB E. HALM, Ph.D., for his two papers on "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliometer," and for other astronomical and mathematical papers published in the *Transactions* and *Proceedings* of the Society within the period, was handed by the Chairman to Professor Dyson, for transmission to Dr Halm, now in Cape Town.

The Chairman, in presenting the Prize, read the following statement explaining the grounds of the award:—

Dr Halm's researches on the rotation of the sun are an important contribution to the study of its mechanical condition. From observations of Sun Spots in the years 1851–1863 Carrington showed that the sun did not rotate as a rigid body, but that the angular velocity in different latitudes diminished from the Equator to latitudes 35 N. and S., the limits of the Sun Spot zone. As, owing to the rotation of the sun, the eastern limb advances and the western limb recedes from the earth, the lines of the spectrum at points on the sun's limb are displaced slightly from the normal positions they occupy in the spectrum of the sun's centre. The

invention of gratings, which have very large dispersion, makes it practicable to measure the rotation spectroscopically, and in the years 1887–1889 Dunér made a beautiful series of observations which determined the law of rotation with great accuracy to within 15° of the sun's poles. The difficulty of explaining the phenomena on dynamical grounds made it desirable to obtain all the additional information observation could supply. With this in mind Dr Halm continued Dunér's observations in the years 1901–1907, to determine whether the law of rotation varied with the frequency of Sun Spots, which have a well-determined but unexplained period of eleven years. He followed Dunér in basing his determination on the comparison of two lines of solar origin with two neighbouring lines caused by absorption in the earth's atmosphere. He differed from him in the employment of a fixed spectroscope fed by a heliostat in place of an equatorial mounting, and still further simplified the conditions of the observation by the use of a heliometer which enabled him to compare simultaneously opposite extremities of any diameter of the sun. By these means considerable increase of accuracy was obtained. The accuracy of the observations was illustrated in an interesting manner by deducing the speed of the earth's rotation from a comparison of morning and evening observations, and the eccentricity of the earth's orbit from the observations of spring and autumn.

As regards the rotation of the sun, Dr Halm found the same equatorial velocity as Dunér, but a greater diminution towards the poles. Another important result brought to light by these researches is a small displacement towards the red, irrespective of rotation, of the lines of the spectrum near the sun's limb, as compared with their positions in the spectrum of the centre of the disc. This displacement, which appears to vary with the Sun Spot cycle, is accounted for by Dr Halm as the result of pressure in the stratum of the sun which the light traverses.

Among Dr Halm's other astronomical papers reference may be made to one on "Line and Band Spectra," in which he shows that these may be included in a single formula; and to one on "Temporary Stars," where he develops in detail a hypothesis of Professor Seeliger's, that a new star becomes visible when a dark body impinges upon and penetrates into a mass of nebular material.

The memoir "On a Group of Linear Differential Equations of the Second Order, including Professor Chrystal's Seiche-equations," is a valuable contribution to the solution of a type of equations which play an important rôle in mathematical physics. It enriches the theory by investigating the solutions for other than integral values of one of the parameters of the equation, and throws a new light on the theoretical importance of the

seiche-functions. It is of fundamental importance for the application of the seiche-functions, on account of the methods it gives for calculating their numerical values; while the tables it contains will be of the utmost service both theoretically and practically.

Dr Halm is now Chief Assistant at the Cape Observatory, and is in consequence unable to be present to receive the prize in person. The Council have accordingly asked Professor Dyson to receive the prize for Dr Halm.

The following Communications were read:—

1. The *Pycnogonida* of the Scottish National Antarctic Expedition. By T. V. Hodgson, F.L.S., Biologist to the National Antarctic Expedition. Communicated by Dr R. H. Traquair, F.R.S., etc.

2. The Marine Mollusca of the Scottish National Antarctic Expedition. By James Cosmo Melvill, M.A., F.L.S., and Rob. Standen, Assistant Keeper, Manchester Museum. Communicated by Professor J. C. Ewart, F.R.S.

3. Preliminary Note on the Internal Structure of *Sigillaria mamillaris*, Brongt., and *Sigillaria scutellata*, Brongt. By Robert Kidston, F.R.S. pp. 203–206.

4. Description of a New Species of Lepidodendron (*L. Pettycurense*), from Pettycur. By Robert Kidston, F.R.S. pp. 207–209.

5. On the Periods of the Elliptic Functions of Weierstrass. By R. T. A. Innes, Director, Government Observatory, Transvaal. pp. 357–368.

6. Hydrachnidæ collected by the Lake Survey. By Wm. Williamson, Esq. Communicated by Sir John Murray, K.C.B. pp. 302–307.

7. Degenerations following Experimental Lesions in the Motor Cortex of the Monkey. By W. A. Jolly, M.B., and Sutherland Simpson, M.D., D.Sc. (From the Physiological Laboratory, University of Edinburgh.) Communicated by Professor E. A. Schäfer, F.R.S. (*With Lantern Illustrations.*) pp. 281–301.

8. Classification of Igneous Rocks according to their Chemical Composition. By H. Warth, D.Sc., late Depy. Superintendent, Geological Survey of India. Communicated by Professor Geikie, F.R.S.

9. Note on Quaternion Integrals. By Dr Heinrich Hermann. Communicated by Dr C. G. Knott ,pp. 371–372.

Messrs Alexander Galbraith, Archibald King, M.A., B.Sc., James Knight, M.A., D.Sc., F.C.S., F.G.S., F.E.I.S., James Murray, and John Thomson Pearce, B.A., B.Sc., were balloted for, and declared duly elected Fellows of the Society.

Donations to the Library of the Royal Society from 1905 to 1907.

I. Transactions and Proceedings of Learned Societies, Academies, etc., received by Exchange of Publications, and List of Public Institutions entitled to receive Copies of the Transactions and Proceedings of the Royal Society of Edinburgh.

T.P. prefixed to a name indicates that the Institution is entitled to receive *Transactions* and *Proceedings*. P. indicates *Proceedings*.

AFRICA (BRITISH CENTRAL).

Zomba.—*Scientific Department.* Meteorological Observations, 1905–6. Fol. (*Presented by H.M. Acting Commissioner and Consul-General.*)

AMERICA (CENTRAL).

Mexico—
T.P. *Sociedad cientifica "Antonio Alzate."* Memorias.
T.P. *Observatorio Meteorologico-Magnetico Central.* Boletin Mensual.
P. *Istituto Geologico.* Boletin.
P. *Academia Mexicana de Ciencias Exactas, Fisicas y Naturales.*
P. Tacubaya.—*Observatorio Astronomico.* Annuario.
P. Xalapa.—*Observatorio Meteorologico Central del Estado Vera Cruz.*

AMERICA (NORTH). (*See* UNITED STATES and CANADA.)

AMERICA (SOUTH).

T.P. Buenos Ayres (Argentine Republic).—*Museo Nacional.* Anales.
Cordoba—
T.P. *Academia Nacional de Ciencias de la Republica Argentina.* Boletin.
T.P. *National Observatory.* Anales.
T.P. La Plata (Argentine Republic).—*Museo de La Plata.*
P. Montevideo (Uruguay).—*Museo Nacional.* Annales (Flora Uruguay).
T.P. Parà (Brazil).—*Museu Paraense de Historia Natural e Ethnographia.*
P. Quito (Ecuador).—*Observatorio Astronomico y Meteorologico.*
Rio de Janeiro (Brazil)—
T.P. *Observatorio.* Annuario.—Boletin Mensal.
P. *Museu Nacional.* Revista (Archivos).
Santiago (Chili)—
T.P. *Société Scientifique du Chili.* Actes.
P. *Deutscher Wissenschaftlicher Verein.*
P. San Salvador.—*Observatorio Astronómico y Meteorólogico.*

AUSTRALIA.

ADELAIDE—

P. *University Library.*

P. *Royal Society of South Australia.* Transactions and Proceedings.

P. *Royal Geographical Society.* Proceedings.

Observatory. Meteorological Observations, 1902–4. 4to. (*Presented.*)

BRISBANE—

P. *Royal Society.* Transactions.

P. *Queensland Branch of the Royal Geographical Society of Australasia.* Queensland Geographical Journal.

P. *Government Meteorological Office.*

P. *Water Supply Department.*

P. GEELONG (VICTORIA).—*Gordon Technical Colleges.*

MELBOURNE—

T.P. *University Library.*

P. *Royal Society of Victoria.* Proceedings.

PERTH, W.A.—

P. *Geological Survey.* Annual Progress Reports.—Bulletins.

Government Statistician's Office. Monthly Statistical Abstract, 1906–7. (*Presented.*)

The Physical Characteristics of the Hardwoods of Western Australia. By G. A. Julius, 1906. 4to. (*From the Government of W.A.*)

Notes on Timbers of Western Australia, etc., 1906. 4to. (*From the Government of W.A.*)

SYDNEY—

T.P. *University Library.*

T.P. *Department of Mines and Agriculture* (*Geological Survey*), *N.S.W.* Records.—Annual Reports.—Palæontology, No. 13: A Monograph of the Silurian and Devonian Corals of N.S.W., etc. Pt. 1. By R. Etheridge. 4to. 1904.—No. 14: Monograph of the Foramenifera of the Permo-Carboniferous Limestones of N.S.W. By Fred. Chapman and W. Howchin. 4to. 1905.

T.P. *Linnean Society of New South Wales.* Proceedings.

T.P. *Royal Society of New South Wales.*

P. *Australian Museum.* Records.—Reports.—Memoirs.—Catalogues. (Special. No. 1.) Nests and Eggs of Birds found breeding in Australia and Tasmania: North. Vol. II. Pt. 1, 1906. 8vo.

AUSTRIA.

CRACOW—

T.P. *Académie des Sciences.* Rozprawy Wydzialu matematyczno-przyrodniezego (Proceedings, Math. and Nat. Sciences Cl.).—Rozprawy Wydzialu filologicznego (Proc., Philological Section).— Rozprawy Wydzialu historyczno-filozoficznego (Proc., Hist.-Phil. Section).—Sprawozdanie Komisyi do badania historyi sztuki w Polsce (Proc., Commission on History of Art

in Poland).—Sprawozdanie Komisyi fizyjograficznej (Proc., Commission on Physiography).—Biblijoteka piserzów polskich (Library of Polish Authors of the 16th century).—Geological Atlas of Galicia; Text, Maps.—Bulletin International.

GRATZ—

T.P. *Naturwissenschaftlicher Verein für Steiermark.* Mittheilungen.

P. *Chemisches Institut der K. K. Universität.*

P. LEMBERG.—*Société Scientifique de Chevtchenko.*

PRAGUE—

T.P. *K. K. Sternwarte.* Magnetische und Meteorologische Beobachtungen. Astronomische Beobachtungen.

T.P. *K. Böhmische Gesellschaft.* Sitzungsberichte: Math. Naturw. Classe; Phil.-Hist.-Philol. Classe.—Jahresbericht.—And other publications.

T.P. *Ceská Akademie Cisaře Františka Josefa pro Vědy Slovesnost a Umění.* Almanach. — Vestnik (Proceedings). — Rozpravy (Transactions): Phil.-Hist. Class; Math.-Phys. Cl.; Philol. Cl.—Historický Archiv.—Bulletin International, Résumé des Travaux présentés.—And other publications of the Academy.

P. SARAJEVO (BOSNIA).—*The Governor-General of Bosnia-Herzegovina.* Ergebnisse der Meteorologischen Beobachtungen.

TRIESTE—

P. *Società Adriatica di Scienze Naturali.*

P. *Museo Civico di Storia Naturale.*

P. *Osservatorio Astronomico-Meteorologico.* Rapporto Annuale.

VIENNA—

T.P. *Kais. Akademie der Wissenschaften.* Denkschriften: Math.-Naturwissenschaftliche Classe; Philosophisch-Historische Classe — Sitzungsberichte der Math.-Naturwissenschaftlichen Classe; Abtheil. I., II.A, II.B, III.; Philosoph.-Historische Classe.—Almanach.—Mittheilungen der Erdbeben Commission.

T.P. *K. K. Geologische Reichsanstalt.* Abhandlungen.—Jahrbücher.—Verhandlungen.

T.P. *Oesterreichische Gesellschaft für Meteorologie.* Meteorologische Zeitschrift.

T.P. *K. K. Zoologisch-Botanische Gesellschaft.* Verhandlungen. — Abhandlungen.

P. *K. K. Naturhistorisches Hofmuseum.* Annalen.

K. K. Central-Anstalt für Meteorologie und Erdmagnetismus. Jahrbücher, Neue Folge. Bd. XLII. 1907. 4to.—Allgemeiner Bericht und Chronik der im Jahre 1904 in Österreich beobachteten Erdbeben. 1906. 8vo.—Bericht über die internationale meteorologische Directoren Konferenz in Innsbrück, 1905. 8vo. 1906. (*Presented.*)

K. K. Militar Geographisches Institut. Astronomisch-Geodätischen Arbeiten. Bd. XXI. Astronomische Arbeiten. 1906. 4to.—Längenbestimmungen. Bd. XIV. 1907. 4to.—Die Ergebnisse der Triangulierungen. Bd. IV. 1906. 4to. (*Presented.*)

Zoologisches Institut der Universität und der Zoologischen Station in Triest. Arbeiten.

BELGIUM.

BRUSSELS—

T.P. *Académie Royale des Sciences, des Lettres et des Beaux Arts de Belgique.* Mémoires Couronnés.—Mémoires Couronnés et Mémoires des Savants Étrangers.—Bulletin, 3^{e} Série : Classe des Sciences ; Classe des Lettres et des Sciences Morales et Politiques.—Annuaire.—Biographie Nationale, XVIII. 2, XIX. 1. 1905–7.

T.P. *Musée Royal d'Histoire Naturelle.* Mémoires.

T.P. *Musée du Congo.* Annales.—*Botanique.* Série V. : Études de systématique et de géographie botaniques sur la Flore du Bas- et du Moyen-Congo, par Ém. de Wildeman. *Zoologie.* Série III. : Faune entomologique de l'Afrique tropicale.—*Ethnographie et Anthropologie.* Série III. : Notes Analytiques sur les collections ethnographiques du Musée du Congo. Série V. : Linguistique.

T.P. *L'Observatoire Royal de Belgique, Uccle.* Annuaire.—Annales Astronomiques. Annales Météorologiques. Annales. — Physique du Globe. — Bulletin Climatologique. Observations Météorologiques.

T.P. *La Société Scientifique.* Annales.

P. *Société Belge d'Astronomie.*

T.P. GHENT.—*University Library.*

T.P. LOUVAIN.—*University Library.*

BOSNIA-HERZEGOVINA. (*See* AUSTRIA.)

BULGARIA.

P. SOFIA. — *Station Centrale Météorologique de Bulgarie.* Bulletin Mensuel. Bulletins Annuaires.

CANADA.

P. HALIFAX (NOVA SCOTIA).—*Nova Scotian Institute of Science.* Proceedings and Transactions.

T.P. KINGSTON.—*Queen's University.*

MONTREAL—

P. *Natural History Society.* Proceedings.

P. *Canadian Society of Civil Engineers.* Transactions.—Annual Reports.

OTTAWA—

T.P. *The Royal Society of Canada.* Proceedings and Transactions.

T.P. *Geological Survey of Canada.* Annual Reports.—Palæozoic Fossils. Vols. 3, 4. 8vo. 1906.—Maps.

P. *Literary and Scientific Society.* Transactions.

T.P. QUEBEC.—*Literary and Historical Society.* Transactions.

TORONTO—

T.P. *University.* University Studies. (*History. Psychological Series. Geological Series. Economic Series. Physiological Series. Biological Series. Physical Science Series.*) Papers from the Chemical Laboratory.

T.P. *Canadian Institute.* Transactions.

P. *Royal Astronomical Society of Canada.* Transactions.—Journal.

CAPE COLONY.

CAPE TOWN—

T.P. *Royal Astronomical Observatory.* Reports.—Annals. Vol. X. Pt. 2. Vol. XII. Pts. 2–4. 1906–07. 4to.—Meridian Observations, 1900–04. 4to.

P. *South African Philosophical Society.* Transactions.

P. *Geological Commission.* Annual Reports. 8vo.

South African Museum. Annals. Vol. IV. Pt. 7. 8vo. (*Presented.*)

CHINA.

HONG KONG—

P. *The Observatory.* Observations and Researches, 1904–06. 4to.

P. SHANGHAI.—*North China Branch of the Royal Asiatic Society.*

DENMARK.

COPENHAGEN—

T.P. *Académie Royale de Copenhague.* Mémoires: Classe des Sciences. 7e Série.—Oversigt. E. Museo Lundii. En Samling af Alfhandlinger. Bd. III. 1. 1906. 4to.

P. *Naturhistorisk Forening.* Videnskabelige Meddelelser.

P. *Danish Biological Station.* Reports.

Conseil Permanent International pour l'Exploration de la Mer. Publications de circonstance. Nos. 28–41. 1905–07. 8vo.—Rapports et Procès-Verbaux de Réunions. Vols. IV.–VI. 1904–06.—Bulletin des Résultats acquis pendants les courses périodiques. Années 1905–06, 1906–07. 4to. (*Presented.*)

Kommissionen for Havundersogelser. Meddelelser: Série Fiskeri, Bind I., Nos. 4–8. Série Plankton, Bind I., Nos. 3, 4. Série Hydrografi, Nos. 7–9. 1905–07. 4to. (*Presented.*)

EGYPT.

T.P. CAIRO—*School of Medicine.* Records, Vols. I.–III. 1901–05. 4to.

ENGLAND AND WALES.

P. BIRMINGHAM.—*Philosophical Society.* Proceedings.

CAMBRIDGE—

T.P. *Philosophical Society.* Transactions and Proceedings.

T.P. *University Library.*

P. CARDIFF.—*University College of South Wales.*

P. ESSEX.—*Essex Field Club.*—*The Essex Naturalist.*

T.P. GREENWICH.—*Royal Observatory.* Astronomical, Magnetical, and Meteorological Observations.—Photo-heliographic Results.—Telegraphic Determinations of Longitude made in 1888–1902. By Sir W. K. M. Christie, 1906. 4to. New Reduction of Groombridge's Circumpolar Catalogue for the Epoch 1810. By Frank W. Dyson and W. G. Thackeray. 4to. 1905.

T.P. Leeds.—*Philosophical and Literary Society.* Reports.

Liverpool—

T.P. *University College Library.*

P. *Literary and Philosophical Society.*

P. *Biological Society.* Proceedings and Transactions.

P. *Geological Society.* Proceedings.

London—

P. *Admiralty.* Nautical Almanac and Astronomical Ephemeris.

T.P. *Anthropological Institute.* Journal.

T.P. *Athenæum Club.*

T.P. *British Association for the Advancement of Science.* Reports.

T.P. *British Museum.* Reproductions from Illuminated Manuscripts. Series 1 and 2. 1907. 8vo.

T.P. *British Museum. Natural History Department.*—Catalogue of Lepidoptera Phalænæ. Vol. VI. Text and Plates.—Catalogue of the Madreporarian Corals, Vols. V., VI. By H. M. Bernard. 1905–06. 8vo.—The History of the Collections contained in the Natural History Depts. Vol. II.—A Synonymic Catalogue of Orthoptera. By W. F. Kirby. Vol. II. 1906. 8vo.—A Synonymic Catalogue of Homoptera. Part I. Cicadidæ. By W. L. Distant. 1906. 8vo.—A Descriptive Catalogue of the Tertiary Vertebrata of the Fayûm, Egypt. By C. W. Andrews. 1906. 4to.—British Blood-Sucking Flies. With Notes by E. E. Austin. 1906. 8vo.—Lists of Casts of Fossils. 5th ed. 1906. 8vo.

T.P. *Chemical Society.* Journal and Abstract of Proceedings.

Clinical Society. Transactions. (*Presented.*)

T.P. *Geological Society.* Quarterly Journal.—Geological Literature.—Abstract of Proceedings.

T.P. *Geological Survey of the United Kingdom.* Summary of Progress.

P. *Geologists' Association.* Proceedings.

T.P. *Hydrographic Office.*

T.P. *Imperial Institute.*

T.P. *Institution of Civil Engineers.* Minutes of Proceedings.—Subject Index.—Charters.—Members.

T.P. *Institution of Electrical Engineers.* Journal.

P. *Institution of Mechanical Engineers.* Proceedings.

T.P. *International Catalogue of Scientific Literature.*

T.P. *Linnean Society.* Journal: Zoology; Botany.—Transactions (Second Series): Botany; Zoology.—Proceedings.

P. *Mathematical Society.* Proceedings. Second Series.

P. *Meteorological Office.* Reports of the Meteorological Council to the Royal Society.—Hourly Readings.—Weekly Weather Reports.—Monthly and Quarterly Summaries.—Meteorological Observations at Stations of Second Order.—The Life History of Surface Air Currents: a Study of the Surface Trajectories of Moving Air. By W. N. Shaw and R. G. K. Lempfert. 1906. 4to.

Mineralogical Society of Great Britain and Ireland. Mineralogical Magazine and Journal. (*Presented.*)

T.P. *Pathological Society.* Transactions.

LONDON—*continued*—

P. *Pharmaceutical Society.* Journal.

T.P. *Royal Asiatic Society.*

T.P. *Royal Astronomical Society.* Monthly Notices.—Memoirs.

T.P. *Royal College of Surgeons.*

T.P. *Royal Geographical Society.* Geographical Journal.

T.P. *Royal Horticultural Society.* Journal.

T.P. *Royal Institution.* Proceedings.

T.P. *Royal Medical and Chirurgical Society.* Transactions.

P. *Royal Meteorological Society.* The Meteorological Record: Monthly Returns of Observations made at the Stations of the Meteorological Society.—Quarterly Journal.

T.P. *Royal Microscopical Society.* Journal. New Series.

P. *Royal Photographic Society.* Photographic Journal.

T.P. *Royal Society.* Philosophical Transactions.—Proceedings.—Year-Book.—Obituary Notices of Fellows of the Royal Society.—Report . . . . on the Pearl Oyster Fisheries of the Gulf of Manaar. By W. A. Herdman. Pts. 4 and 5. 4to. 1905–07.—Mediterranean Fever Reports. Pts. 2, 5–17. 8vo. 1907.

T.P. *Royal Society of Literature.* Transactions.—Reports. Christabel. By Samuel Taylor Coleridge. Illustrated by a facsimile of the manuscript and by textual and other notes. By Ernest K. Coleridge, 1907. 4to.

T.P. *Royal Statistical Society.* Journal.

T.P. *Society of Antiquaries.* Proceedings.—Archæologia; or Miscellaneous Tracts relating to Antiquity.

T.P. *Society of Arts.* Journal.

Society of Chemical Industry. Journal. (*Presented.*)

Society for Psychical Research. Journal. Vols. VI.–XVII. 1893–94–1903–04. 8vo. London. (*Presented by W. C. Crawford, Esq.*)

Solar Physics Committee. Annual Report.—Spectroscopic Comparison of Metals. London, 1907. 4to.

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——— Journal of Philology. (*Presented.*)
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——— Ergänzungshefte.
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——— d'Hygiène Publique et de Médecine Légale.
——— des Sciences Naturelles. Zoologie et Paléontologie.
——— des Sciences Naturelles. Botanique.
Annals and Magazine of Natural History (Zoology, Botany, and Geology).
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——— of Mathematics.
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Anthropologie (L').
Archiv für Naturgeschichte.

Archiv for Mathematik og Naturvidenskab. (*Presented.*)
Archives de Biologie.
——— de Zoologie Expérimentale et Générale.
——— des Sciences Biologiques. (*Presented.*)
——— des Sciences Physiques et Naturelles.
——— Italiennes de Biologie.
Arkiv for Matematik, Astronomi och Fysik. (*Presented.*)
——— for Kemi, Mineralogi och Geologi. (*Presented.*)
——— for Botanik. (*Presented.*)
——— for Zoologi. (*Presented.*)
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Bericht über die Wissenschaftlichen Leistungen in der Naturgeschichte der niederen Thiere. Begründet von R. Leuckart.
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——— des Sciences Mathématiques.
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Centralblatt für Bakteriologie und Parasitenkunde.
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Dingler's Polytechnisches Journal.
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Electrical Engineer. (*Presented.*)
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English Mechanic and World of Science.
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Gazette Médicale d'Orient. (*Presented.*)
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Indian Antiquary.
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——— de Mathématiques Pures et Appliquées.
——— de Pharmacie et de Chimie.
——— des Savants.
——— für die Reine und Angewandte Mathematik (Crelle).
——— für Praktische Chemie.
——— of Anatomy and Physiology.
——— of Botany.
——— of Pathology and Bacteriology.
——— of Physical Chemistry. (*Presented.*)
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Nyt Magazin for Naturvidenskaberne. (*Presented.*)
Observatory.
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Petermann's Mittheilungen aus Justus Perthes' Geographischer Anstalt.
Philosophical Magazine. (London, Edinburgh, and Dublin.)
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——— Politique et Littéraire. (Revue Bleue.)
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Saturday Review.
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Times.
Zeitschrift für die Naturwissenschaften.
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Ray Society's Publications.

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Fauna und Flora des Golfes von Neapel und der angrenzenden Meeres-Abschnitte. Herausgegeben von der Zoologischen Station zu Neapel.

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ABSTRACT

OF

THE ACCOUNTS OF JAMES CURRIE, ESQ.,

As Treasurer of the Royal Society of Edinburgh.

SESSION 1906-1907.

I. ACCOUNT OF THE GENERAL FUND.

CHARGE.

1. Arrears of Contributions at 1st October 1906 . . .				£198	9	0
Less Contribution included in Commutation Fees . .				3	3	0
				£195	6	0
2. Contributions for present Session:—						
1. 162 Fellows at £2, 2s. each . .	£340	4	0			
137 Fellows at £3, 3s. each . .	431	11	0			
	£771	15	0			
Less 2 Contributions included in Commutation Fees . .	6	6	0			
	£765	9	0			
2. Fees of Admission and Contributions of twenty new Resident Fellows at £5, 5s. each	105	0	0			
3. Fees of Admission of ten new Non-Resident Fellows at £26, 5s. each .	262	10	0			
4. Commutation Fees in lieu of Future Contributions of 2 Fellows . .	67	14	6	1200	13	6
3. Interest received—						
Interest, less Tax	£372	14	2			
Annuity from Edinburgh and District Water Trust, less Tax . . .	49	17	6	422	11	8
4. Society's Transactions and Proceedings sold				73	16	10
5. Annual Grant from Government				600	0	0
Amount of the Charge . .				**£2492**	**8**	**0**

DISCHARGE.

1. Rent of Society's Apartments for Year, less Tax				£285	0	0
2. Insurance, Gas, Electric Light, Coal, Water, etc.:—						
Insurance	£10	13	0			
Gas	1	4	9			
Electric Light	2	13	6			
Coal	4	17	0			
Water	2	2	0			
Income Tax	15	0	0			
				36	10	3
3. Salaries:—						
General Secretary	£100	0	0			
Librarian	150	0	0			
Assistant Librarian	54	3	4			
Office Keeper	35	0	0			
Doorkeeper	12	0	0			
Treasurer's Clerk	25	0	0			
				376	3	4
4. Expenses of Transactions:—						
Neill & Co., Ltd., Printers	£333	15	7			
Do. (Ben Nevis)	58	8	0			
M'Farlane & Erskine, Lithographers	98	8	6			
W. & A. K. Johnston, Ltd., do.	3	5	0			
Hislop & Day, Engravers	11	17	3			
A. S. Huth, do.	35	0	0			
Orrock & Son, Bookbinders	74	0	2			
				614	14	6
5. Expenses of Proceedings:—						
Neill & Co., Ltd., Printers	£310	13	3			
M'Farlane & Erskine, Lithographers	13	19	2			
W. & A. K. Johnston, Ltd., do.	2	0	0			
Hislop & Day, Engravers	1	7	10			
				328	0	3
6. Books, Periodicals, Newspapers, etc.:—						
Otto Schulze & Co., Booksellers	£132	5	10			
James Thin, do.	48	3	8			
R. Grant & Son, do.	10	14	0			
Kegan Paul & Co., do.	2	5	4			
Wm. Green & Sons, do.	1	11	0			
International Catalogue of Scientific Literature	17	0	0			
Robertson & Scott, News Agents	3	7	0			
Orrock & Son, Bookbinders	31	5	3			
Post Office Directory	0	7	6			
Egypt Exploration Funds, Subscription	3	3	0			
Ray Society, do.	1	1	0			
Palæontographical Society, do.	1	1	0			
Journal de Conchyliologie	0	15	0			
				252	19	7
Carry forward				£1893	7	11

DISCHARGE—*continued.*

Brought forward . .		£1893 7 11
7. OTHER PAYMENTS:—		
Neill & Company, Ltd., Printers . .	£84 15 6	
R. Blair & Son, Confectioners . .	28 11 0	
Lantern Exhibitions, etc., at Lectures .	14 15 10	
Expenses of Deputation to London to Government	39 6 5	
Lindsay, Jamieson & Haldane, Auditors	6 6 0	
National Telephone Co.	7 7 4	
Petty Expenses, Postages, Carriage, etc. .	85 5 9	
		266 7 10
8. IRRECOVERABLE ARREARS of Contributions written off .		8 8 0
9. ARREARS of CONTRIBUTIONS outstanding at 1st October 1907:—		
Present Session	£110 5 0	
Previous Sessions	111 6 0	
		221 11 0
Amount of the Discharge .		**£2389 14 9**

Amount of the Charge	**£2492 8 0**
Amount of the Discharge	**2389 14 9**
Excess of the Charge	**£102 13 3**
FLOATING BALANCE DUE BY THE SOCIETY at 1st October 1906	£145 14 11
Deduct Excess of the Charge as above	102 13 3
Floating Balance due by the Society at 1st October 1907	**£43 1 8**

Being—

Balance due to the Union Bank on Current Account	£45 6 8	
Less Balance due by Treasurer, ...	2 5 0	
		£43 1 8

II. ACCOUNT OF THE KEITH FUND

To 1*st October* 1907.

CHARGE.

1. BALANCE due by the Union Bank, and on hand at 1st October 1906		£88 4 6
2. INTEREST RECEIVED :—		
On £896, 19s. 1d. North British Railway Company 3 per cent. Debenture Stock for year to Whitsunday 1907, less Tax	£25 11 4	
On £211, 4s. North British Railway Company 3 per cent. Lien Stock for year to Lammas 1907, less Tax .	6 0 4	
		31 11 8
		£119 16 2

DISCHARGE.

1. Dr Thomas H. Bryce, money portion of Prize for 1903–1905	£47 5 8	
2. Alex. Kirkwood & Son, Engravers, for Gold Medal	16 0 0	
	£63 5 8	
3. BALANCE due by the Union Bank at 1st October 1907	56 10 6	
		£119 16 2

III. ACCOUNT OF THE NEILL FUND

To 1*st October* 1907.

CHARGE.

1. Balance due by the Union Bank and on hand at 1st October 1906	£45	19	8
2. Interest Received:—			
On £355 London, Chatham and Dover Railway Company $4\frac{1}{2}$ per cent. Arbitration Debenture Stock for year to 30th June 1907, less Tax . . .	15	3	6
	£61	3	2

DISCHARGE.

Nil.

Balance due by the Union Bank at 1st October 1907	£38	7	11			
Dividend Warrants, uncashed	22	15	3			
				£61	3	2

IV. ACCOUNT OF THE MAKDOUGALL-BRISBANE FUND

To 1*st October* 1907.

CHARGE.

1. Balance due at 1st October 1906:—		
By the Union Bank of Scotland on Deposit Receipt	£135 0 0	
By the Union Bank of Scotland on Current Account and on hand. .	38 15 8	
		£173 15 8
2. Interest Received on £365 Caledonian Railway Company 4 per cent. Consolidated Preference Stock No. 2 for year to 30th June 1906, less Tax .	£13 17 4	
On Deposit Receipt with Union Bank .	7 10 3	
		21 7 7
		£195 3 3

DISCHARGE.

1. Dr Jacob E. Halm, money portion of Prize for 1904–1906.		£14 0 0	
2. Alex. Kirkwood & Son, Engravers, for Gold Medal		16 0 0	
		£30 0 0	
3. Balance due by Union Bank of Scotland at 1st October 1907:—			
On Deposit Receipt	£135 0 0		
On Current Account	30 3 3		
		165 3 3	
			£195 3 3

V. ACCOUNT OF THE MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND

To 1*st October* 1907.

CHARGE.

Sum on Deposit Receipt with the Union Bank of Scotland at 1st October 1906	£197	2	5
Interest on above	10	19	4
	£208	1	9

DISCHARGE.

Nil.

Above Sum on Deposit Receipt with the Union Bank of Scotland at 1st October 1907	£208	1	9

VI. ACCOUNT OF THE GUNNING-VICTORIA JUBILEE PRIZE FUND

To 1*st October* 1907.

(Instituted by Dr R. H. Gunning of Edinburgh and Rio de Janeiro.)

CHARGE.

	£	s.	d.
1. Balance due by Union Bank and on hand at 1st October 1906	£45	17	10
2. Interest received on £1000 North British Railway Company 3 per cent. Consolidated Lien Stock for year to Lammas 1907, less Tax	28	10	0
	£74	7	10

DISCHARGE.

Nil.

	£	s.	d.
Balance due by the Union Bank of Scotland on Current Account at 1st October 1907	£74	7	10

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 1*st October* 1907.

1. GENERAL FUND—

1. £2090, 9s. 4d. three per cent. Lien Stock of the North British Railway Company at 83 per cent., the selling price at 1st October 1907	£1735	1	9
2. £8519, 14s. 3d. three per cent. Debenture Stock of do. at $84\frac{7}{8}$ per cent., do.	7231	2	1
3. £52, 10s. Annuity of the Edinburgh and District Water Trust, equivalent to £875 at 170 per cent., do.	1487	10	0
4. £1811 four per cent. Debenture Stock of the Caledonian Railway Company at $112\frac{1}{2}$ per cent., do.	2037	7	6
5. £35 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at 115 per cent., do.	40	5	0
6. Arrears of Contributions as per preceding Abstract of Accounts	221	11	0
	£12,752	17	4
Deduct Floating Balance due by the Society as per preceding Abstract of Accounts	43	1	8
AMOUNT . .	£12,709	15	8

Exclusive of Library, Museum, Pictures, and Furniture of the Society's apartments at the Royal Institution.

2. KEITH FUND—

1. £896, 19s. 1d. three per cent. Debenture Stock of the North British Railway Company at $84\frac{7}{8}$ per cent., the selling price at 1st October 1907	£761	6	7
2. £211, 4s. three per cent. Lien Stock of do. at 83 per cent., do.	175	5	8
3. Balance due by the Union Bank of Scotland . . .	56	10	6
AMOUNT	£993	2	9

STATE OF FUNDS—*continued.*

3. NEILL FUND—

1. £355 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at 115 per cent., the selling price at 1st October 1907	£408	5	0
2. Balance due by the Union Bank of Scotland (£38, 7s. 11d.), and uncashed dividend warrants in hand (£22, 15s. 3d.)	61	3	2
AMOUNT . . .	£469	8	2

4. MAKDOUGALL-BRISBANE FUND—

1. £365 four per cent. Consolidated Preference Stock No. 2 of the Caledonian Railway Company at 107½ per cent., the selling price at 1st October 1907 .	£392	7	6
2. Sum on Deposit Receipt with the Union Bank of Scotland	135	0	0
3. Balance due by do. on Current Account . . .	30	3	3
AMOUNT . . .	£557	10	9

5. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—

Sum on Deposit Receipt with the Union Bank of Scotland at 1st October 1907	£208	1	9

6. GUNNING-VICTORIA JUBILEE PRIZE FUND—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £1000 three per cent. Consolidated Lien Stock of the North British Railway Company at 83 per cent., the selling price at 1st October 1907	£830	0	0
2. Balance due by the Union Bank of Scotland on Current Account	74	7	10
AMOUNT . . .	£904	7	10

EDINBURGH, 18*th October* 1907.—We have examined the six preceding Accounts of the Treasurer of the Royal Society of Edinburgh for Session 1906–1907, and have found them to be correct. The securities of the various Investments at 1st October 1907, as noted in the above Statement of Funds, have been exhibited to us.

LINDSAY, JAMIESON & HALDANE,
Auditors.

VIDIMUS of ESTIMATED INCOME of THE GENERAL FUND FOR SESSION 1907–1908.

1. INTEREST :—						
On £8519, 14s. 3d. Railway Debenture Stock at 3 per cent.				£255	11	10
On £2090, 9s. 4d. Railway Lien Stock at 3 per cent.				62	14	4
On £1811 Railway Debenture Stock at 4 per cent.				72	8	8
On £35 Railway Debenture Stock at $4\frac{1}{2}$ per cent.				1	11	6
				£392	6	4
2. ANNUITY from the Edinburgh and District Water Trust				52	10	0
				£444	16	4
Deduct Income Tax at 1s. per £				22	4	10
				£422	11	6
3. ANNUAL CONTRIBUTIONS :—						
Of 164 Fellows at £2, 2s. each	£344	8	0			
Of 139 Fellows at £3, 3s. each	437	17	0			
				782	5	0
4. ANNUAL GRANT from Government				600	0	0
5. SALES of Society's Transactions				30	0	0
TOTAL ESTIMATED INCOME,				£1834	16	6

Exclusive of Fees of Admission and Contributions of New Fellows who may be admitted during the Year.

INDEX.

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4. Additions to a Paper after it has been finally handed in for publication, if accepted by the Council, will be treated and dated as separate communications, and may, or may not, be printed immediately after the original paper.

5. Brief Abstracts of Transactions Papers will be published in the Proceedings, provided they are sent along with the original paper.

6. Separate Issue of Reprints; Author's Free and Additional Copies.—As soon as the final revise of a Transactions paper has been returned, or as soon as the sheet in which the last part of a Proceedings paper appears is ready for press, a certain number of separate copies or reprints, in covers bearing the title of the paper and the name of the author, are printed off and placed on sale. The date of such separate publication will be printed on each paper.

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7. Index Slips.—In order to facilitate the compilation of Subject Indices, and to secure that due attention to the important points in a paper shall be given in General Catalogues of Scientific Literature and in Abstracts by Periodicals, every author is requested to return to the Secretary, along with his final proof, a brief index (on the model given below), of the points in it which he considers new or important. These indices will be edited by the Secretary, and incorporated in Separate Index Slips, to be issued with each part of the Proceedings and Transactions.

MODEL INDEX.

Schäfer, E. A.—On the Existence within the Liver Cells of Channels which can be directly injected from the Blood-vessels. Proc. Roy. Soc. Edin., vol. . 1902, pp. .

Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Liver,—Injection within Cells of.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .